International Journal of Fracture 73: R79–R81, 1995.

ON THE FORM OF FATIGUE CRACK GROWTH FORMULAE

D.L. Steadman, R.L. Carlson and G.A. Kardomateas
School of Aerospace Engineering, Georgia Institute of Technology
Atlanta, Georgia 30332-0150, USA
Tel: (404) 894-8198; Fax: 404-894-2760

A number of fatigue crack growth prediction formulae have been proposed which require experimental data to calculate empirical constants. Unfortunately, these formulae can result in unwieldy conversion factors when changing from one system of dimensional units to another. When working with tables of experimental data, this awkward conversion can lead to numerical errors. Our objective is to propose the use of modified crack growth equations which allow the user to perform this conversion with a minimum of difficulties.

The mode I stress intensity factor defines a functional relationship containing both the loading and the geometry of the crack. The desire to formulate a general crack growth law led to the development of the Paris equation,

$$\frac{da}{dN} = C(\Delta K)^n \qquad (1)$$

This equation represents a simple relationship between the rate of crack gowth and the range of the stress intensity factor. The constants C and n are empirically derived material properties. A straight line fit on a log-log plot of experimental data leads to values for these parameters for a particular material.

The difficulties referred to are the result of the dimensions of the C parameter. The dimensions of C are the same as the term below, i.e.,

$$(da/dN)(\Delta K)^{-n}$$

In SI units, these dimensions are commonly expressed as:

$$(\text{m/cycles})(Pa\sqrt{m})^{-n}$$

Typical values of n range from slightly less than 2 to about 4.5. For example, for aluminum 7075-T6, n is equal to 2.836, from [2]. For this material, the dimensions for C are:

$$[(\text{cycles})(Pa)^{2.836}(m)^{0.418}]^{-1}$$

Different powers of n lead to different powers of the dimensional units. In a data tabulation, therefore, the dimensional units of C can differ for each material listed. When values for C are tabulated, it is not surprising that the implicit dimensional units are omitted.

The computations required for converting from one system of units to another is awkward because the dimensions of C depend on the material property n; in other words, there is not a unique conversion factor. When it is necessary to translate data values of C from one system of units to another for many materials, the conversion factor calculation can be tedious and lead to mistakes.

This problem can be eliminated through the use of dimensionless ratios; thus, the Paris equation can be written in the form

$$\frac{da}{dN} = \hat{C}\left(\frac{\Delta K}{K_0}\right)^n \tag{2}$$

where $\hat{C}=CK^n_0$, and K_0 is a constant stress intensity value, such as the critical stress intensity factor. Then $\hat{C}$ is in units of length/cycle. With this formulation, there exists a unique conversion factor from one system of measure to another which is independent of material.

Although the Paris Law equation has been accepted for crack growth prediction for many applications, it does not capture many important features of crack growth. In its linear fit to experimental data, it ignores important features at the beginning and end of the crack life. The first is that there exists a threshold ΔK below which the crack does not grow. This threshold behavior is represented by a downward turn of the da/dN curve as ΔK approaches small values. The second feature is a transition to unstable growth as the level of stress intensity approaches the fracture toughness. When the maximum stress intensity factor approaches a critical value, the crack growth rate becomes unbounded. Finally, a third feature is the growth rate change for different values of the stress ratio $R=K_{min}/K_{max}$. For a given ΔK, the growth rate tends to increase with an increase in R.

Henkener et al. [1] have described a growth equation which captures these features for use in the NASA/FLAGRO crack growth prediction program. This relationship is the modified Forman equation,

$$\frac{da}{dN} = \frac{C(1-f)^n \Delta K^n \left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{(1-R)^n \left(-\frac{\Delta K}{(1-R)K_{Ic}}\right)^q} \tag{3}$$

where C,n,p and q are empirically derived constants for each material, and f is the crack opening function for plasticity induced crack closure. With this relationship, the growth rate is zero for $\Delta K=\Delta K_{th}$, when the stress intensity is equal to its threshold value, or when the closure function f is equal to 1. In addition, the growth rate increases with increasing R ratio and becomes unbounded as ΔK approaches K_{Ic}.

This equation represents a change from the Forman equation of earlier FLAGRO codes [2]. In the current formulation, the Paris equation is corrected by a non-dimensional parameter that includes the plasticity closure, threshold, load ratio and instability effects. However, it also exhibits the same conversion shortcoming as the Paris equation. Following the procedure adopted for (2), a form of (3) can be written as:

$$\frac{da}{dN}=\frac{\hat{C}\left(\frac{\Delta K}{K_0(1-R)}\right)^n(1-f)^n\left(1-\frac{\Delta K_{th}}{\Delta K}\right)^p}{\left(1-\frac{\Delta K}{(1-R)K_{Ic}}\right)^q}$$

This form also exhibits the threshold, load ratio, plasticity and instability features, but contains only dimensionless ratios. The constant K_0 could be chosen as a reference value, which could be either a critical stress intensity factor or the threshold value of the range of stress intensity factor for R=0. With this modified formulation, the conversion between SI units in millimeters and standard English units in inches is:

$$\hat{C}_{\text{English}}=25.4\hat{C}_{SI}.$$

This conversion factor is the same for all materials.

REFERENCES

[1] J.A. Henkener, V.B. Lawrence and R.G. Forman, in *Fracture Mechanics: Twenty-Third Symposium, ASTM STP 1189* (1993) 474-497.
[2] R.G. Forman et al., *Fatigue Crack Growth Computer Program NASAFLAGRO, JCS-22267*, NASA Lyndon B. Johnson Space Center, Houston, Texas (1989).

23 November 1995