Effects of compressive load excursions on fatigue crack growth*

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(Received 17 November 1992; revised 18 May 1993)

The variable-amplitude loading encountered in service often includes compressive excursions. It has been a common practice to ignore these excursions for fatigue crack growth analyses. Recently, experimental data on both smooth bars and cracked specimens with intermittent negative R ratio loadings have indicated that the effects of compressive excursions are not negligible.

(Keywords: crack closure; variable-amplitude loading; compressive excursions; crack surface asperities)

A number of computer codes have been developed for determining fatigue crack growth histories for variable-amplitude loading. A feature of many of the service load spectra is the presence of intermittent compressive excursions. Since it has seemed reasonable to believe that no contribution to crack growth is developed during a compressive excursion, it has been a common analytical practice to exclude the affects of negative loads.[1] Thus, for a negative R cycle the stress intensity factor range is set equal to the maximum stress intensity factor.

The effect of compressive load excursions has been the subject of a number of experimental programmes during the past several years. A review of the results of these investigations indicates that it is incorrect to assume that compressive cycles do not contribute to fatigue crack growth. Since it follows that the use of this assumption can lead to non-conservative predictions, the behaviour involved needs to be examined. The objectives of the discussion presented here are to describe the features of the behaviour and to identify mechanisms that could cause the effects that have been observed.

EXPERIMENTAL RESULTS

The effects of negative R ratio loadings have been the subject of a number of investigations.[2–14] Compressive loading excursions have been investigated in tests on both smooth bar specimens and cracked specimens. Zaiken and Ritchie[2] conducted fatigue crack growth tests on compact tensile specimens of a cast I/M 7150 aluminium alloy. They found that crack growth could occur for loading levels below the threshold stress intensity range after the application of large compression overloads. They attributed this behaviour to a flattening of previously formed roughness asperities, which reduced the effective stress intensity range by closure obstruction.

Pompetzki *et al*[3] have conducted extensive investigations of the effects of overloads on fatigue damage in smooth bars of the aluminium alloy 2024–T351 and of SAE 1045 steel. They applied fully reversed ($R = -1$), constant-amplitude load cycles to specimens with circular cross-sections to obtain uniaxial stress amplitude versus cycles to complete failure to establish the baseline behaviour. As conducted, the tests included both short and long crack phases of the lifetimes.

In additional experiments they repeated the previous tests, but applied intermittent compressive overloads. Three sets of tests were conducted, in which the numbers of cycles between the intermittent overloads differed. All the intermittent overload test curves were below the baseline curve for stress versus cycles to failure. The level of the curves decreased with decreasing cycles between overloads, and they inferred from the results that compressive overloads increased the accumulated damage. Both the aluminium alloy and the steel exhibited this behaviour. The authors suggested that the observed behaviour resulted from a decrease in the crack closure opening stress which resulted in an increase in the crack growth driving force. The reduction in opening stress was attributed to a reduction in the height of the crack tip wake by compressive yielding during the compressive overload.

Yu *et al*[4] have presented test results on the aluminium alloy 2024–T351 in the near threshold region. They found that crack growth curves shifted to lower threshold values and greater crack growth rates as the minimum stress became more compressive. The R value and range of stress intensity factor did not provide a correlation with the crack growth behaviour.

In additional experiments, which provide insight into the processes involved, Yu *et al*[4] examined the

* Dedication: We dedicate this paper to the memory of the late Dr C.J. Beevers. He was a valued friend and a source of stimulation and encouragement.

0142-1123/94/020141–06

effects of single, intermittent compressive excursions in tests which were otherwise loaded under fixed positive R values. The tests were conducted on centre-crack plate specimens. The number of cycles between the intermittently applied loads was varied from test to test. The number of cycles between the compressive load cycles ranged from 1 to infinity for six values. Introduction of the compressive cycles substantially shifted the growth rate curves, with the highest rates being developed for one excursion per cycle (all negative R cycles), and the lowest for infinite cycles between excursions (no negative R cycles). The shift behaviour was observed even when only one compressive cycle was applied between 1000 positive R cycles. Topper[5] has reported similar data for CSA G40.21 steel and also for a variety of other metals.[6,7]

The variation in response for three alloys was investigated by Kemper *et al.*[8] The alloys – age-hardened aluminium 2024, pure copper and the powder metallurgy aluminium alloy IN-905XL – had different microstructures, deformation behaviour and mechanical properties, and the authors demonstrated that the responses to negative R loading differed significantly. They concluded from their experimental results that the ASTM recommendation of using only the tensile portion of cyclic loading was valid only when obstruction to closure was absent. Surface roughness measurements indicated that this applied only to the ultra-fine-grained, mechanically alloyed Al IN-905XL, for which the fracture surface was very flat. It also appeared to be valid for the soft copper under very high compressive loadings. It was not valid for the Al 2024, for which very high compressive loads were not able to reduce the level of closure obstruction.

Tack and Beevers[9] obtained fatigue crack growth data for three bearing steels for ratios of R between 0.1 and −2.5. The specimens had square cross-sections with corner cracks. For all the steels the fatigue crack growth rates for a given ΔK were greater for the negative R ratio tests than for the $R = 0.1$ tests. The increase in growth rates was greatest between $R = 0$ and $R = 1.0$ and exhibited little increase for more negative R values. This suggests that the compressive effects may be subject to a saturation phenomenon. It was also observed that the crack tips remained open throughout the loading cycles and that the first crack face contacts occurred at some distances from the tip.

Herman *et al*[10] and Hertzberg *et al*[11] have shown that special low-closure, maximum stress intensity test data provide an upper-bound estimate of crack growth following compressive excursions. They also attribute the observed behaviour to the crushing of asperities in the crack wake.

Henkener *et al*[12] conducted a fatigue crack growth experiment on an Al–Li–Zr alloy four-point-bend specimen, which was interrupted by the application of a uniform compressive load. Upon resumption of the bending fatigue test, the rate of crack growth increased drastically. They attributed the observed effect to a crushing of closure obstructing asperities which, in turn, led to an increase in the range of the effective intensity factor.

Swain *et al*[13] have presented results of a study of short fatigue crack growth in 4340 steel. Tests were conducted for $R = 0.5$, 0 and −1. Specimens had rectangular cross-sections with semicircular edge notches that led to the formation of semi-elliptical surface cracks. Short crack growth below the long crack threshold was observed only for $R = -1$; *ie* short crack growth did not occur for positive R ratios.

Table 1 Yield and ultimate strengths

Alloy	0.2% yield strength (MPa)	Ultimate strength (MPa)
Waspaloy	860	1250
IN-9052	379	448
M50 NiL	1200	1462

Most of the experimental results that have been reported have dealt with the effect of compressive excursions on the rate of crack growth versus the range of the stress intensity factor plots. Recently, results have been presented that focus directly on crack growth behaviour.[14] Data were presented on three alloys: Waspaloy, aluminium powder metallurgy alloy IN-9052, and the bearing steel M50 NiL. The specimens had square cross-sections with corner cracks, and loading was uniaxial. The 0.2% yield strengths and ultimate strengths are given in *Table 1*.

Crack growth was measured for tests in which constant values of maximum stress and minimum stress with positive R were applied. The tests were then interrupted, and although the maximum stress was maintained, the minimum stress was reduced to a negative, compressive value so that R was negative. After an interval, the initial loading was resumed. *Figure 1* illustrates the type of loading that was applied.

An examination of the data for Waspaloy in *Figure 2* reveals that the rate of crack growth (the slope) is discontinuous at each change in loading condition. For these tests $\sigma_{max} = 300$ MN m^{-2} and the initial crack length $a_0 = 1.20$ mm. Also, it is clear that the rate of growth for $R = -2$ loading is substantially greater than that for $R = 0.1$. It also appears that the final slope for the initial $R = 0.1$ phase is slightly greater than the initial slope for the final $R = 0.1$ loading. This suggests that the interposed $R = -2$ loading may have introduced a transient retardation behaviour upon resumption of the $R = 0.1$ loading.

The relative slopes during the $R = 0.1$ and $R = -2$

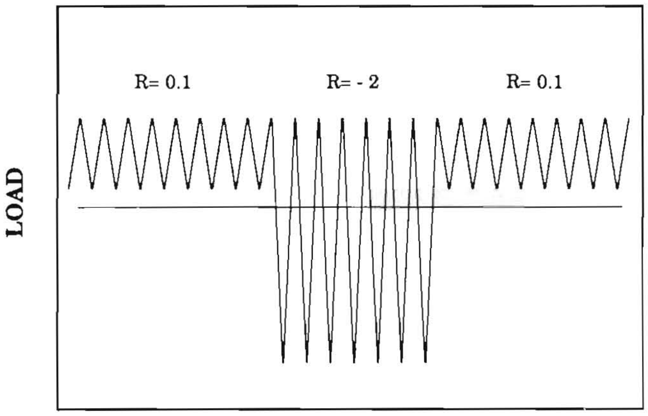

Figure 1 Loading sequences

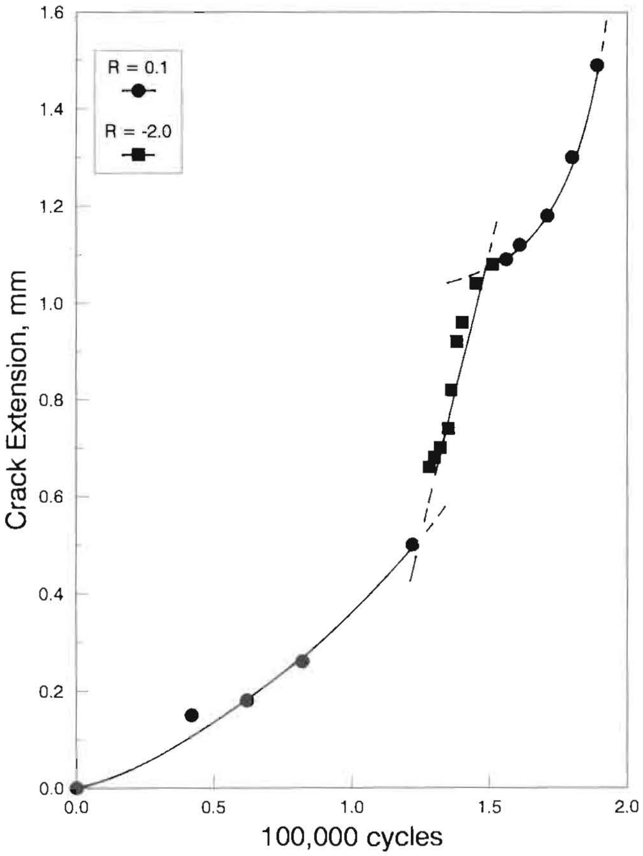

Figure 2 Data for Waspaloy: σ_{max} = 300 MN m^{-2}, a_0 = 1.20 mm

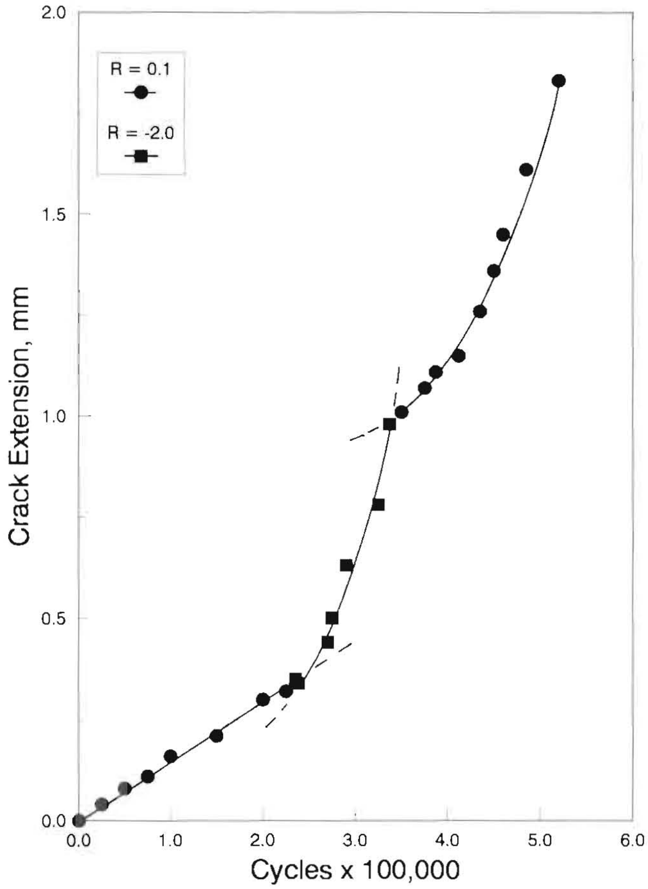

Figure 3 Data for M50 NiL: σ_{max} = 108 MN m^{-2}, a_0 = 1.62 mm

loadings in *Figure 3* for M50 NiL are analogous to those observed in *Figure 2* for Waspaloy. However, the final slope for the first $R = 0.1$ phase is slightly lower than the initial slope for the second $R = 0.1$ loading, so there is no clear indication of a retardation transient for M50 NiL.

The loading sequence for the IN-9052 test data presented in *Figure 4* differed from that for the results of *Figures 2* and *3*. Here, the initial and final loading were under $R = -2$ and the intermediate phase was under $R = 0.1$ loading. Again, the slopes at the loading changes are discontinuous, and the rates for $R = -2$ are distinctly greater than those for the intervening loading of $R = 0.1$. The slopes at the end of the first $R = -2$ loading and that at the beginning of the second $R = -2$ phase, when compared, do not indicate the presence of a transient effect.

DISCUSSION

The results described indicate that compressive loading excursions should not be ignored. If, for example, loading sequences of the type depicted in *Figure 1* were analysed by assuming that compressive effects could be ignored, the value of $R = 0.1$ would be used throughout an analysis of the crack growth history. The test results presented indicate clearly that the latent errors in such an approach cannot be considered to be negligible.

Closure effects

The features of the experimental results that have been reported indicate that the observed behaviours are primarily associated with closure effects.

Of the metals tested by Kemper *et al*,[8] two could be analysed by use of the ASTM recommendation;[1] *ie* the compressive excursions could be ignored and the effective range of the stress intensity factor could be set equal to K_{max}. For IN-905 XL the crack surfaces were very flat, so closure obstruction was minimal. The relative roughness of the fracture surfaces is dependent on microstructure and deformation characteristics, as noted by Kemper *et al.*[8] Copper was the second material, and although its fracture surfaces were not flat, closure obstruction appeared to be eliminated when sufficiently high compressive loads were applied. This suggests that the effective heights of the asperities were reduced by inelastic deformation. Since closure contact is often observed to occur on facets inclined to the crack path, however, the reduction in the effective misfit height may be a result of a shearing action rather than a compressive crushing.

Both Kemper *et al.*[8] and Tack and Beevers[9] observed a saturation effect in which increases in compressive load beyond a certain level did not result in additional increases in crack growth rate. Reduction of asperity heights could reasonably be subject to a limiting inelastic deformation as the asperity cross-sectional areas are increased under increasing compressive loading. Strain-hardening effects could also be expected to contribute to the saturation behaviour. It may further be observed that the type of discrete contact associated with roughness asperities could be expected to provide for more 'efficient' crushing than a continuous wake layer.

If the effect of compressive excursions could be

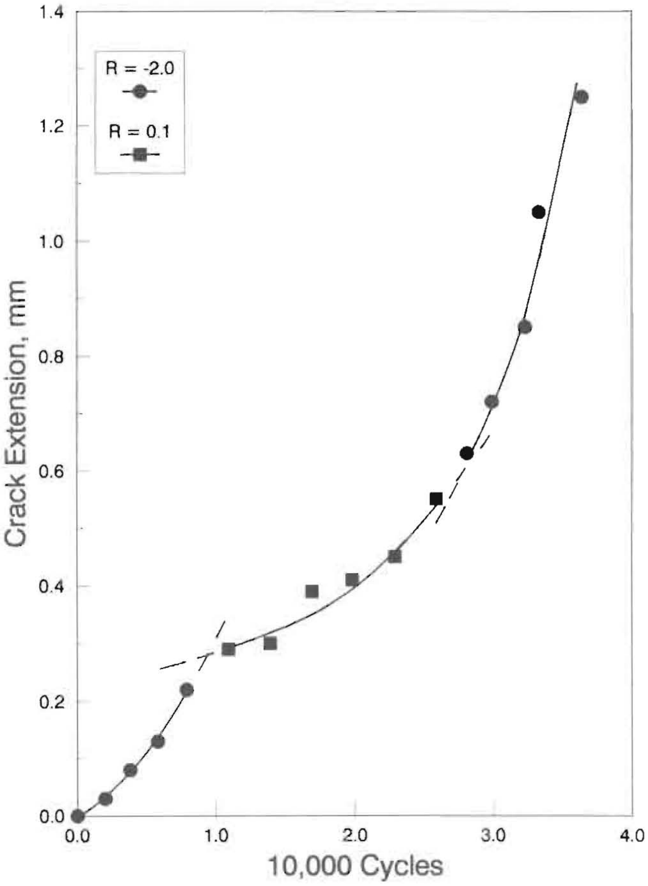

Figure 4 Data for IN-9052: σ_{max} = 100 MN m^{-2}, a_0 = 1.55 mm

neglected, going from R = 0.1 to R = −2.0 loading should not result in the change in slopes seen in the plots of *Figures 2* and *3*. At least a part of the change may, however, be attributed to a difference in the effective stress intensity factor range for the two loading conditions. This can be shown by reference to features of a discrete-asperity model for closure obstruction,[15,16] which is illustrated in *Figure 5* as a plot of the variation of the mode I stress intensity factor with external load. With no closure obstruction the loading path cycles along the line OA. If, during unloading from point A, closure obstruction is

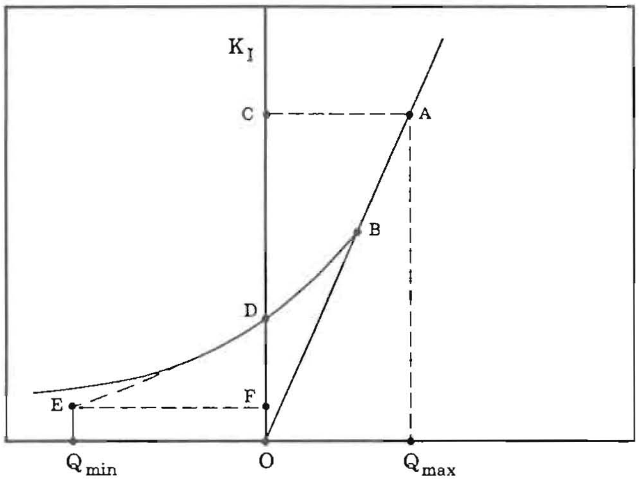

Figure 5 Effect of closure obstruction on stress intensity factor

encountered at point B, the model for a single asperity results in a straight-line load path that moves downwards and to the left of B, but above line OB. For two asperities, it can be shown that two straight-line segments will be developed. For a number of asperities an increasing number of contacts will occur during unloading, and the curve to the left of point B represents this behaviour. If the heights of the asperities are inelastically reduced, it may be anticipated that unloading would drop below the elastic solution, as shown by the dashed curve.

For purposes of illustration consider the two loading conditions for R = 0 and R = −2. For cyclic loading between O and Q_{max} (R = 0) the range of the effective stress intensity factor would be measured on the K axis from point D to point C in *Figure 5*. K is greater than zero because of closure obstruction. For loading between Q_{min} and Q_{max}, the range of the effective stress intensity factor would be measured from point F to point C. Since the magnitude of the distance from F to C is greater than that from D to C, it would follow that the rate of crack growth for R = −2 should be greater than that for R = 0. The difference depends upon the distance from F to D.

From the features of the discrete-asperity model, it can be concluded that positive values of stress intensity can be developed with externally applied compressive forces. Negative values of stress intensity factor have no meaning, however, and the model indicates that it is not, in fact, an issue. It is also clear, moreover, that the effective range of stress intensity factor should not, for compressive excursions, be taken as the difference between the maximum stress intensity factor and the value at point B, the so-called opening stress intensity factor. The effective stress intensity range developed, as described above, with the use of a discrete-asperity model that accounts for inelastic crushing, could account for the behaviour observed in compressive excursion tests. Quantitative details of such a model, however, remain to be developed.

Another alternative has been proposed. Some investigators have effectively introduced a fictitious negative, minimum stress intensity factor for correlation purposes. Swain *et al*[13] have, for example, in computing ΔK for R = −1, set $\Delta\sigma = 2\sigma_{max}$. Although this adjusts data in the proper direction, it does not appear to be a sound practice.

Secondary mechanisms

Attributing compressive excursion behaviour exclusively to a closure obstruction mechanism ignores the possible presence of mechanisms within the cyclic plastic zone. The transient behaviour reported by Carlson *et al*[14], after a change in the loading conditions illustrated in *Figure 1*, is not surprising when compressive excursion effects are recognized. Within each loading phase, nominally steady-state conditions have been developed. With an abrupt change in loading, both closure behaviour and the cyclic plastic zone size must undergo an accommodation to new loading conditions. The 'steady state' cyclic plastic zone size for R = −2 loading in *Figure 2* would be expected to be larger than that for R = 0.1 loading. In abruptly going from R = −2 to R = 0.1 the fatigue crack must, therefore, traverse the R = −2 cyclic plastic zone before that for R = 0.1 can be established. A transient response should, therefore, be expected. The abrupt change from $R = 0.1$ to $R = -2$, as in *Figure 4,* is different because the second loading would produce a cyclic plastic zone that would envelop the cyclic plastic zone formed earlier.

Another factor that may contribute to the nature of the crack growth is the issue of cyclic softening versus cyclic hardening. An examination of the ultimate strength to yield strength ratios of *Table 1* indicates that whereas Waspaloy would be expected to harden cyclically, the other two alloys are at the upper boundary for cyclic softening.[17]

In a fundamental paper in 1983, Forsyth[18] suggested that the topography of a fracture surface near the crack tip, as well as the externally applied loads, was important to an understanding of crack advance. Modelling surface features such as asperities, which act as obstructions to closure, leads to a partitioning of the crack tip stress state into two components: a component caused by external forces, which may be classified as global loads, and one resulting from asperity contact forces, which may be termed local loads. Since there are a variety of types of local contact forces that can be developed, different crack tip stress states are possible. The consequences of the various possibilities can be illustrated by reference to features observed in an examination of fatigue crack paths. For the alloys of *Table 1*, the surface roughness excursions associated with the microstructures ranged from the order of 20 μm for the Waspaloy to about 5 μm for the IN-9052 and the M50 NiL.

Several mechanisms that may be operative during compressive excursions have been suggested.[19] The analytical assumption that the crack surfaces are perfectly flat would produce a uniform, compressive stress state in such a cracked body. Tack and Beevers,[9] however, observed that even under maximum compressive loading, complete closure did not occur, and that voids were present. When the presence of such a void is recognized, it follows that the local stress will not be a simple, uniform compression. Thus a void at the crack tip could result in a very large effective compressive stress concentration. A very large compressive stress could produce localized compressive yielding. Upon unloading, there would then be a tensile residual stress in front of the crack tip. It can be expected that residual stresses produced by this mechanism would vary along the crack front, and any contribution to the total stress would have an integrated effect. The mechanism suggested is analogous to that which has been used in experiments designed to initiate cracks. Suresh and Brockenbrough[20] and Tack and Beevers[9] have applied compressive loads to notched specimens to generate residual tensile stress fields that increase the local sensitivity to crack initiation under subsequent cyclic loading. The scales (micro versus macro) for these examples are different, but the mechanisms are similar.

The models discussed to this point have focused on mechanisms that affect the effective range of the mode I stress intensity factor. The local loads developed between impinging fracture surfaces are not, however, exclusively restricted to mode I effects. Micrographs of fracture surfaces reveal inclined jogs or steps along the crack path. Suresh[21] has examined the state developed at the tip of kinked and forked cracks and calculated effective stress intensity factors that are functions of the mode I and II stress intensity factors at the crack tips. The crack profile model used for the kinked crack is shown in *Figure 6a*. Suresh associated the kinked and forked cracks with tensile overloads. He concluded from his analyses that a resulting reduction in the effective stress intensity factor range contributed to the retardation observed after the application of a tensile overload. Although he was not concerned with the effects of compressive excursions, his models do focus attention on how mode II contributions can be developed by cracks that are not perfectly flat.

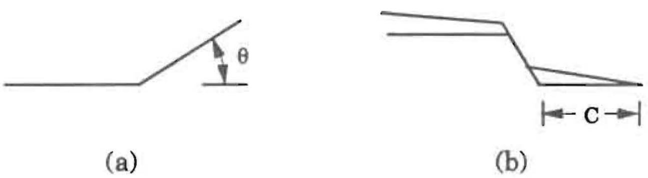

Figure 6 Crack-branching models

One of the consequences of the misfit developed between the surfaces of branched cracks is the development of contact friction. This can occur not only under compressive loads, but also under tensile loads during closure. The presence of crack interface friction has, in fact, been found to affect fatigue crack growth under combined mode I and II loading.[22] Friction effects are also present in the torsional loading of notched, circular bars. Here, circumferential cracks are developed, and because the cracks are not flat, local mode I effects are superimposed on global mode III effects.[23]

A model that includes both crack branching and friction effects has been proposed,[24] and its features are shown in *Figure 6b*. An analysis of this model indicates that the inclination angle of the facet step, its distance from the crack tip, and the friction coefficient affect the values of the stress intensity factors. The loading paths for the stress intensity factors are shown in *Figures 7a* and *7b*. Loading for K_I and K_{II} occurs along the paths A–B–C. Unloading occurs along C–B–D. Point B represents the load above which there is no fracture surface contact. Note

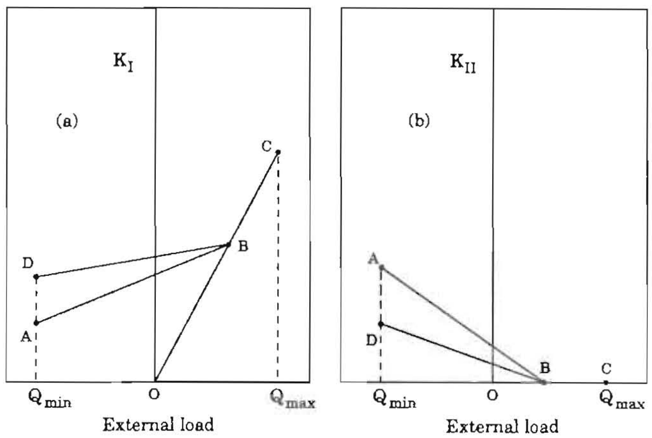

Figure 7 Variations of stress intensity factors for mixed mode model

that K_{II} is zero above the 'opening' load. During closure, the loading and unloading paths differ. This is due to a reversal of the direction of the friction force when the relative directions of the sliding surfaces are reversed, and a 'jump' from point D to point A occurs. The 'loop' represents a frictional form of hysteresis.

Under normal mode I loading, the value of K_I decreases with decreasing external load. The model exhibits this type of response, but K_{II} increases with decreasing load. The maximum value of K_{II} then occurs at the minimum load or the largest applied compression. Thus, whereas K_{II} effects may not be significant for tension–tension tests, they may become a factor in tests with compressive excursions. K_I and K_{II} are out of phase with one another. It then follows that the loading is non-proportional; *ie* the ratio of K_{II} to K_I is not constant during a cycle. Non-proportional low-cycle fatigue loading is usually found to be more damaging than in-phase loadings.[25] Some insight into the complexity of the behaviour may be gained by considering the cyclic plastic zone. Under mode I loading, the rate of crack growth has been considered to be a function of the cyclic plastic zone size. Since the shapes for plastic zones for modes I and II are different, however, not only sizes but also the shapes of these zones can be expected to be a factor.

SUMMARY AND CONCLUSIONS

An examination of experimental data from tests with compressive loading excursions indicates that a variety of responses are possible. The features of the behaviour and possible operative mechanisms may be summarized as follows.

1. Obstruction to closure depends upon the characteristics of fracture surface roughness, which depends on microstructural features and deformation properties. The characteristics include nominally flat surfaces, 'soft' asperities and 'hard' asperities.
2. The ASTM recommendation that compressive excursions be neglected in the analysis of crack growth may be acceptable for flat fracture surfaces and surfaces with soft asperities, but not for surfaces with comparatively hard asperities.
3. There appears to be a saturation level of compressive load, and increases in compression beyond this value do not result in increases in crack growth rate.
4. For some metals a loading change from negative R to positive R appears to produce a retardation transient.
5. The primary mechanism operative during compressive excursions is an inelastic reduction in effective closure obstruction heights. Secondary mechanisms may involve abrupt changes in the size of the cyclic plastic zone, the development of residual tensile stresses at the crack tip, and a combined non-proportional mode I and mode II loading state at the crack tip.

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