

The effects of overloads in fatigue crack growth

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Several theories have been proposed to explain the transient fatigue crack growth decelerations and accelerations which follow overloads. The mechanisms that have been proposed to explain retardation after a tensile overload, for example, include residual stress, crack deflection, crack closure, strain hardening, and plastic blunting/resharpening. These mechanisms are reviewed in the light of recent experimental results, and implications with regard to their applicability are examined. It is suggested that no single mechanism can be expected to represent observed effects over the entire range of da/dN versus ΔK ; eg, behaviour ranging from the near threshold region to the Paris region.

Key words: crack growth; overloads; discrete asperities model

The loading conditions developed in structural components in aircraft are aptly described as variable amplitude loading. A comprehensive treatment of the causes and the effects of these conditions includes an integrated consideration of flight load spectra, component load spectra, crack tip stress analysis and a crack growth law. Some of the difficulties of the problems involved have been clearly identified recently by Partl and Schijve¹ who reconstructed crack growth from fractographic observations for two flight simulation loadings. The incentive for initiating their research was their conclusion that a basic correlation between crack growth and load history must be developed. Previously, Schijve² had concluded that 'it cannot be said that a generally applicable and reliable crack growth prediction model is now available'.

The difficulties encountered in developing a crack growth prediction model can be recognized by isolating attention to the transient effects which have been observed after a single overload. The mechanisms that have been proposed to explain the transient retardation after a single overload, for example, include residual stress,³ crack deflection,⁴ crack closure,⁵ strain hardening⁶ and plastic blunting/resharpening⁷. Of these mechanisms, two have been the subjects of the most intensive examination and application. These are the effects of residual stress and crack closure. Willenborg *et al*⁸ and Wheeler⁹ proposed models which account for the effect of residual stress. Modifications of these models have been adopted for use in a number of fatigue crack growth programs. Crack closure, which involves the development of obstructions to crack closure upon unloading, is manifested in different forms. Research studies for developing an understanding of these complex mechanisms are still being conducted.

The objective of this paper is to present the results of an examination of recent experimental results which contribute to an understanding of roles of mechanisms that have been proposed. It is concluded that a number of issues need to be resolved before a generally acceptable crack growth prediction model for variable amplitude loading can be developed.

Recent experimental results

The transient retardation which follows a tensile overload has been the subject of a number of investigations. To clearly focus on the transient response produced, investigators have conducted experiments in which a constant stress intensity range, ΔK , is maintained except for a single, tensile overload.

Ward-Close and Ritchie¹⁰ conducted tensile overload experiments on compact tension specimens which were machined from IMI 550 titanium alloy plate. All tests were performed under predominantly plane strain conditions. The test results examined here were for specimens with a fine grained α/β microstructure. In their experiments the stress intensity factor range of $\Delta K = 15 \text{ MPa}\sqrt{\text{m}}$ with $R = 0.1$ was maintained constant. Single 100% and 150% tensile overloads for which $K_{OL} = 2K_{max}$ and $2.5 K_{max}$, respectively, were applied.

McEvily and Yang¹¹ conducted similar experiments on the aluminium alloy 6061-T6. For their tests $\Delta K = 8.0 \text{ MPa}\sqrt{\text{m}}$, $R = 0.05$ and overloaded $\Delta K = 16.0 \text{ MPa}\sqrt{\text{m}}$ with K_{min} unchanged.

Damri and Knott also have reported similar results¹² for a 50D low carbon structural steel. For these tests $\Delta K = 12.0 \text{ MPa}\sqrt{\text{m}}$ and the overload stress intensity factor $K_{OL} = 27.0 \text{ MPa}\sqrt{\text{m}}$.

During the application of the overload, a plastic zone, depending on the magnitude of the overload and the yield strength, is formed in front of the crack tip. In the interior the plastic zone is developed under plane strain conditions. On the surface plane stress conditions are approached. Plastic zone sizes for the titanium alloy for the 100% overload case, the aluminium alloy and the low carbon steel are listed in Table 1.

It should be noted that the plane stress conditions at the surface cannot be fully realized because of restraint from the interior material. Fleck¹³ has observed that for thin specimens the computed plane stress and measured values were in good

Table 1. Plastic zone sizes

Material	Plane stress plastic zone* (mm)	Plane strain plastic zone** (mm)
Titanium alloy ¹⁰	0.37	0.12
Aluminium alloy ¹¹	1.13	0.38
Low carbon steel ¹²	4.03 (1.95)	1.34 (0.65)

$$* \frac{1}{\pi} \left(\frac{K_{OL}}{\sigma_y} \right)^2 \text{ (plane stress)}$$

$$** \frac{1}{3\pi} \left(\frac{K_{OL}}{\sigma_y} \right)^2 \text{ (plane strain)}$$

K_{OL} is the overload stress intensity factor and σ_y is the yield strength.

agreement. For thick specimens, however, he observed that the measured values were between the computed plane stress and the plane strain values.

An additional qualification should also be noted. The plastic zone size computations were based on the assumption that the material responses were perfectly plastic. A measure of how closely this assumption was satisfied may be checked by examining the strain hardening capacities of the given materials. These results are presented in Table 2.

From Table 2 it can be concluded that the use of σ_y to compute plastic zone sizes for the titanium and aluminium alloys should provide reasonably acceptable results. Clearly, this is not the case for the low carbon steel. To examine the effect of strain hardening for the low carbon steel, the plastic zone sizes have been recalculated using an average of the yield strength and tensile strength for σ_y in equations below Table 1. The values obtained for the low carbon steel are given in parentheses in Table 1.

A measure of the duration of retardation is the distance that a crack advances during the transient behaviour following an overload. This has been called the 'delay distance'. The experimentally measured delay distances for the three alloys are listed in Table 3.

The compressive residual stress would be expected to be eliminated when the crack has passed through the overload plastic zone. A comparison of the measured delay distances of Table 3 with the plastic zone sizes of Table 1 reveals that the compressive residual stress mechanism cannot be the primary reason for the retardation for the titanium alloy. It may, however, be an operative mechanism for the aluminium and steel alloys.

Table 2. Mechanical properties

Material	Yield strength, σ_y (MPa)	Tensile strength, σ_0 (MPa)	σ_0/σ_y
Titanium alloy	984	1130	1.15
Aluminium alloy	276	311	1.13
Low carbon steel	240	450	1.88

Table 3. Delay distances

Material	Measured delay (mm)	Willenborg delay (plane stress) (mm)	Willenborg delay (plane strain) (mm)
Titanium alloy	0.88	0.18	0.06
Aluminium alloy	0.35	0.55	0.18
Low carbon steel	>2.00*	1.79 (0.87)	0.60 (0.29)

*The post-overload crack growth rate had not achieved the pre-overload rate for the data points plotted.¹²

The Willenborg model has been used to describe retardation after an overload. Data in Table 1 can be used to estimate the delay distances that result from the use of the Willenborg model. The residual stress intensity factor, K_R , of the model can have only positive values. The predicted delay distances can, therefore, be determined by setting $K_R = 0$. The experimentally measured delay distances and the Willenborg delay distances can be compared in Table 3. The discrepancy between the delay distances for the titanium alloy again indicates that another mechanism must be responsible for retardation. A comparison for the aluminium alloy indicates that the Willenborg model may be acceptable. It would appear that for the low carbon steel, the Willenborg model would provide conservative results, *ie* the predicted delay distances are shorter than the measured lengths.

Since the Willenborg model has received considerable attention, a review of its features may contribute to an understanding of its applicability. The residual stress intensity factor, K_R , as used, results in a change in the stress intensity factor ratio, R . By definition,

$$K_R = \Phi \left[K_{OL} \left(1 - \frac{a}{\rho_{OL}} \right) - K_{max} \right] \quad (1)$$

where

$$\Phi = \frac{1 - (K_{th}/K_{max})}{S - 1}$$

with a being the crack extension after the overload, ρ_{OL} the overload plastic zone size, K_{th} the threshold stress intensity factor and S the shut-off ratio.

It may be noted that the bracketed term in Equation (1) could be taken in other forms. For example, the bracket term could be written as

$$\left[K_{OL}^2 \left(1 - \frac{a}{\rho_{OL}} \right) - K_{max}^2 \right]^{0.5} \quad (2)$$

From Equation (1), $K_R = 0$ gives

$$a = \rho_{OL} \left(1 - \frac{K_{max}}{K_{OL}} \right) \quad (3)$$

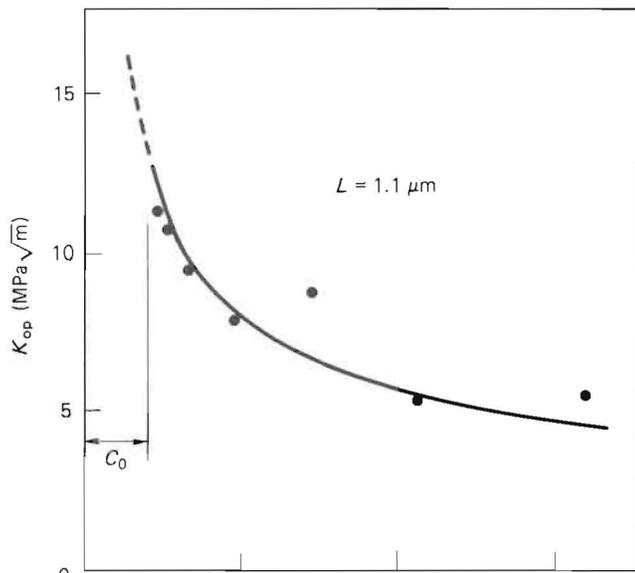
as the limiting value of a for the effect of residual stress. The corresponding limiting value from Equation (2) is

$$a = \rho_{OL} \left[1 - \left(\frac{K_{max}}{K_{OL}} \right)^2 \right] \quad (4)$$

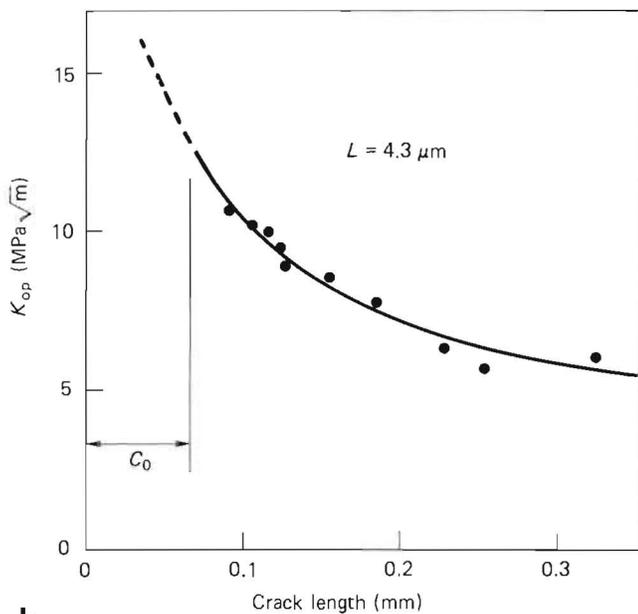
For an overload ratio of $K_{max}/K_{OL} = 0.5$, the limiting values of a for Equations (3) and (4) are, respectively, $0.5\rho_{OL}$

and $0.75\rho_{OL}$. Thus, the modified form would give an increase in the predicted delay length. The fact that alternative forms could be used is a consequence of the fact that K_R is not the result of a solution to the mechanics problem but rather a rationally evolved result.

For the tests conducted by Ward-Close and Ritchie¹⁰ and McEvily and Yang,¹¹ obstruction to closure was observed, and the opening values of the stress intensity factor, K_{OP} , were determined for crack extension after overloads were applied. The data of Ward-Close and Ritchie are presented in Fig. 1. The data in Fig. 1(a) are for an overload of 100%. Those of Fig. 1(b) are for an overload of 150%. As plotted, the overloads occurred at the origins. During the interval between the origin and the first data point, a transient crack growth acceleration or delayed retardation was observed. This



a



b

Fig. 1 Opening stress intensity factor versus crack extension (a) after 100% overload and (b) after 150% overload for Ref. 10 data

behaviour had also been observed previously by Nowack *et al*¹⁴ and by Paris and Hermann.¹⁵

This immediate, post-overload behaviour was attributed to a crack tip stretching which reduced closure contact pressures between previously formed obstructions and, by effectively increasing the stress intensity factor range, resulted in a brief crack growth acceleration period. Ultimately, the crack extended through the stretched ligament and formed an asperity which resulted in the K_{OP} data points shown in Fig. 1. It should be noted that the immediate post-overload transient cannot be rationalized by use of residual stress models of the Willenborg type. The solid curves and the parameters C_0 and L shown in Fig. 1 will be discussed in the section on Application of the Model.

Obstruction to closure has been observed to occur in different forms. In sheets the obstruction is formed at the specimen faces at the free ends of the crack tip front. For dominantly plane strain conditions obstructions on the interior crack surfaces in the form of surface roughness can be developed.¹⁶

The experiments of McEvily and Yang¹¹ were designed to examine the form of closure obstruction after a tensile overload. They presented evidence which indicated that even though crack closure obstruction prior to an overload was of the plane strain form, obstruction after an overload occurred at the specimen faces, *ie* under plane stress conditions. Data points for K_{OP} obtained by McEvily and Yang¹¹ on the aluminium alloy 6061-T6 are presented in Fig. 2. The trend of the data points is similar to that observed by Ward-Close and Ritchie.¹⁰ To examine the character of the obstruction form resulting from the overload, McEvily and Yang¹¹ repeated their experiment, but after the overload, they removed surface layers from the specimen by machining. Upon resumption of testing, no retardation transient was observed. They suggest that the results of Ward-Close and Ritchie¹⁰ are subject to the same interpretation; *ie* the obstructions to closure after the overloads occur at the side faces of the specimens.

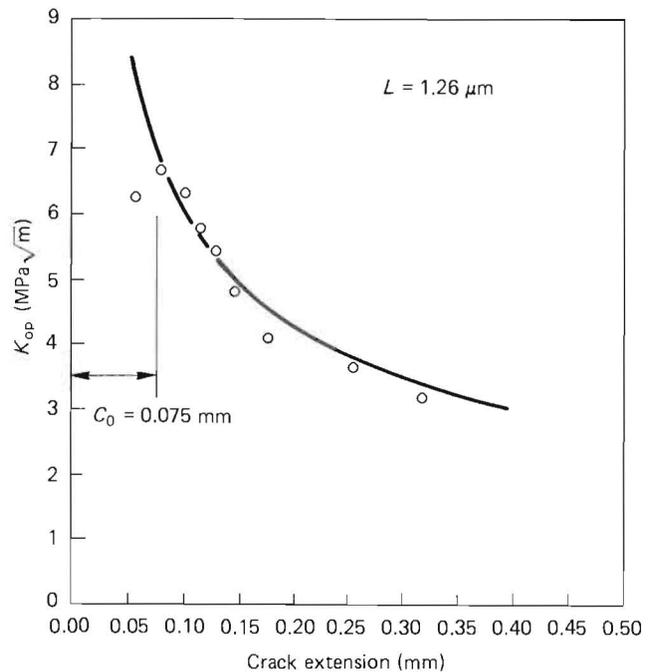


Fig. 2 Opening stress intensity factor versus crack extension for Ref. 11 data

McEvily and Yang¹¹ also conducted experiments on the closure behaviour developed for multiple overloads. From their results they concluded that during the transient period following the start of overloading, there was a transition from a surface obstruction to a through-the-thickness obstruction. These results clearly indicate that the form of closure obstruction is subject to the details of loading.

The results of Damri and Knott¹² differed in some details from those of Ward-Close and Ritchie and those of McEvily and Yang. First, Damri and Knott did not observe a delayed retardation. Retardation began immediately after the application of a tensile overload. Perhaps this meant that the stretched crack tip ligament fractured either during the overload, or just a few cycles afterwards. The overload asperity could then have initiated retardation without a delay transient.

They then repeated their experiment, but, after the overload, they subjected the specimen to a thermal stress relief at 650 °C for two hours. Upon resumption of steady state loading, they observed that there was no retardation transient. They concluded from their results that the retardation observed in the initial experiment was primarily due to the residual compression stress state in front of the crack tip.

In an additional test Damri and Knott¹² subjected a specimen to the same stress relief treatment after the retardation transient was very nearly completed. Upon resumption of loading, initial crack growth rates nearly an order of magnitude greater than the steady state values were observed. Within about half a millimetre of crack extension, the rate then decreased to the steady state value. In this experiment the crack had extended very nearly through the overload plastic zone before the thermal treatment, so the compressive residual stress zone remaining would be expected to have been substantially reduced in size. Since the thermal treatment immediately after the overload restored the crack growth rate to its steady state value, it would be expected that the delayed thermal treatment in the second test should have likewise merely restored the growth rate to its steady state value without the initial, increased rate.

Damri and Knott¹² indicate that the increased rate of growth observed for the thermal treatment applied after retardation was nearly complete resulted from a decrease in the closure load. They do not, however, cite the mechanism which is responsible for the decrease.

In reviewing the results of Damri and Knott,¹² the possibility that strain hardening may have played a role should be examined. It was concluded from the results in Table 2 that the low carbon steel had a substantial capacity for strain hardening.

Fleck¹³ has suggested that overload strain hardening effects would be secondary in explaining retardation, and he presents two reasons for this belief. First, he observes that the strain hardening mechanism could not explain delayed retardation. Damri and Knott,¹² however, did not observe delayed retardation, so this argument does not apply to this case. Secondly, he cites results of several investigators¹⁷⁻¹⁹ who observed that cracks grew faster in prestrained, strain hardened materials than in materials in the as-received state. This comparison, however, may not be applicable to the results of Damri and Knott. In the cases cited the specimens were homogeneously strain hardened. In two of the studies^{17,18} prestraining was accomplished by stretching. In the third¹⁹ prestraining was introduced by cold rolling. The crack growth rate in a very small enclave of strain hardened material loaded through a surrounding, 'soft' material may

not behave in a manner analogous to the results cited by Fleck.

Before dismissing the role of strain hardening in retardation behaviour, the variety of possible responses in the process zone in front of the crack tip should be considered. This zone can, for example, be considered to be undergoing low cycle fatigue, and either cyclic hardening or cyclic softening can result from such loading. It has, for example, been observed²⁰ that if $\sigma_0/\sigma_y > 1.4$, hardening occurs. If $\sigma_0/\sigma_y < 1.2$, softening occurs. Reference to Table 2 would indicate that the titanium and aluminium alloys would, for example, soften, whereas the low carbon steel would harden. It may also be expected that a thermal treatment could produce a significant recovery in a material for which $\sigma_0/\sigma_y = 1.88$.

It does not, on the basis of results which are available, appear to be possible to make a firm generalization about the possible role of strain hardening in retardation behaviour. The anomalous behaviour observed by Damri and Knott may, however, possibly be the result of a more simple explanation. The temperature and time for the thermal treatment could have resulted in not only a stress relief but also a recovery or softening. This latter would reduce the yield stress and result in an increase in crack tip opening displacement. Since this would reduce K_{OP} , the observed acceleration in crack growth rate after the thermal treatment may then have been a consequence. If this explanation is correct, however, it implies that a different interpretation of the results observed when the thermal treatment was applied immediately after the overload is necessary. With the compressive residual stress being eliminated, two opposing effects would be necessary. These could be a decrease in crack growth rate due to closure obstruction being cancelled by a decrease in K_{OP} due to softening.

The experimental measurements of K_{OP} for the titanium and aluminium alloys indicate that retardation was affected by obstructions to closure. Results for these alloys will be analysed by use of a closure model in the sections which follow.

Closure models

The generation of crack growth programs which include the effects of closure obstruction requires the development of closure models. Newman²¹ has provided a good description of the features of a modified Dugdale strip model which has been applied by a number of researchers. This model has been used by Ward-Close *et al*²² to analyse results from the paper by Ward-Close and Ritchie.¹⁰

Experimental results obtained by Buck *et al*²³ and by Ward-Close and Ritchie¹⁰ have indicated that the primary crack face contacts which obstruct closure are immediately adjacent to the crack tip. A discrete asperities model²⁴⁻²⁶ provides a relatively simple means of isolating attention to the near crack tip region of closure contact. This does not mean that segments of the crack face more distant from the crack tip are neglected. It simply means that the more distant asperities are a part of the past crack advance history which does not influence current behaviour. The features of this model can be described by reference to Fig. 3. Obstruction to closure is developed by the discrete asperities on the fracture surface. The geometric features of an asperity and its distance from the crack tip at the right in Fig. 3 are model parameters. These should be considered as effective values. It will be shown that the values of the primary parameters can be determined from experiments.

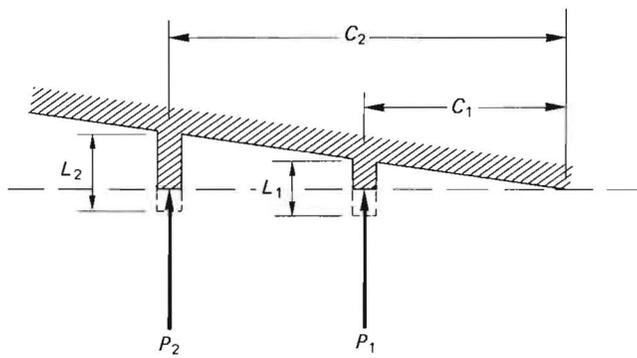


Fig. 3 Discrete asperities model

The features of the model are used in conjunction with the elasticity solution for concentrated forces acting on a crack face. This problem is illustrated in Fig. 4. The stress intensity factor, K , developed and the displacement, u , at the load point are also given. The solution for multiple asperities has been developed by use of superposition.²⁵ The equation for the opening stress intensity factor for a single asperity is

$$K_{OP} = \frac{LG}{2(1-\nu)} \left(\frac{2\pi}{C} \right)^{0.5} \quad (5)$$

where G is the elastic modulus of rigidity and ν is the Poisson's ratio.

The features of the model can be illustrated by reference to the plot of stress intensity factor versus external load in Fig. 5. For no obstruction to closure, or no asperities, the loading cycle moves along the straight line from A to B. The corresponding stress intensity range is then measured between A and E. For a single asperity, loading proceeds along D-C-B, and clearly, the range of the stress intensity factor is reduced, being measured from D to E. As the crack grows, the asperity recedes from the crack tip, and the value of K_{OP} decreases. It may be noted that Brahma *et al*²⁸ have proposed a 'discontinuous closure' model which can be used to qualitatively describe the crack growth response after an overload. The discrete asperity model shown in Fig. 3 has the same features as the Brahma *et al* model, but it provides a basis for obtaining quantitative results.

Additional features of the model can be illustrated by reference to the dashed lines. For two asperities, loading proceeds along G-F-C-B. If asperities weld to one another,

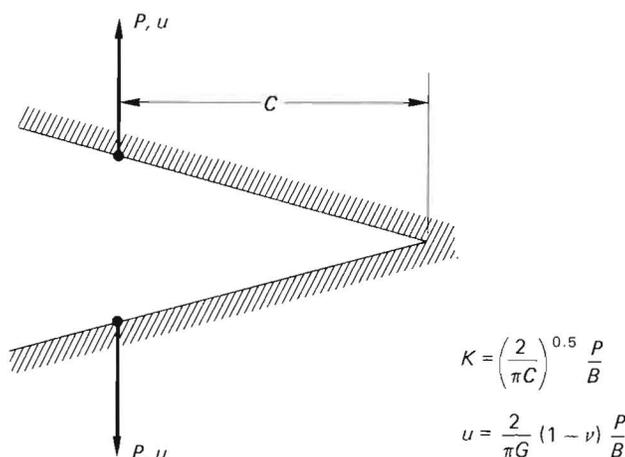


Fig. 4 Concentrated forces on crack faces problem

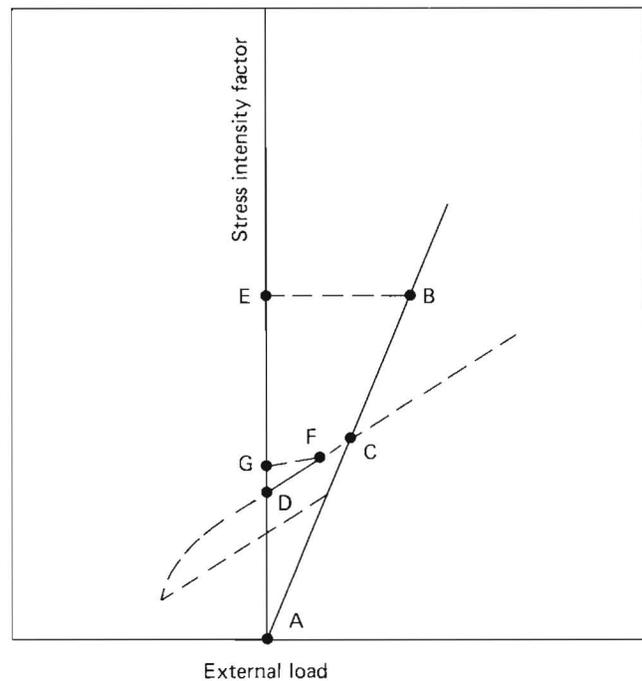


Fig. 5 Closure model for stress intensity factor versus external load

then loading is extended from D-C to the dashed line to the right. This corresponds to the development of a tensile force in the asperities to the right of point C. This results in a further reduction in the range of stress intensity factor and properly describes the difference in crack growth rates between tests in an inert atmosphere, where welding can occur, and an active atmosphere where welding does not occur.²⁷

The model can also be extended to the left of the origin, *ie* for cases in which the external loading contains a compressive overload. If the compressive overload is sufficiently large, it may be anticipated that the asperity heights will be reduced by inelastic deformation. This is illustrated by the downward curve of the extended dashed line. Reloading would then occur along the lower straight line returning to the right. The subsequent loading cycle would then exhibit an increased stress intensity range which would result in the increased crack growth rate which has been observed after a compressive overload.

In the next section the discrete asperities model will be used to analyse the data of Ward-Close and Ritchie¹⁰ and those of McEvily and Yang.¹¹

Application of the model

When closure conditions are developed, the crack growth rate is determined by the range of the stress intensity factor. Thus, the equation $\text{eff} \Delta K = K_{\max} - K_{OP}$ indicates that K_{OP} must be determined. The discrete asperities model will be used to demonstrate the correlation between Equation (5) and the data of Ward-Close and Ritchie¹⁰ and McEvily and Yang.¹¹

The variable C in Equation (5) represents the distance between an asperity and the crack tip. This corresponds to the abscissa variable of Fig. 1. From Ref. 10, Young's modulus $E = 107$ GPa. Assuming material isotropy and a Poisson's ratio of 0.3, an estimate of the rigidity modulus is $G = 41.2$ GPa. Using these property values and an asperity misfit value of $L = 1.1 \mu\text{m}$ in Equation (5) yields the curve shown in Fig. 1(a) for an overload of 100%. An obstruction misfit value

of $L = 4.3 \mu\text{m}$ gives the curve of Fig. 1(b) for an overload of 150%. These curves describe how K_{OP} decreases as the distance increases between the tip of the growing crack and the asperity which results from the overload. An examination of these plots reveals that the data points are well correlated by the use of Equation (5).

In the experiments analysed, retardation did not begin until values of C of about 0.04 mm and 0.65 mm after the overloads were achieved. This immediate, post-overload behaviour was attributed to a crack tip stretching which relieved the contact pressure on asperities developed prior to the overload. Ultimately, the crack extended through the stretched ligament, and the asperity causing retardation was formed. The values of C_0 in Fig. 1 represent initial, effective distances from the crack tips.

The data of McEvily and Yang¹¹ for K_{OP} have been analysed. For their aluminium alloy 6061-T6 specimen $G = 26.5 \text{ GPa}$ for a value of Poisson's ratio of 0.3. For a value of $L = 1.26 \mu\text{m}$ in Equation (5), the solid curve of Fig. 2 is obtained. Again, Equation (5) provides a good correlation with the data. A value for $C_0 = 0.075 \text{ mm}$ represents the initial, effective distance of the asperity from the crack tip at the beginning of retardation.

Discussion

The experimental results of Ward-Close and Ritchie¹⁰ and McEvily and Yang¹¹ demonstrate that the retardation transient observed after a tensile overload can be attributed to the obstruction to closure developed by the overload. The discrete asperity model can describe the opening stress intensity variation after the overload. The primary parameter in the model is the obstruction misfit, L , which can be determined from experimental data.

Two distinct closure models have been proposed. These are the modified Dugdale strip model²¹ and the discrete asperity model applied here. The objective of both models is the same, *ie* to determine, quantitatively, the magnitude of the obstruction misfit which prevents complete closure. The strip type models proceed by continuously computing the thickness of a perfectly plastic layer, which is of the order of microns, and introducing a crack extension criterion. Three-dimensional effects are incorporated by use of a constraint factor.* Both models are based on results obtained from deformable body mechanics. As presented here, however, the misfit parameters of the discrete asperity model are determined from measurements from special experiments.

The discrete asperity model is two-dimensional and therefore may be considered to be more applicable to through-the-thickness closure. The correlation with the McEvily and Yang¹¹ data, however, suggests that L can be considered the 'effective' value of obstruction which props the crack open. It would appear that it can represent an experimentally integrated measure of closure obstruction.

The results which have been presented here are limited in scope and represent an exploratory examination of the use of a relatively simple method for describing the effect of an overload on asperity induced closure. It would, in future

work, be of interest to examine possible correlations of the parameters L and C_0 with such basic quantities as crack tip opening displacement. An examination of the limited data of Ward-Close and Ritchie¹⁰ indicates that it may be possible to develop a simple relationship between L and C_0 with K^2 , which is proportional to crack opening displacement, for overloads up to 100%.²⁶ A marked non-linearity which is observed for K^2 above 100% overloads may be related to the details of the crack extension mechanism. Ward-Close and Ritchie¹⁰ observed that for the higher loads, there was, in addition to crack tip stretching and blunting, tearing or crack extension during the overload cycle. This added complication of incremental crack advance during the overload cycle must, therefore, be considered.

As the asperity produced by the overload recedes from the crack tip and loading at a lower, constant ΔK resumes, additional asperities are produced. If L_1 represents the overload asperity height and L_2 a subsequent asperity height, a condition in which L_1 is much greater than L_2 would develop. Then near tip contact would occur, but it would not be on the asperity closest to the crack tip. Nearest crack tip asperity contact would not develop until K_{OP} for the most recently produced asperity was larger than the current K_{OP} for the overload asperity.

A qualitative description of how K_{OP} varies as the crack extends is presented in Fig. 6. Constant ΔK is maintained from A to B. At B, a tensile overload is applied and the crack tip stretching decreases the misfit obstruction of asperities formed previously, and the K_{OP} has a reduced value as shown between B and C. In the transition between B and C the stretched crack tip ligament is fractured to form a stretched asperity which, by increasing the obstruction misfit, results in an abrupt increase in K_{OP} at C. As this asperity recedes from the crack tip, K_{OP} decreases.

Crack tip stretching is involved in the formation of asperities. A doubling of the stress intensity factor, for example, increases the crack tip opening displacement, CTOD, by a factor of three. The formation of asperities also requires a fracture of the stretched ligament, however, and this process can be also expected to be microstructure dependent.

In the process of asperity production under variable amplitude loading, a sequence of scalloped curve segments

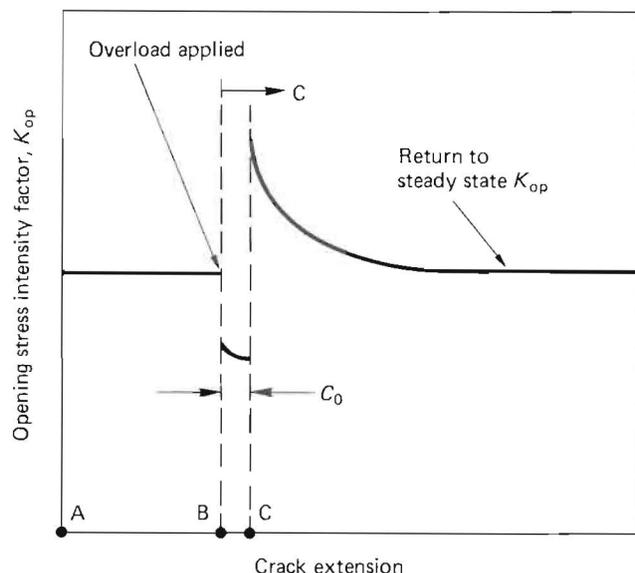


Fig. 6 Comparison of crack growth rates after tensile overload

*It may be recalled that the results of McEvily and Yang¹¹ indicated that closure obstruction was of a plane strain form before an overload and of a plane stress form during retardation. This suggests that a 'constraint factor', which specifies the prevailing effective yield strength, may vary under variable amplitude loading.

derived from curves of the type shown in Fig. 1 can be visualized. The operative K_{OP} curve would be the highest one for the current crack length.

The transient behaviour following a tensile overload can rationally be attributed to either a residual stress effect or to closure obstruction. It is not, however, necessary to choose one model to the exclusion of the other. Certainly, both surface roughness obstructions to closure and residual stresses in front of the crack tip due to the overload can be expected to be present in the near threshold region. A combined procedure for determining effective stress intensity factors is possible. Thus consider, as an example, a formulation in which

$${}_{\text{eff}}K_{\text{max}} = K_{\text{max}} + K_{\text{R}} \quad (6)$$

$${}_{\text{eff}}K_{\text{min}} = K_{OP} - K_{\text{R}} \quad (7)$$

for $K_{OP} - K_{\text{R}} > 0$, and

$${}_{\text{eff}}K_{\text{min}} = 0 \quad (8)$$

for $K_{OP} - K_{\text{R}} < 0$. It also follows that

$${}_{\text{eff}}R = \frac{{}_{\text{eff}}K_{\text{min}}}{{}_{\text{eff}}K_{\text{max}}} \quad (9)$$

In the near threshold region in which closure obstruction is clearly present, K_{OP} may dominate. When the crack faces do not impinge upon one another during a cycle, K_{R} probably dominates. A proper partitioning of K_{OP} and K_{R} should be the subject of future research. It should, however, be noted that the interaction between K_{OP} and K_{R} may well be complex. Suppose that both closure obstruction and residual compressive stresses in front of the crack tip are present. Then, if the local compressive stresses were relieved, the pressure across the asperities would be increased. This, in turn, would signify an increase in K_{OP} . The two effects are not, therefore, uncoupled. The extent of this coupling probably varies with the level of loading, but at present it is unknown. It should also be noted that other mechanisms, such as strain hardening, may play a role in retardation behaviour.

Conclusions

A review of the available crack growth prediction methods was performed to assess their limits and regions of validity. Although current fatigue life estimation procedures attribute retardation to a single mechanism, multiple effects need be considered especially in the near threshold region. The following conclusions can be specifically drawn:

- (i) Available crack growth prediction models do not properly account for all of the mechanisms which are operative under variable amplitude loading.
- (ii) A discrete asperities model can be used as a diagnostic tool to describe the variation in the opening stress intensity factor after a tensile overload which results in an obstruction to closure. Crack tip closure measurements of the type made by Ward-Close and Ritchie¹⁰ and by McEvily and Yang¹¹ can be used to determine the two parameters of the discrete asperities model.
- (iii) The discrete asperities model has features which can rationalize the acceleration transient after a compressive overload, and the difference in crack growth rates observed in inert and active atmospheres.
- (iv) When obstruction to closure develops after a tensile overload, retardation can continue beyond the crack

tip plastic zone. This indicates that retardation models based on residual stress effects alone cannot describe the transient behaviour observed. Fracture surface features, which are microstructure dependent, must also be considered.

- (v) Ultimately, a model which incorporates effects due to more than one mechanism may be required to describe the full range of possibilities under variable amplitude loading.

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