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A Model for Quasi-steady Asymmetric Fully Plastic Crack Growth

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ABSTRACT

The ductility of asymmetrically cracked structures, as may occur near welds, is considerably decreased due to the presence of a single shear band and the crack progressing into prestrained and predamaged material. Such an effect may have important consequences in the design and maintenance of pressure vessels. For the fully plastic state, assuming strain increments following a power law relation, damage due to hole growth varying linearly with strain and quasi-steady crack growth leads to a closed form solution for the crack growth in such cases of combined shear and tension. The results show a progressively higher crack advance per unit far-field displacement as the crack grows along the ligament, an effect which has also been observed experimentally.

NOMENCLATURE

c	Crack advance distance.
E	Young's modulus.
J	J-integral.
k	Shear strength.
K_M^P	Far-field stress intensity factor.
M^P	Mode 1 mixity parameter.
n	Strain hardening exponent.
n_s	A hardening coefficient.

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u Far-field displacement.
u_1 Initiation displacement.
W Work per unit volume.

α Material parameter.
γ Principal shear strain.
ε Equivalent strain.
η Damage.
θ_f Growing crack orientation.
θ_s Shear band orientation.
ρ Mean inclusion spacing.
σ Mean normal stress.
$\bar{\sigma}$ Equivalent stress.
σ_0 Yield stress in simple tension.
σ_1 Flow stress at unit strain.
τ Principal shear stress.

INTRODUCTION

Predicting the ductility of pressure vessels and structures is important for their design and maintenance, especially in the presence of cracks and in the fully plastic state which is the desirable one before fracture. In the typical symmetric case the crack tends to advance into the relatively undamaged region between two plastic shear bands. However, if a crack is near a weld or shoulder, loading into the plastic range can eliminate one of the bands and thus give a single asymmetric shear band extending from the crack tip (Fig. 1). The resulting crack propagation into the previously damaged material gives less ductility than the typical symmetric case. Preliminary tests on carbon steel specimens indeed gave deformation from maximum load to instability only half of that for a symmetric specimen. Fractography indicates that the crack grows by a combination of sliding off and void growth along the shear band; essentially the near tip stress and strain causes damage by hole nucleation and growth from

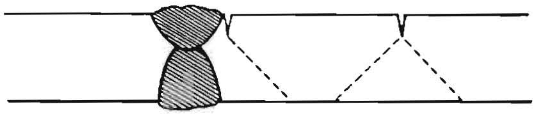

Fig. 1. Symmetric and asymmetric shear band configurations.

inclusions. The assumption of a nonhardening material leads to a single slip band with a constant displacement discontinuity and a consequent infinite strain across it, whereas any strain hardening would impose infinite stress across the band and cause the band to fan out. In the following we present a model for predicting crack growth by employing the mixed mode stress and strain fields of Shih[1] to analytically represent the damage and strain in such mixed mode I and II asymmetric configurations.

ANALYSIS

Shih[1] extended the HRR[2,3] singularity by giving the dominant singularity solutions governing the asymptotic behavior of the stress and strains at the crack tip of a stationary crack for the complete range of loadings between Mode I and II. The power hardening relation between plastic strain and stress was assumed to be:

$$\varepsilon^{\mathrm{P}} = \alpha\left(\frac{\sigma}{\sigma_0}\right)^{n_s - 1}\frac{\sigma}{E} \tag{1}$$

where σ_0 is the yield stress in simple tension, α a material constant and n_s the strain hardening coefficient. The dominant singularity was expressed in terms of the far-field stress intensity factor K_M^P and the Mode I mixity parameter M^P as:

$$\sigma_{ij} = \sigma_0 K_M^P r^{-1/(n_s+1)}\tilde{\sigma}_{ij}(\theta, M^P) \tag{2}$$

$$\varepsilon_{ij} = \frac{\alpha\sigma_0}{E}(K_M^P)^{n_s} r^{-n_s/(n_s+1)}\tilde{\varepsilon}_{ij}^P(\theta, M^P) \tag{3}$$

$$u_i = \frac{\alpha\sigma_0}{E}(K_M^P)^{n_s} r^{1/(n_s+1)}\tilde{u}_i(\theta, M^P) \tag{4}$$

Since our problem is fully plastic, the amplitude of the singularity will be expressed in terms of the path independent integral, which is given by:

$$J = \frac{\alpha\sigma_0^2}{E} I_{n_s}(M^P)(K_M^P)^{n_s - 1} \tag{5}$$

with $I_{n_s}(M^P)$ a numerical constant determined from the singularity analysis, which depends on the strain hardening coefficient n and the near-field M^P. Since we are interested in the fully plastic regime, it is more convenient to use a material law in terms of the flow stress at unit strain instead of the yield stress:[4]

$$\sigma = \sigma_1 \varepsilon^n \tag{6}$$

Thus $n = 1/n_s$ and $\sigma_1 = [(\sigma_0^{(1-n)/n} E)/\alpha]^n$ and we can rewrite the dominant singularity equations as:

$$\sigma_{ij}(r, \theta) = \left[\frac{J}{\sigma_1 I_{1,n}(M^p) r}\right]^{n/(n+1)} \tilde{\sigma}(\theta, 1/n, M^p) \tag{7}$$

$$\varepsilon_{ij}(r, \theta) = \left[\frac{J}{\sigma_1 I_{1,n}(M^p) r}\right]^{1/(n+1)} \tilde{\varepsilon}_{ij}(\theta, 1/n, M^p) \tag{8}$$

$$\frac{u_i}{r} = \left[\frac{J}{\sigma_1 I_{1,n}(M^p) r}\right]^{1/(n+1)} \tilde{u}(\theta, 1/n, M^p) \tag{9}$$

The mixity parameter, M^p, is determined by taking the relative flank-to-flank displacement of the near tip singular field and the far slip line to be at the same direction:

$$\tan(\theta_s - \theta_f) = \frac{u_2(\pi - \theta_f, M^p) - u_2(-\pi - \theta_f, M^p)}{u_1(\pi - \theta_f, M^p) - u_1(-\pi - \theta_f, M^p)} \tag{10}$$

Now the path independent integral J can be evaluated from Fig. 2 directly from its definition. Consider the crack running at an average angle θ_f while the shear band is at an angle θ_s. Express the work per unit volume W as from the nonhardening case:

$$W = k\gamma \tag{11}$$

where k is the shear strength and γ is the shear strain. The shear strain can be written for a relative displacement u across the shear band of infinitesimal width δt as:

$$\gamma = \frac{u}{\delta t} \tag{12}$$

and since $dx_2 = \delta t \cos(\theta_s - \theta_f)$ we find:

$$J = \frac{ku}{\cos(\theta_s - \theta_f)} \tag{13}$$

The rate of damage accumulation can be approximated by the

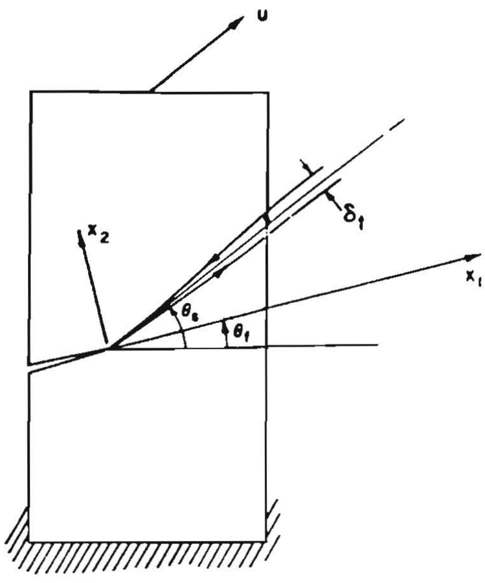

Fig. 2. *J*-integral parameters.

equation of McClintock.[5] In terms of the two largest principal stress components and the strain hardening exponent:

$$\frac{d\eta}{d\bar{\varepsilon}} = \frac{\sinh\left[(1-n)(\sigma_a+\sigma_b)/(2\bar{\sigma}/\sqrt{3})\right]}{(1-n)\ln F_l} \tag{14}$$

where F_l is the hole growth ratio (defined as the ratio of the mean hole spacing over the inital mean radius). Define F_η as the rate of damage per unit principal shear strain by:

$$F_\eta = d\eta/d\gamma = \sqrt{3}\, d\eta/d\bar{\varepsilon} \tag{15}$$

Far from the crack tip the loading of Fig. 2 gives a triaxiality $\sigma/\tau = (\sigma_a+\sigma_b)/(2\bar{\sigma}/\sqrt{3}) = 1$ for nonhardening plasticity. Closer to the tip the normal strain will tend to relax the normal stress. At the same time the shear strain will increase. Since the near tip triaxiality depends on the orientation of the crack relative to the shear band, we can take F_η to be constant depending on the (assumed constant) direction of the growing crack. Between the two limiting cases of $\sigma/\tau = 0, 1$ and for $F_l = 1{\cdot}3$ and $n = 1/13$, F_η varies between 0 and 2·52. Substitution of eqns (15) and (13)

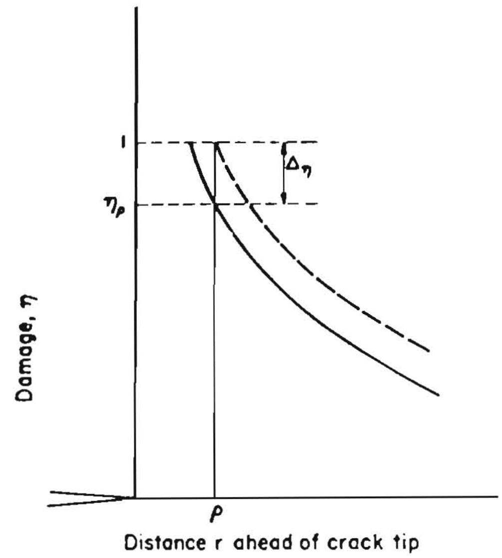

Fig. 3. Damage variation ahead of the crack.

the displacement per unit crack growth $du/d\xi$ over prior positions of the crack and the damage increment $(d\eta/dc)$ required for further growth by $dc = \rho$:

$$1 = \left[\int_0^c \frac{d\eta}{du}\frac{du}{d\xi}d\xi\right] + \left(\frac{d\eta}{dc}\right)dc$$
$$= \left[\int_0^c \frac{F_\eta F_\xi}{u_i}\left(\frac{u_i}{c+\rho-\xi}\right)^{1/(n+1)}\frac{du}{d\xi}d\xi\right] + \frac{F_\eta F_\xi}{u_i}\left(\frac{du}{dc}\right)\left(\frac{u_i}{\rho}\right)^{1/(n+1)}\rho \quad (20)$$

Taking the displacement per unit crack growth $du/d\xi$ nearly constant at du/dc (quasi-steady growth), which is a reasonable assumption since most of the damage occurred as the crack approached its current value, allows restating eqn (20) in a form which can be integrated:

$$1 = \frac{du}{dc}\frac{F_\eta F_\xi(n+1)}{nu_i}\left[-u_i^{1/(n+1)}(c+\rho-\xi)^{n/(n+1)}\Big|_0^c + \frac{n}{n+1}\left(\frac{u_i}{\rho}\right)^{1/(n+1)}\rho\right] \quad (21)$$

Introducing the crack growth length and displacement in normalized form:

$$c^* = c/\rho$$
$$u^* = u/\rho \quad (22)$$

into eqn (8) yields the damage as a function of the far-field displacement and the distance r from the crack tip:

$$\eta = \left[\frac{ku}{\sigma_1 I_{1/n}(M^p) r \cos(\theta_s - \theta_f)}\right]^{1/(n+1)} F_\eta \bar{\gamma} \tag{16}$$

with the dimensionless principal shear strain $\bar{\gamma}$:

$$\bar{\gamma} = 2(\bar{\varepsilon}_{rr}^2 + \bar{\varepsilon}_{r\theta}^2)^{1/2} \tag{17}$$

The initiation displacement and the direction of crack growth can be calculated by considering the crack advancing to the direction requiring the least far-field displacement to reach the critical damage of unity. The critical strain for fracture, achieved at one inclusion spacing $r = \rho$, can be found from the strain equation (8) together with eqn (13). A computer program has simulated the above equations for the case of a shear band at $\theta_s = 45°$ and for material characteristics $\sigma_1/k = 3$ and $n = 1/13$ and resulted in a crack growth direction θ_f of about $36{\cdot}3^0$, initiation displacement $u_i/\rho = 0{\cdot}714$ and triaxiality $\sigma/\tau = 0{\cdot}38$. The parameter F_η is thus $F_\eta = 0{\cdot}85$ for a typical hole growth ratio $F_t = 1{\cdot}3$. Our main object is nevertheless to study the growing crack, whichever its orientation is.

Consider thus the crack growing from initial length c_i through previous lengths ξ to a current length c. The damage at ρ ahead of the current crack tip is less than 1, as Fig. 3 shows, so a further displacement is required for further crack advance. The damage increment $d\eta$ for the displacement increment du across the shear band can be expressed in terms of the current crack tip $c + \rho - \xi$ for low strain hardening ($n \approx 0$) according to the following inverse power law, where, since the exponent in $u^{1/(n+1)-1}$ is close to zero, we substitute u_i for u as a first approximation:

$$\begin{aligned} d\eta &= \frac{F_\eta \bar{\gamma}}{n+1}\left[\frac{ku_i}{\sigma_1 I_{1/n}(M^p) r \cos(\theta_s - \theta_f)}\right]^{1/(n+1)} \frac{du}{u_i} \\ &= F_\eta F_\gamma \left(\frac{u_i}{c + \rho - \xi}\right)^{1/(n+1)} \frac{du}{u_i} \end{aligned} \tag{18}$$

with

$$F_\gamma = \left[\frac{k}{\sigma_1 I_{1/n}(M^p) \cos(\theta_s - \theta_f)}\right]^{1/(n+1)} \bar{\gamma} \tag{19}$$

Take as a fracture criterion that the damage reaches unity at a structural distance ρ ahead of the current crack length c. This damage is found by summing the integral of the damage increments per unit displacement and

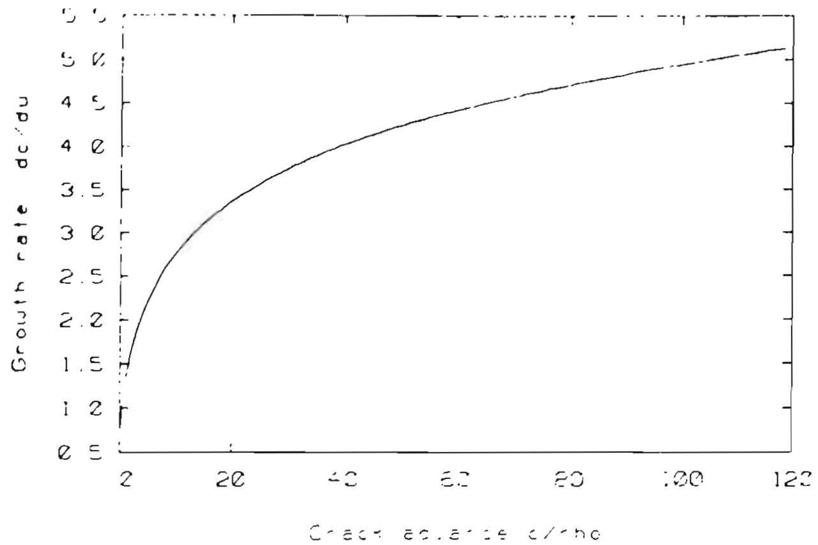

Fig. 4. Quasi-steady crack growth rate vs. crack advance.

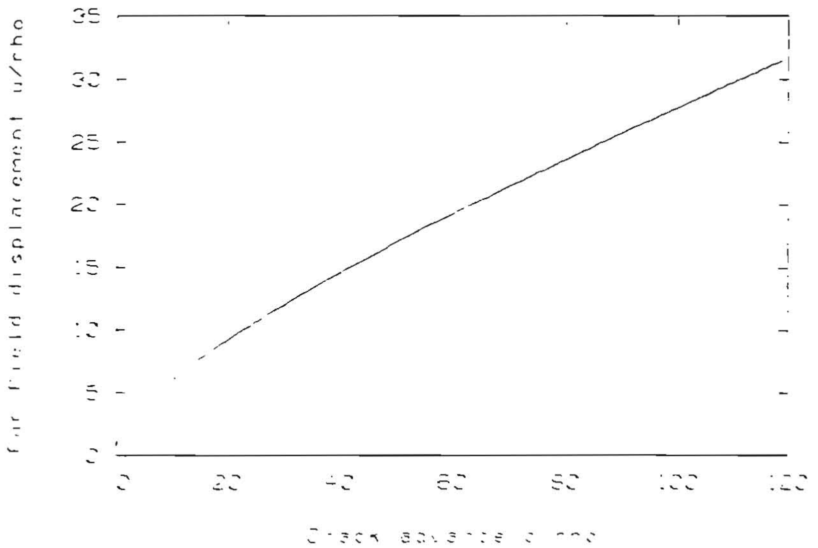

Fig. 5. Far-field displacement vs. crack advance.

we can solve eqn (21) for the crack growth rate:

$$\frac{dc^*}{du^*} = \frac{F_\eta F_\gamma (n+1)}{n} \times \left[\left(\frac{c^*+1}{u_i^*} \right)^{n/(n+1)} - (u_i^*)^{-n/(n+1)} + \left(\frac{n}{n+1} \right) (u_i^*)^{-n/(n+1)} \right] \qquad (23)$$

Thus the crack advance per unit displacement starts at $F_\eta F_\gamma (1/u_i^*)^{n/(n+1)}$ and continually increases with c^*, although at a decreasing rate, as is seen from Fig. 4 which gives the crack growth rate vs. crack advance distance for the example case of $n = 1/13$, $u_i^* = 0{\cdot}714$ and $F_\gamma = 0{\cdot}88$, $F_\eta = 0{\cdot}85$ considered here. In Fig. 5 the far-field displacement u^* vs. crack growth c^* is plotted. Equation (23) can be used to assess the size effects. For example for a prototype-specimen ratio of 100 and a ligament of $\frac{1}{2}$m in the prototype structure, the crack growth rate after growth in both cases by 1/10th of the ligament is about 2·3 times bigger in the large prototype than in the small specimen. Thus small laboratory tests can be used to derive the parameters F_η, n, F_γ, u_i/ρ, and thus produce an 'engineering' equation that can be used to estimate the displacement to cause fracture in the actual structure. Figure 6 shows the large-scale crack growth. Notice that an infinite crack growth rate is not predicted; however, instability (which

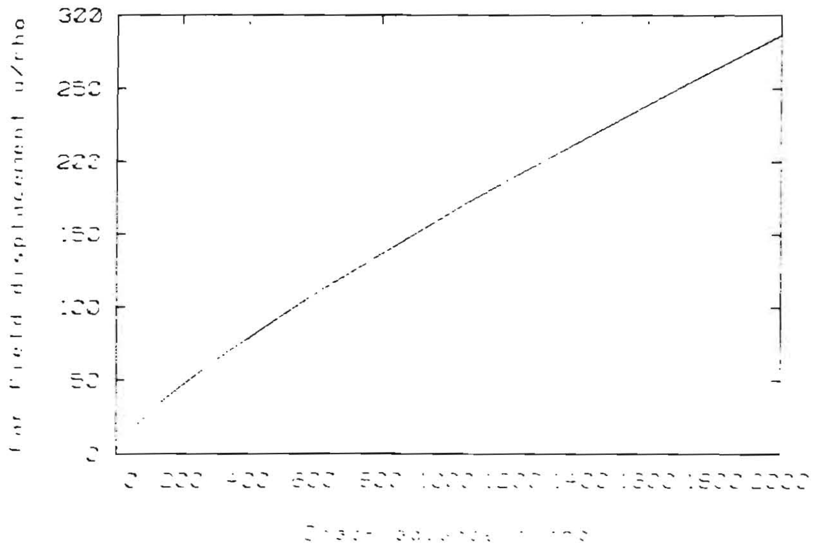

Fig. 6. Large-scale crack growth.

is mainly coupled to the surroundings) is likely to occur due to the high values of dc/du.

CONCLUSIONS

In asymmetrical singly grooved tensile specimens simulating weld defects the crack progresses into pre-strained material with less ductility than in the symmetrical case where the crack grows into new material between two shear bands. A quasi-steady model is presented to predict the crack growth by hole coalescence in regions dominated by an HRR-type strain hardening singularity.[2,3] The analysis gives the crack growth per unit displacement and the displacement as functions of the crack advance. The crack growth per unit displacement is found to increase continuously with crack growth according to eqn (23), although at a decreasing rate. The availability of such an equation allows also estimating the size effects.

ACKNOWLEDGEMENT

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