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EXPERIMENTAL STUDY OF THE *T*-CRITERION IN DUCTILE FRACTURES

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Abstract—In the present paper, the newly introduced *T*-criterion was applied in the case of the fracture of a ductile material. Polycarbonate of Bisphenol A (PCBA) specimens containing slant internal cracks were tested and the experimentally obtained angles of crack propagation, as well as the respective fracture loads were compared to corresponding predictions of the *T*-criterion. It was found out that the predicted and measured quantities are in good agreement, especially in cases where existing other fracture criteria fail. The prediction of the *T*-criterion, that for small angles of crack inclination there must be angles of propagation absolutely larger than 90°, was clearly verified. Also, the fracture load measured, for each of the same small angles of inclination, was more than three times lower, as compared to the values given by other criteria in use, and in excellent agreement with *T*-criterion predictions.

INTRODUCTION

THE SLANT-ANGLED crack problem has been extensively studied in the past few years by introducing criteria which could determine the direction of propagation, as well as the respective critical failure-load. The basic problem arises from the situation sketched in Fig. 1. A crack of length 2a is inclined by an angle β to the direction of loading axis and one seeks both the direction ϑ_0 along which the crack begins to grow and the magnitude of the external load P_f at which crack-growth starts.

Historically, the first fracture criterion is that of maximum tangential or hoop stress, since Griffith[1] had already stated that "the crack will open at a plane normal to the direction of maximum stress". In this direction the first study was done by Erdogan and Sih[2]. According to this criterion the crack will propagate along a plane normal to the maximum tangential stress σ_ϑ. Propagation will start when the maximum σ_ϑ reaches a critical value, which is considered as a material constant.

Later, in a series of papers[3–5], Sih proposed a new criterion based on the local strain-energy density (*S*-criterion). According to this criterion the minimum value of strain-energy density around the crack-tip defines the direction of crack propagation and its value, compared to a material constant, is an indication of whether the crack propagates or not.

Both criteria involve the application of linear fracture mechanics, the stress field being calculated over a constant distance r around the crack-tip. This distance r has both a mathematical and a physical meaning. From a mathematical point of view, the exact value of r does not affect the predicted angle ϑ_0 of propagation, only in the case when singular expressions are used for the stress components. When more terms, or the exact expressions for stresses, are used, the value of ϑ_0 varies depending on r[6–7].

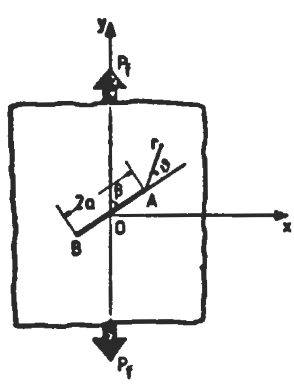


Fig. 1. A slant-angled crack under uniaxial tensile load.

On the other hand, the radius r has, also, a physical meaning. Really there must be a certain area around the crack-tip called the *core region*, inside which the use of linear-elasticity expressions for stresses is forbidden, since the material is no-more linear elastic inside this area.

On this subject, recently, Theocaris and Andrianopoulos [8] introduced a new fracture criterion. the T-criterion, suitable for ductile fracture, where as the core region the Mises elastic–plastic boundary is used. A limiting case of the T-criterion is the S-criterion, suitable only for brittle fracture. According to the T-criterion, angle ϑ_0, for small angles β, is absolutely much larger, when compared to the predictions of all the other criteria and the corresponding fracture loads much smaller.

The purpose of this paper is to study experimentally the behavior of ductile cracked materials, fractured under static load, in order to decide which of the different predictions of the abovementioned criteria are valid for different angles β of inclination of the initial crack.

THEORETICAL CONSIDERATIONS

Referring to a slant-cracked specimen indicated in Fig. 1, the stress components around the crack-tips are given by:

$$\sigma_x = \frac{K_I}{(2\pi r)^{1/2}}\left(\cos\vartheta/2 - \frac{1}{2}\sin\vartheta\sin 3\vartheta/2 - \mu(2\sin\vartheta/2 + \frac{1}{2}\sin\vartheta\cos 3\vartheta/2)\right) = \frac{K_I}{(2\pi r)^{1/2}} f_x(\vartheta)$$

$$\sigma_y = \frac{K_I}{(2\pi r)^{1/2}}\left(\cos\vartheta/2 + \frac{1}{2}\sin\vartheta\sin 3\vartheta/2 + \frac{1}{2}\mu\sin\vartheta\cos 3\vartheta/2\right) = \frac{K_I}{(2\pi r)^{1/2}} f_y(\vartheta) \qquad (1)$$

$$\tau_{xy} = \frac{K_I}{(2\pi r)^{1/2}}\left(\frac{1}{2}\sin\vartheta\cos 3\vartheta/2 + \mu(\cos\vartheta/2 - \frac{1}{2}\sin\vartheta\sin 3\vartheta/2\right) = \frac{K_I}{(2\pi r)^{1/2}} f_{xy}(\vartheta)$$

where (r, ϑ) are the polar coordinates around the crack tip, $\mu = K_{II}/K_I$ and K_I, K_{II} the mode $-I$ and $-II$ stress intensity factors, given by

$$K_I = \sigma_0(\pi a)^{1/2}\sin^2\beta$$

$$K_{II} = \sigma_0(\pi a)^{1/2}\sin\beta\cos\beta. \qquad (2)$$

The elastic energy per unit volume is expressed by

$$T = \frac{\partial W}{\partial V} = \frac{1}{8G}(k(\sigma_x + \sigma_y)^2 + (\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2) \qquad (3)$$

where $k = (3-\nu)/(1+\nu)$ for generalized plane-stress and $k = (3-4\nu)$ for plane strain conditions, ν Poisson's ratio and G the shear modulus of the material.

The Mises yield condition is expressed as

$$\frac{1}{6G}(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2) = T_{D,0} = \text{CONST} \qquad (4)$$

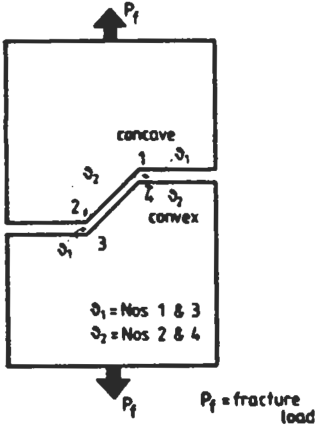
where $T_{D,0}$ is the distortional part of the total strain-energy density T. According to the Mises condition $T_{D,0}$ is considered as a material constant. Combining eqns (1) and (4), after some algebra, we obtain

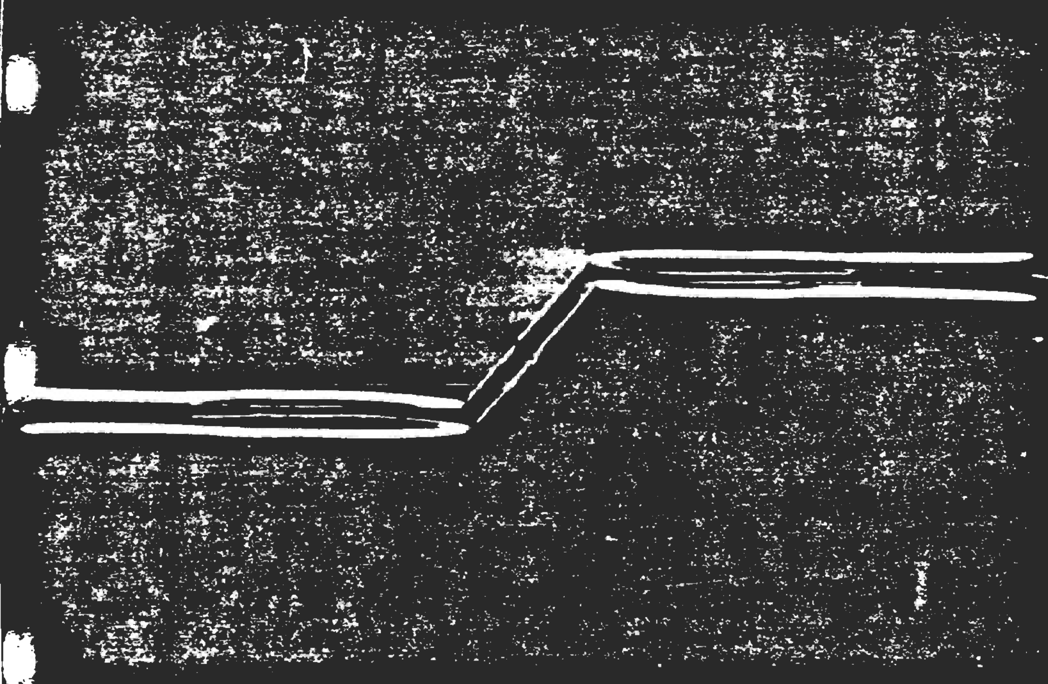
$$r = \frac{(1+\nu)K_I^2}{6\pi E T_{D,0}}(f_1^2(\vartheta) + f_2^2(\vartheta) - f_1(\vartheta)f_2(\vartheta)) = r(\vartheta) \qquad (5)$$

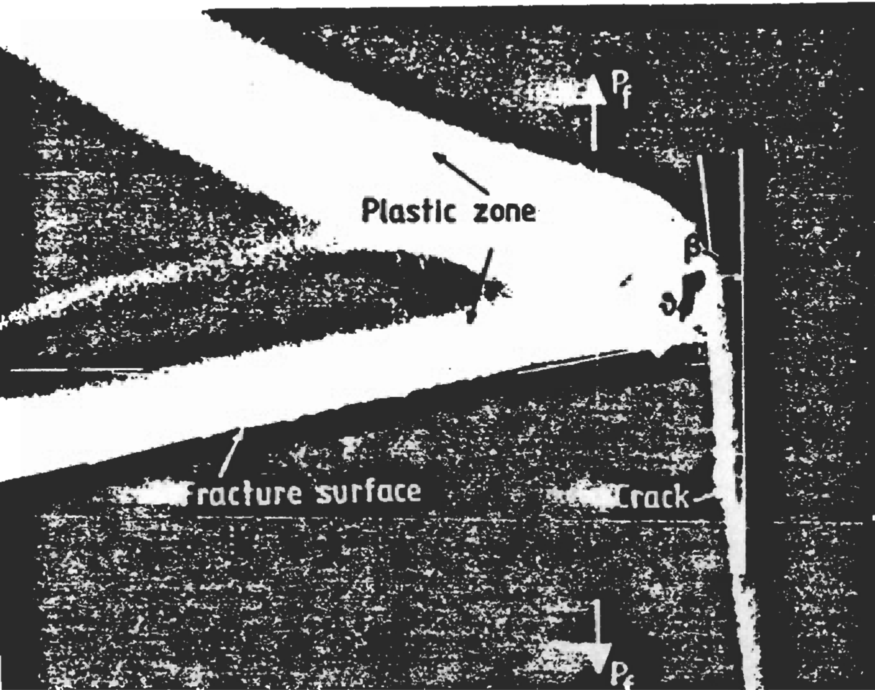
where

$$f_{1,2}(\vartheta) = \frac{1}{2}(f_x(\vartheta) + f_y(\vartheta)) \pm ((f_x(\vartheta) - f_y(\theta))^2 + 4f_{xy}^2(\vartheta))^{1/2}. \qquad (6)$$

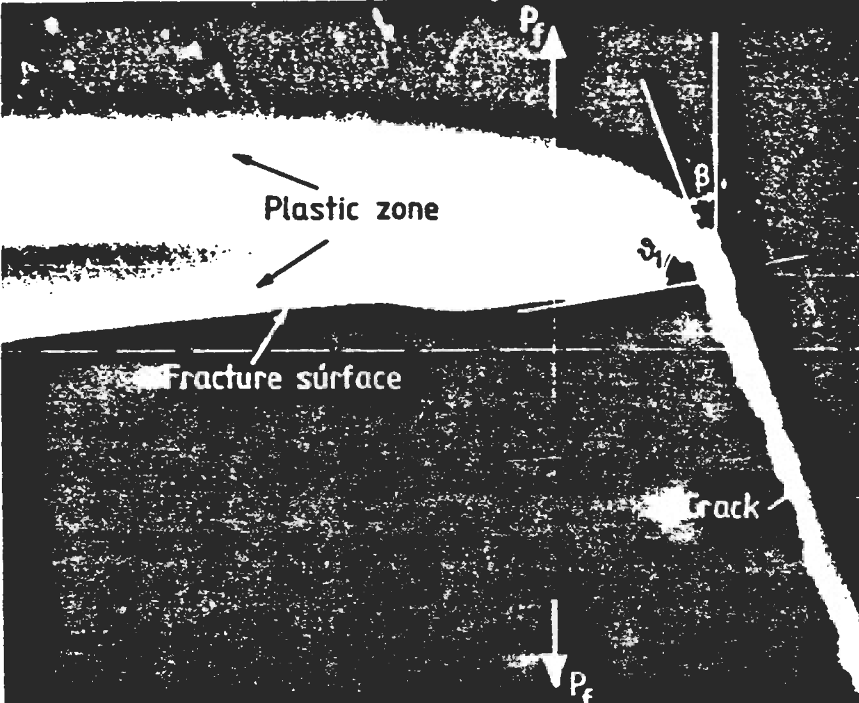
According to the T-criterion the strain-energy density T is evaluated along the Mises elastic–plastic

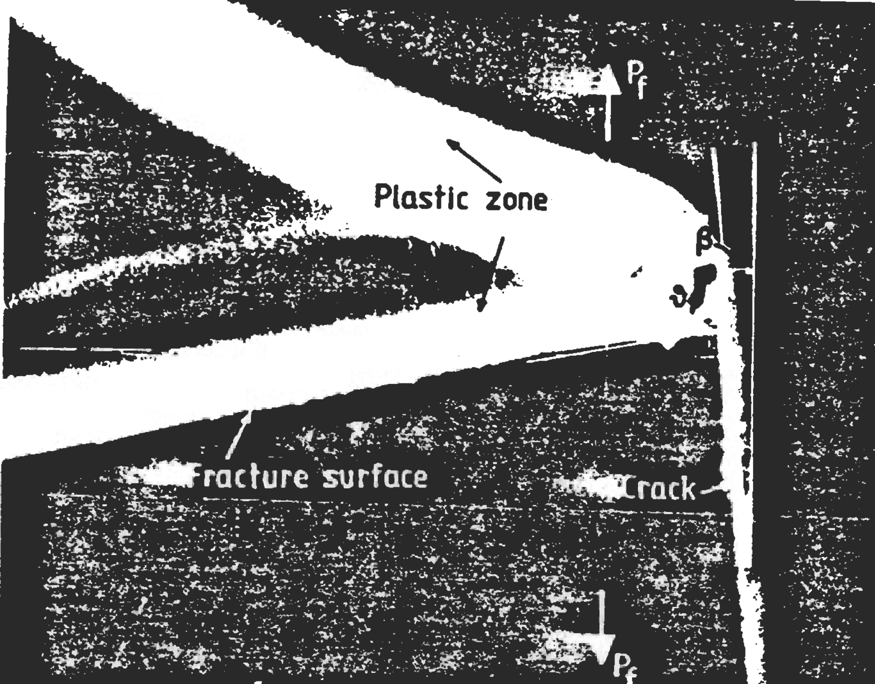

P_f
concave
1
ϑ_1
ϑ_2
2
4
ϑ_2
convex
ϑ_1
3
ϑ_1 = Nos 1 & 3
ϑ_2 = Nos 2 & 4
P_f
P_f = fracture load





Pf
Plastic zone
β
δ
Fracture surface
Crack
Pf


P_f
β
Plastic zone
ϑ_1
Fracture súrface
Crack
P_f



P_f
Plastic zone
β
Fracture surface
Crack
P_f

boundary given by eqn (5) and the *maximum* value of T defines the expected angle ϑ_0 of initiation of propagation of the crack. However, searching for maximum values of T along the elastic–plastic boundary defined by the Mises yield condition is equivalent to a searching for maximum values of T_V, since T_D is constant along the curve of evaluation. T_V stands for the dilatational component of the total strain-energy density T and T_D for its distortional part. The result that cracks propagate along the direction of maximum dilatational strain-energy density reflects the experimental fact that under static load the crack propagates by coalescing with voids existing or created during deformation of the specimen around the crack tip, a theory which is considered as the principal mechanism for crack propagation in ductile materials. It is reasonable to assume that the volume of these voids is exclusively depending on the normal stresses, i.e. on T_V. Thus, it is interesting noting that only the T-criterion is in conformity with a macroscopic configuration of the process of plastic deformation, which undoubtedly contributes to the fracture of materials, even also the brittle ones.

EXPERIMENTAL RESULTS

The experiments were performed on polycarbonate (PCBA) specimens with dimensions $0.3 \times 0.09 \times 0.0015\ m^3$. A central slant crack of length $2a = 0.016$ m was cut by a 0.0003 m wide saw at the required angles of inclination β. In all cases the ends of the cracks were smoothed by a diamond wire of 0.0002 m diameter in order to eliminate any initial directional preference. A quantity of ten specimens for each angle β were tested. The whole region of $\beta = 10°$ (90°) 10° was covered, with the addition of angles $\beta = 5°$ and $\beta = 45°$. The angle ϑ_0 of initiation of propagation was measured after magnifying 25 times the picture of the specimen in a projector. It must be noted that ϑ_0 was measured where it was possible, since there existed cases where an initial angle of propagation could not be accurately defined.

In each specimen four angles were measured as it is indicated in Fig. 2. Due to the ductile type of fracture, there is no unique fracture surface. We defined the Nos. 1 and 3 angles as located in the concave region and the Nos. 2 and 4 as in the convex one. Due to the symmetry, the values of angles 1 and 3 do not differ substantially between them, and the same holds for the angles 2 and 4. So, in the following we speak only for the angle ϑ_1 defined as concave and ϑ_2 defined as convex.

On the measurements obtained, a statistical analysis was performed by calculating the mean value and the standard deviation. Then, we listed the measurements in ascending order and made a frequency analysis in order to resume the central tendency of them. The respective histogram and cumulative frequency diagram for ϑ_1 are shown in Fig. 3(a) and for the fracture load in Fig. 3(b), both for $\beta = 10°$. In the whole process we used the Chauvenett criterion [9], which provides a consistent basis to reject or retain data. It came out that we had to reject only one measurement.

Finally, the coefficient of variation was computed. Its value for both angles ϑ_1, ϑ_2 took values between 2.7 and 12.2% (mostly about 4%). The corresponding values for the fracture load were 0.4 and 5.3% (mostly about 1.7%). Table 1 contains in summarized form the statistical results for the angle ϑ_1 and the fracture load P_f. These results strongly support the reliability of the experiments.

In Fig. 4 the mean values of the experimentally obtained angles of propagation ϑ_1 and ϑ_2 are plotted along with the theoretical predictions of the T-, S- and σ_θ-criteria. It is out of doubt that the concave angle ϑ_1 of propagation is in good agreement to the theoretical predictions of T-criterion, especially at small angles β, where the predictions of the other two criteria differ significantly from those of T-criterion. As it was expected, for large values of β, experimental results and predictions of all the criteria do not differ substantially. Also, the prediction of T-criterion that for $\beta < 10°$ the crack diverges from its initial direction by angles absolutely greater than 90°, is clearly confirmed, since for $\beta = 5°$, ϑ_1 was equal to $-91.3°$.

Concerning the convex angle ϑ_2, its values do not agree with any of the three criteria, even for large values of β where their predictions more or less coincide. Thus, for the present, the information carried by angle ϑ_2 is under question and ϑ_1 is considered as the angle of crack propagation. However, it should be noted that an almost constant difference between respective ϑ_1 and ϑ_2-angles was measured of the order of 13°, shown in Fig. 4. Furthermore, the qualitative difference between ϑ_1 and ϑ_2 (they are defined in different geometries) does not allow averaging.

Figure 5 shows the fracture load P_f for various angles β, normalized to its value for $\beta = 90°$, along with the theoretical predictions of T-, S- and σ_θ-criteria. As it is shown in this figure, the predictions of T-criterion almost coincide with the experimental results in the whole β-range. On the contrary, the predictions of the other two criteria are confirmed experimentally only for large values of β, while for small β's their predictions strongly overestimate the fracture load.

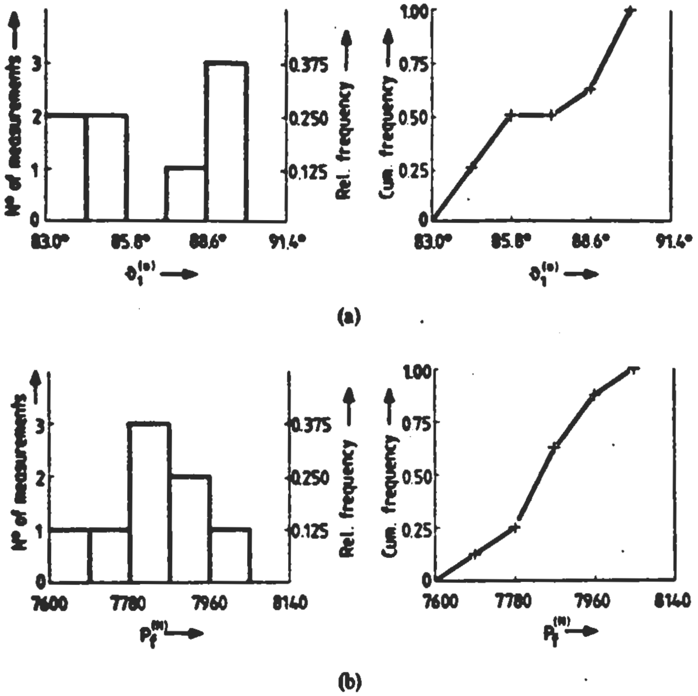


Fig. 3. Histogram of number of measurements and cumulative-frequency diagrams for: (a) angle of propagation ϑ_1 and (b) fracture load P_f for $\beta = 10°$.

Table 1. Statistical data for angle of propagation ϑ_1 and fracture load P_f (P_f in N).

Angle β	Number of measurements	Mean value	Standard deviation	Coef. of variation
5°	8	-91.3°	3.5°	3.8%
	8	8325	233	2.8%
10°	8	-86.8°	2.8°	3.2%
	8	7828	133	1.7%
20°	7	-77.1°	2.1°	2.7%
	6	7083	115	1.6%
30°	6	-69.5°	1.9°	2.7%
	5	6450	113	1.7%
40°	10	-59°	2.7°	4.6%
	6	6342	102	1.6%
45°	8	-54.5°	2.5°	4.7%
	9	6290	89	1.4%
50°	6	-53.6°	4.5°	8.4%
	6	6247	45	0.7%
60°	6	-42.5°	4.2°	9.8%
	6	5884	108	1.8%
70°	8	-31.1°	3.8°	12.2%
	8	5815	127	2.1%
80°	7	-18°	1°	5.8%
	8	5780	308	5.3%
90°	0	0°	0°	0
	2	5775	237	4.1%

The main conclusion which may be derived from the experimental curve is that the fracture load is slightly varying vs angle β. This result is in conformity with the T-criterion predictions. Furthermore, this prediction may be considered as rational, since the application of a roughly constant external load in two similar specimens with different angles β of inclination of their cracks results in the creation of a small plastic zone for small β and a large one for large β, which correspond to exactly the same T_D-values along the elastic–plastic boundary and comparable values of $T = T_D + T_V$. Thus, the next step of an unstable crack propagation starts at significantly different distances r from the crack-tip but for almost equal T- or T_V-levels.

The experimental results given in Fig. 5 imply that as a critical quantity for crack initiation may serve the dilatational strain-energy density T_V and not the minimum radius of the elastic–plastic zone, as it was stated in Ref. [8]. Thus, it is necessary to modify a corollary of the T-criterion, by stating that: "*In order to create crack initiation of propagation, the maximum value of the dilational strain-energy density* T_V

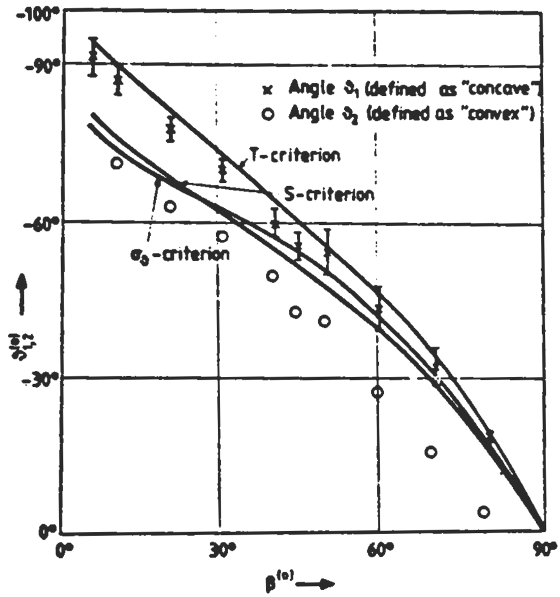


Fig. 4. Experimentally obtained angles ϑ_1 and ϑ_2 of propagation and theoretical predictions from T-, S- and σ_θ-criteria for PCBA specimens, vs angle β of crack inclination.

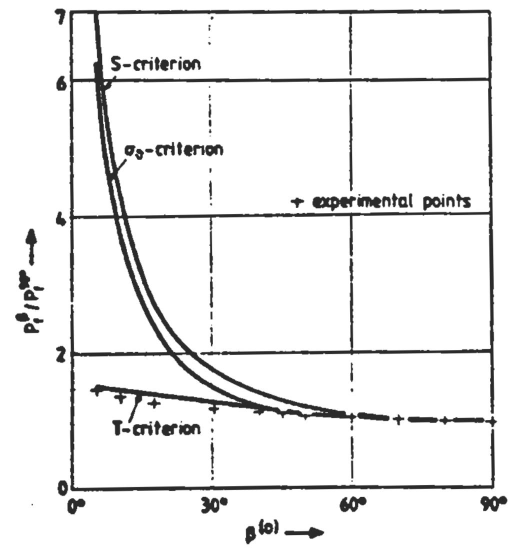


Fig. 5. Variation of the normalized fracture load P_f^β / P_f^{90} vs angle β, as it is predicted by T-, S- and σ_θ-criteria and experimentally obtained values for PCBA specimens.

must be equal to, or greater than, a critical value $T_{V,cr}$". When developing the T-criterion[8], the minimum radius of the Mises elastic–plastic boundary was replacing the critical value of the strain-energy density S, introduced in the S-criterion instead of T_V, but experiments indicate it is preferable to consider T_V as the critical quantity for initiation of propagation of the slant crack.

In a conceptual manner, the T-criterion implies that from the total strain-energy density imposed by the external load, the T_D-component prepares the material around the crack-tip for fracture by forming a plastic zone, and the T_V-component provokes crack propagation. It means that other post-initial propagation phenomena, like crack velocity, crack arrest or bifurcation, must depend on T_V-components mainly, at least in ductile materials.

As it was previously stated, the S-criterion is a limiting case of T-criterion, if one considers that the Mises yield area gradually reduces to a circular (and smaller) one, as the mode of fracture transits from ductile to brittle. But, still a strong qualitative difference exists between the two criteria. Namely, there is no way, in the case of S-criterion, to define the radius r of the core region. On the contrary, in the case of T-criterion, the transition from ductile to brittle fracture can be described by applying a kind of weight function to relation (5), in order (i) to smooth the shape of the initial elastic–plastic boundary and (ii) to reduce the size of the elastic–plastic area.

Previous existing experimental evidence coincides inherently with the developed considerations by indicating that, as the radius of the core region is increasing progressively (by introduction of some amount of plasticity) the fracture angle ϑ_0 is progressively increasing. A typical example is the curves given by Sih and Kipp[10] in Fig. 2 of their paper, where for normalized radii α of the core zone equal to $\alpha = 0$, 0.1 and 0.2, the respective $\vartheta_0 = f(\beta)$ curves lie the one above the other with increasing α's. Thus, the Mises elastic–plastic boundary, which is an upper envelope of the above radii, corresponds to the upper envelope of the curves $\vartheta_0 = f(\beta)$.

In Fig. 6 two typical photographs are given showing some remarkable situations. In Fig. 6(a) the existence of angle of propagation absolutely greater than 90° ($\vartheta_0 = -94°$) for $\beta = 5°$ is indicated. In Fig. 6(b) the case where $\beta = 20°$ is given for which $\vartheta_0 = -80°$. It is worthwhile noting that, for any angle of inclination β of the initial crack, the crack when propagated, had the tendency to become a clearly transverse crack, in conformity with existing theoretical and experimental studies [11] (see Fig. 2b). It is of interest also to indicate the plastically deformed lips of both cracks creating two furrows along these lips. The study of this phenomenon explained analytically by Bui [12] deserves a special treatment.

CONCLUSIONS

A limited attention was focused up-to-now in the experimental study of ductile fracture, although brittle fracture may be considered as a limiting case of the ductile one. Also, the assumptions of brittle fracture, in reality, violate the natural fact that plasticity always exists. These reasons drived us to introduce in a previous paper [8] a new fracture criterion, the T-criterion. It appears that elastoplastic materials show a clear preference to this criterion. Really, the T-criterion requires absolutely high angles of propagation for small angles of crack inclination, and relatively low fracture loads.

The experiments performed in PCBA in this paper showed such a type of behavior. It was also noticeable that the mechanism of ductile fracture (i.e. voids coalescing to the main crack) is physically connected to the dilatational component of the total strain-energy density, which is considered as the critical quantity for crack initiation according to the T-criterion. Finally, T-criterion may be more flexible in dynamic crack propagation, since dynamic loads or crack velocities affect the plastic zone around the crack-tip and this influence can be encountered in applying this criterion.

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