Pergamon

Int. J. Solids Structures Vol. 34, No. 4, pp. 451–459, 1997
Published by Elsevier Science Ltd
Printed in Great Britain
0020–7683/97 $17.00 + .00

PII: S0020–7683(96)00018–2

ENERGY RELEASE RATE AND STRESS INTENSITY FACTORS FOR DELAMINATED COMPOSITE LAMINATES

IZHAK SHEINMAN† and GEORGE A. KARDOMATEAS
School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia, U.S.A.

(*Received* 1 *May* 1995; *in revised form* 1 *January* 1996)

Abstract—A procedure of total energy release rate and stress intensity factors is developed for general non-homogeneous laminated composite laminates. The total energy release rate is obtained by using the *J*-integral for a one dimensional model of plane stress, plane strain and cylindrical bending. Decomposition of it into mode I and mode II, by which the mode mixity calculation is carried out, is based on the assumption of equivalent orthotropic properties through the laminate thickness. The process is straightforward and can be used as a criterion for delamination onset and growth of one dimensional structural model under general loading in the pre- and post-buckling states. Published by Elsevier Science Ltd.

INTRODUCTION

The extensive use of composites in the last decade for high performance, low weight structures motivates the need for modeling and prediction of their structural behavior. One of the most severe problems concerning laminated composites is the formation of delaminated zones which may grow under service loading. These delaminations, when compressed, may lead to a deterioration in the load carrying capacity, due to local buckling (Simitses *et al.* (1985), Shienman *et al.* (1989)) and possibly crack propagation (Chai *et al.* (1981), Sallam and Simitses (1985), Kardomateas (1990)). Local buckling in itself does not imply the ultimate load, and usually, if the delamination does not grow, the laminate is capable of carrying on in a post-buckling mode under higher loading (Sheinman and Soffer (1991)). With delamination growth, the carrying load capacity is reduced further, which detracts from the high potential of the composites. Therefore an extensive research is needed to gain insight into the factors that influence the energy release rate and the mode I and mode II stress intensity factors, which in turn control the delamination onset and growth phenomenon.

Both subjects of energy release rate and stress intensity factors have been studied by many researchers since the early 1960s. Most of them confined their research to homogeneous isotropic materials, some of them to homogeneous bimaterial or to orthotropic (Yin and Yang (1984), Suo and Hutchinson (1990), Hutchinson and Suo (1992), Kardomateas (1993) (1994)), very few to anisotropic materials (see, for example, Suo (1990)) and almost none to non-homogeneous laminated composites with mixed mode macrocracks. A comprehensive review on mixed mode cracking is given by Hutchinson and Suo (1992).

The difficulty in non-homogeneous laminated composites is due to the fact that the process is three-dimensional in character. Consequently most (if not all) papers concerning this issue use the numerical finite element analysis (see for example O'Brien (1982), (1984), Whitcomb (1984), Law (1984), Suo *et al.* (1991), Bao *et al.* (1992), and Davila and Johnson (1993)). The 3-dimensional analysis using finite elements is sensitive to the element length and, even for very simple cases, a large number of elements through the thickness are required. Furthermore, the element refinements do not guarantee the convergence and accuracy of the numerical solution due to the singular nature of the boundary layer in the crack tip neighborhood.

† On sabbatical leave at the Technicon Israel Inst. of Technology.

The total energy release rate can be computed exactly, even for non-homogeneous laminated composites. However, using it as a criterion for delamination onset and growth (like Yin and Wang (1984) or Sallam and Simitses (1985)) is limited when mixed mode is involved (see Hutchinson and Suo (1992)). The interface toughness is not a single material parameter, rather it is a function of the relative amount of mode II to mode I acting on the interface. The criterion for initiation of crack advance along the interface, when the crack tip is loaded in a mixed mode state (either for monotonic or cyclic loading—see Kardomateas *et al.* (1995)) is characterized by the mode II to mode I ratio. Hence the motivation for decomposition of the total energy release rate into its modes.

The purpose of the present work is to develop a set of new formulae for decomposition of the total energy release rate into its mode I and mode II, by which the stress intensity factors are derived. It may be used as a criterion for the onset of delamination growth in non-homogeneous laminated composites. The emphasis here is on delamination between the composite layers, rather than cracking through the layers. The procedure does not require a three-dimensional analysis and is used for any non-homogeneous laminated composite with small thickness-to-span ratio, for which it is assumed that pre-deformation plane sections and normals remain plane and normal after deformation. It is based on an exact formulation of the total energy release rate and on the assumption of orthotropic material properties through the thickness (that assumption is used by all the works of three-dimensional analysis by finite element). These formulas are straightforward and can be used as a criterion for the onset of growth in any general analytical or numerical procedure.

For developing the algorithm, the forces and moments at the crack tips are needed. They can be obtained by any analytical or numerical process (see for example Sallam and Simitses (1985) or Sheinman and Soffer (1991)). The model is confined here to a one-dimensional procedure for a laminated composite of thickness t and with possibly a parallel plane crack between the laminated layers at a distance h from the top, see Fig. 1, for which the constitutive equations in the x-y plane are required. Then, by using the J-integral, the exact total energy release rate is obtained. Next, we assume an equivalent orthotropic behavior in the x-z plane for the stress singularity asymptotic solution, yielding the energy release rate in terms of its modes. Finally, comparing it to the total energy release rate gives the stress intensity factors.

CONSTITUTIVE RELATION

Let (x, y) be the in-plane coordinates of the reference surface and z the normal coordinate (through the thickness of the delaminated composite, Fig. 1). For small thickness-to-span ratios, the Kirchhoff-Love hypothesis can be used and the stress-strain relation for each lamina j is:

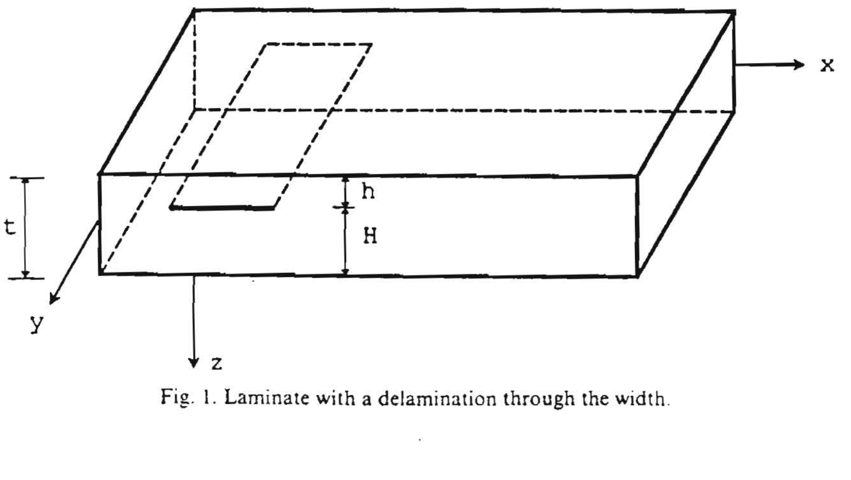

Fig. 1. Laminate with a delamination through the width.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_j = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{13} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{23} \\ \bar{Q}_{13} & \bar{Q}_{23} & \bar{Q}_{33} \end{bmatrix}_j \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_j \tag{1}$$

where

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \bar{\varepsilon}_{xx} \\ \bar{\varepsilon}_{yy} \\ \bar{\gamma}_{xy} \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ 2\kappa_{xy} \end{Bmatrix} \tag{2}$$

$\{\bar{\varepsilon}\}$ and $\{\kappa\}$ are the strain of the reference surface and the change of curvature vectors, respectively. $\bar{Q}_{ij}$ are the laminate transformed stiffnesses.

Under the classical laminate theory, the force-strain relation can be written as:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon} \\ \kappa \end{Bmatrix} \tag{3}$$

$\{N\} = \{N_{xx}, N_{yy}, N_{xy}\}$ is the membrane force vector, and $\{M\} = \{M_{xx}, M_{yy}, M_{xy}\}$ is the bending moment vector. A, B and D are matrices of order 3×3 given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int \bar{Q}_{ij}(1, z, z^2)\,dz. \tag{4}$$

Using a one-dimensional model (in the x-z plane) leaves only two dependent variables (displacement in axial (u) and normal (w) directions, respectively) and the strain and change of curvature can be given in terms of the axial force (N_{xx}) and bending moment (M_{xx}) (needed for the total energy release rate) as:

$$\begin{aligned} \bar{\varepsilon}_{xx} &= \alpha_1 N_{xx} + \alpha_2 M_{xx} \\ \kappa_{xx} &= \alpha_3 N_{xx} + \alpha_4 M_{xx} \end{aligned} \tag{5}$$

Two different one-dimensional models are considered:

(a) Classical beam model for which one can get the α coefficients directly from eqn (3) as:

$$\begin{aligned} \alpha_1 &= \frac{D_{11}}{A_{11}(D_{11} - B_{11}^2/A_{11})} \\ \alpha_2 = \alpha_3 &= \frac{-B_{11}}{A_{11}(D_{11} - B_{11}^2/A_{11})} \\ \alpha_4 &= \frac{1}{D_{11} - B_{11}^2/A_{11}}. \end{aligned} \tag{6}$$

Here $(A_{11}, B_{11}, D_{11}) = \int c_{xx}(1, z, z^2)\,dz$ where

$$\begin{aligned} c_{xx} &= \bar{Q}_{11} + \delta(c_1\bar{Q}_{12} + c_2\bar{Q}_{13}) \\ c_1 &= \frac{(\bar{Q}_{23}\bar{Q}_{13} - \bar{Q}_{33}\bar{Q}_{12})}{(\bar{Q}_{22}\bar{Q}_{33} - \bar{Q}_{23}^2)} \\ c_2 &= \frac{(\bar{Q}_{12}\bar{Q}_{23} - \bar{Q}_{22}\bar{Q}_{13})}{(\bar{Q}_{22}\bar{Q}_{33} - \bar{Q}_{23}^2)} \end{aligned} \tag{7}$$

$$\delta = \begin{cases} 0 \text{ for assumption of plate sides constrained } \varepsilon_{yy} = \gamma_{xy} = 0 \text{ (referred as plane strain)} \\ 1 \text{ for assumption of plate sides free } \sigma_{yy} = \tau_{xy} = 0 \text{ (referred as plane stress).} \end{cases}$$

It should be noted here that the constitutive equations based on $\sigma_{yy} = \tau_{xy} = 0$ for each lamina would be inconsistent. Instead, one should set the resultants N_{yy}, N_{xy} and M_{xy} to zero which is applied in the cylindrical bending theory (described next).

(b) Cylindrical bending model (see Sheinman (1989)). Here the strain and bending moment vectors can be written as:

$$\{\varepsilon\} = [a]\{N\} - [b]\{x\}$$
$$\{M\} = [b]^{\mathrm{T}}\{N\} + [d]\{x\} \tag{8}$$

where

$$[a] = [A]^{-1};\quad [b] = [A]^{-1}[B];\quad [d] = [D] - [B][A]^{-1}[B].$$

From eqn (8) one can get the following α coefficients (eqn (5)):

$$\alpha_1 = \{[I + A^{-1}B(D - BA^{-1}B)^{-1}B]A^{-1}\}_{11}$$
$$\alpha_2 = \{-A^{-1}B(D - BA^{-1}B)^{-1}\}_{11}$$
$$\alpha_3 = \{-(D - BA^{-1}B)^{-1}BA^{-1}\}_{11}$$
$$\alpha_4 = \{(D - BA^{-1}B)^{-1}\}_{11} \tag{9}$$

The subscript 11 means the first term of the matrix. It is seen that the αs are determined not only by A_{11}, B_{11}, D_{11} but also by A_{ij}, B_{ij}, D_{ij} $(i,j = 1,2,3)$ For symmetric laminated layup ($B_{ij} = 0$) with $v = 0$, the αs of eqn (9) coincide exactly with these of eqn (6), but for nonsymmetric layup ($B_{ij} \neq 0$), a significant disagreement between the approaches can be observed, depending on the stacking combination and on the orientation. In deriving the cylindrical bending model (the most suitable for representing a one dimensional laminated configuration), the effect of lamina orientation should be taken into account by using eqns (9). To illustrate the different approaches, the example of stacking combination $\alpha/0/-\alpha$, taken from Sheinman (1989), is considered here for comparing the α_1 and α_4 (for $\alpha/0/-\alpha$ $B_{11} = B_{22} = B_{12} = B_{33} = 0, B_{13}, B_{23} \neq 0$ so $\alpha_2 = \alpha_3 = 0$). α_1 and α_4 are plotted in Figs 2 and

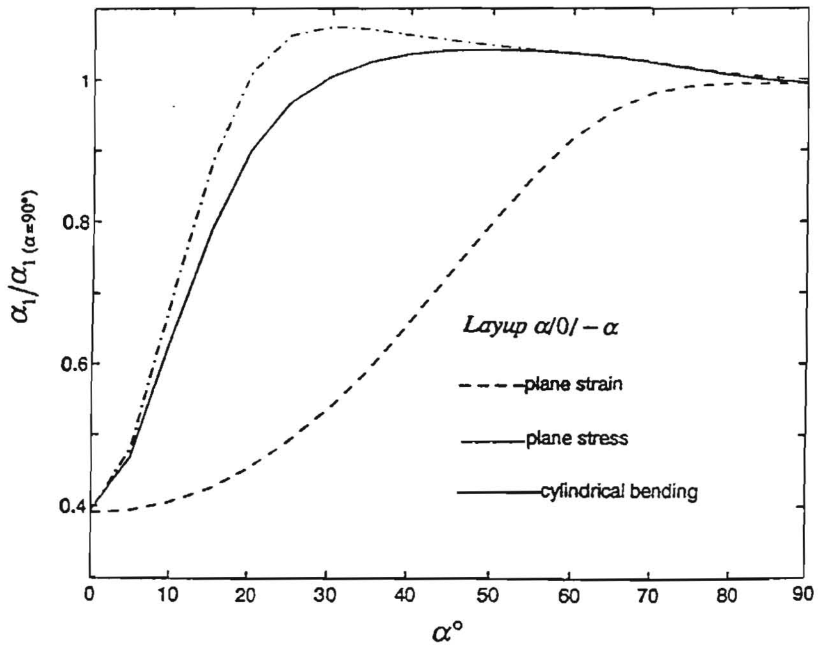

Fig. 2. Axial force coefficient α_1 vs fiber orientation α for the $\alpha/0/-\alpha$ layup.

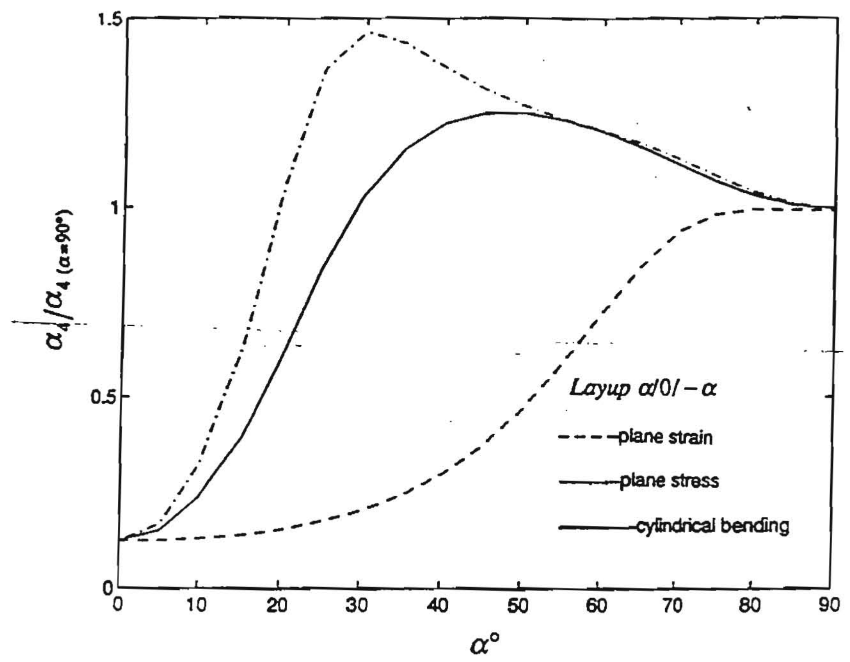

Fig. 3. Bending moment coefficient α_4 vs fiber orientation α for the $\alpha/0/-\alpha$ layup.

3, respectively, vs the orientation angle α. It is observed that the cylindrical bending model is in between the plane strain ($\varepsilon_{yy} = \gamma_{xy} = 0$) and plane stress ($\sigma_{yy} = \tau_{xy} = 0$).

ENERGY RELEASE RATE

The delamination subdivides the one-dimensional model into four regions (see Fig. 4, zr_i denotes the location of the reference surface of region i) represented by equilibrium equations, continuity requirements at the crack tips and boundary conditions at the ends. The strain energy release rate and the stress intensity factors are based on the J-integral and depend on the stress variation only. The stress itself can be based on either a linear or a nonlinear analysis. For the delaminated composite laminates under axial compression the geometrically nonlinear procedure is called for. A general geometrical procedure, based on the above constitutive relations, can be developed (see for example Sheinman and Soffer

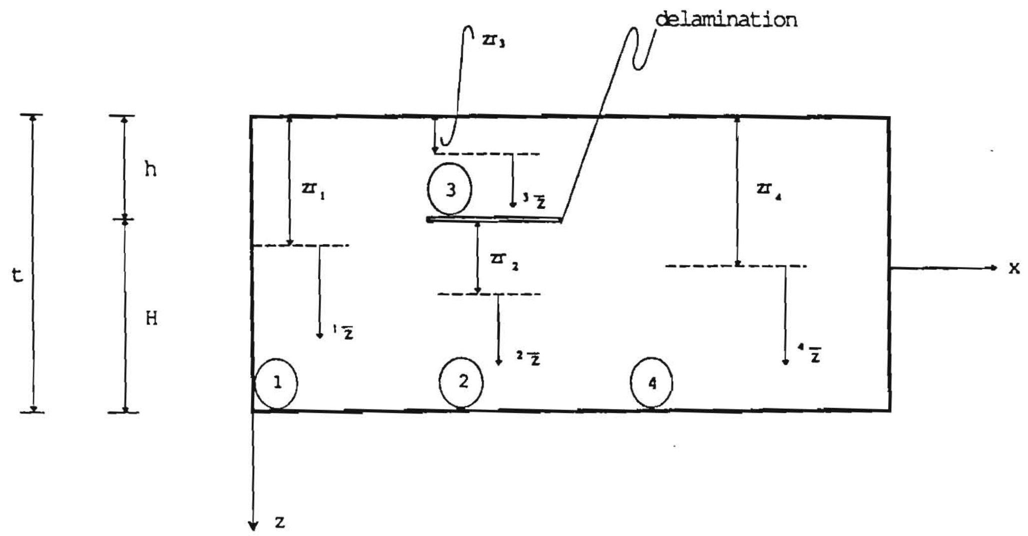

Fig. 4. Geometry and sign convention.

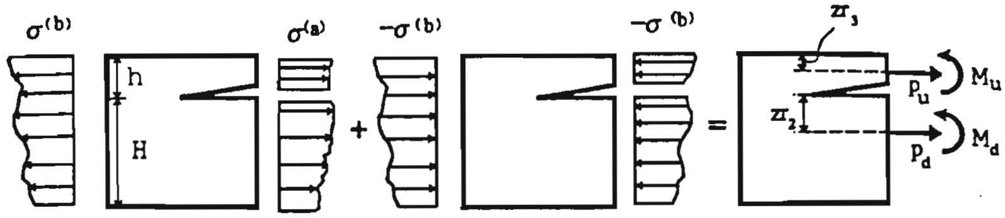

Fig. 5. Stress superposition scheme.

(1991)) and be used for the stress state before $\sigma^{(b)}$ and after $\sigma^{(a)}$ the crack tip for each load level.

For the delamination onset and growth the relevant stress resultants at the crack tips are calculated by the stress superposition as illustrated in Fig. 5 for a typical tip creating regions 2 and 3 next to region 1, where zr_2 and zr_3 are the reference surfaces of regions 2 and 3 derived from:

$$\Sigma \int_{\bar{z}} c_{xx} \bar{z} \, \mathrm{d}\bar{z} = 0, \tag{10}$$

$\bar{z}$ is the normal coordinate from the reference surface.

The stress resultants are given by

$$[P_d, M_d] = \int_H (\sigma^{(a)} - \sigma^{(b)})[1, \bar{z}] \, \mathrm{d}\bar{z}$$

$$[P_u, M_u] = \int_h (\sigma^{(a)} - \sigma^{(b)})[1, \bar{z}] \, \mathrm{d}\bar{z} \tag{11}$$

the subscripts d and u refer to the lower (d) and upper (u) regions, respectively.

P_d and M_d can also be expressed in terms of P_u and M_u by applying the equilibrium conditions:

$$P_d = -P_u$$

$$M_d = -M_u + P_u(h + zr_2 - zr_3) \tag{12}$$

The energy release rate, for the system of Fig. 5, can be computed exactly by the J-integral (Rice, 1968):

$$G = \int_\Gamma \mathrm{d}J = \frac{1}{2}\int_{\bar{z}} (\sigma^{(a)} - \sigma^{(b)})(\bar{\varepsilon}_{xx} + \bar{z}\kappa_{xx}) \, \mathrm{d}\bar{z} \tag{13}$$

Using eqn (5), the energy release rate (eqn (13)) yields:

$$G = \frac{1}{2}\left[\frac{P_u^2}{R_1} + \frac{M_u^2}{R_2} + \frac{2G_{pm}}{\sqrt{R_1 R_2}}\right] \tag{14}$$

where

$$1/R_1 = \alpha_{1d} + \alpha_{1u} - (\alpha_{2d} + \alpha_{3d})(h + zr_2 - zr_3) + \alpha_{4d}(h + zr_2 - zr_3)^2$$

$$1/R_2 = \alpha_{4d} + \alpha_{4u}$$

$$G_{pm} = P_u M_u \sin\gamma$$

$$\sin\gamma = \sqrt{R_1R_2}/2[\alpha_{2d}+\alpha_{2u}+\alpha_{3d}+\alpha_{3u}-2\alpha_{4d}(h+zr_2-zr_3)]. \tag{15}$$

Here, α_{id} and α_{iu} ($i = 1,2,3,4$) are obtained by eqn (6) (for plane stress or plane strain) or by eqn (9) (for cylindrical bending), using eqn (4) referred to the reference surfaces $\bar{z}_i$.

The energy release rate given by eqn (14) is the most general exact expression for any non-homogeneous anisotropic material. For homogeneous material (non-laminated or laminated with equal laminate properties), the coefficients are reduced exactly to the expression given by Suo and Hutchinson (1990) (see eqn 2.7 there).

Finally, the energy release rate can be written by its magnitude as:

$$\begin{aligned} G &= |G|e^{i\phi_1} \\ |G| &= \frac{1}{\sqrt{2}}\left|\frac{P_u}{\sqrt{R_1}} \mp ie^{i\gamma}\frac{M_u}{\sqrt{R_2}}\right| \\ \phi_1 &= \mathrm{arctg}\left[\frac{\mp M_u\cos\gamma}{P_u\sqrt{R_2/R_1} \pm M_u\sin\bar{\gamma}}\right] \\ \bar{\gamma} &= |\gamma| \end{aligned} \tag{16}$$

The upper sign is for the case of $G_{pm} > 0$ and the lower for $G_{pm} < 0$

STRESS INTENSITY FACTORS

The stress intensity factors are computed from the stress fields in the neighborhood of the crack tip. Since the crack is positioned in the x-z plane, the relevant stress field is analyzed on this plane. For that purpose the Kirchhoff assumptions are violated and each lamina (k) is considered as an orthotropic material for which the constitutive relation is

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{zz} \\ \tau_{xz} \end{Bmatrix}_k = \begin{bmatrix} s_{11} & s_{12} & \\ s_{12} & s_{22} & \\ & & s_{33} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{zz} \\ \gamma_{xz} \end{Bmatrix}_k \tag{17}$$

where

$$\begin{aligned} s_{11} &= \frac{c_{xx}(1-v_{xy}v_{yx})}{1-v_{xz}v_{zx}} \\ s_{22} &= \frac{E_{zz}}{1-v_{xz}v_{zx}} \\ s_{12} &= v_{xz}s_{22} = v_{zx}s_{11} \\ s_{33} &= G_{xz} \end{aligned}$$

E_{zz}, G_{xz}, v_{xz} are the modulus of elasticity, shear modulus and Poisson's ratio in the x-z domain. For most practical purposes they are assumed to be the properties of the matrix itself ($E_{zz} = E_{22}$ G_{xz}, $v_{xz} = v_{12}$) (see Kriz and Stinchcomb (1979), Whitcomb (1984)).

Each lamina may have different properties, thus $s_{ij} = s_{ij}(z)$. Using the smeared technique and assuming that the entire section of the layups maintains its equivalent orthotropic behavior† yields the properties as:

† Generally, it is an anisotropic material. However, since each lamina is an orthotropic one, for most practical cases an equivalent orthotropic properties assumption can be used, which simplifies the calculation of the stress intensity factors.

$$\bar{s}_{ij} = \frac{1}{t}\sum_k s_{ij}^k t_k \tag{18}$$

k denotes lamina sequence, t_k the lamina thickness and t the total thickness.

Following Suo (1990) (see also Sih *et al.* (1965)), the energy release rate, for an orthotropic material can be written in terms of the stress intensity of mode one (K_I) and mode two (K_{II}) as:

$$G = p_{11} n(\lambda^{-3/4} K_I^2 + \lambda^{-1/4} K_{II}^2) \tag{19}$$

where $[p] = [\bar{s}]^{-1}$ is the compliance matrix and

$$\lambda = p_{11}/p_{22}$$

$$\rho = \frac{2p_{12} + p_{33}}{2\sqrt{p_{11}p_{22}}}$$

$$n = \sqrt{\frac{1+\rho}{2}}.$$

The magnitude of the energy release rate eqn (19) can also be found from:

$$\begin{aligned} G &= |G| e^{i\phi_2} \\ |G| &= \sqrt{p_{11}n}|\lambda^{-3/8} K_I + i\lambda^{-1/8} K_{II}| \\ \phi_2 &= \mathrm{arctg}[\lambda^{1/4} K_{II}/K_I] \end{aligned} \tag{20}$$

By equating eqns (20) and (16), and bearing in mind that they can differ by phase angle shift ω, one can get the expressions for the stress intensity factors:

$$\begin{aligned} K_I &= \frac{\lambda^{3/8}}{\sqrt{2p_{11}n}}\left[\frac{P_u}{\sqrt{R_1}}\cos\omega \pm \frac{M_u}{\sqrt{R_2}}\sin(\omega+\bar{\gamma})\right] \\ K_{II} &= \frac{\lambda^{1/8}}{\sqrt{2p_{11}n}}\left[\frac{P_u}{\sqrt{R_1}}\sin\omega \mp \frac{M_u}{\sqrt{R_2}}\cos(\omega+\bar{\gamma})\right] \end{aligned} \tag{21}$$

The upper sign is for the case of $G_{pm} > 0$ and the lower for $G_{pm} < 0$. $w = \phi_2(\omega) - \phi_1$ is a function of the material and geometric parameters. It can be obtained by numerical solution of the integral equation for the anisotropic plane elasticity problem (see Suo and Hutchinson (1990)). However, one may use the approximated expression $\omega = 52.1 - 3h/H$ (in degrees) as is suggested by Suo (1990).

CONCLUSIONS

Based on the J-integral, an exact formulation for the strain energy release rate (G) of any non-homogeneous delaminated laminate is derived. Decomposition of the strain energy release rate into mode I and mode II is based on the assumption of equivalent orthotropic properties for the anisotropic material. The approach of equivalent material properties may yield better results for the stress intensity factors than using numerical three-dimensional analysis.

Acknowledgements—The financial support of the Office of Naval Research, Ship Structures S&T Division, Grant N00014-91-J-1892 and of the Army Research Office through the Georgia Tech Center of Excellence for Rotary Technology (CERT), Grant DAAL 03-93-9-0002, and the interest and encouragement of the Grant Monitors Dr Y. D. S. Rajapakse and Dr G. Anderson, are both gratefully acknowledged.

REFERENCES

Bao, G., Ho, S., Suo, Z. and Fan, B. (1992). The role of material orthotropy in fracture specimens for composites. *Int. J. Solids Structures* **29**, 1105–1116.

Chai, H., Babcock, C. D. and Knauss, W. G. (1981). One dimensional modeling of failure in laminated plates by delamination buckling. *Int. J. Solids Structures* **17**, 1069–1083.

Davila, C. G. and Johnson, E. R. (1993). Analysis of delamination initiation in post-buckling dropped-ply laminates. *AIAA* **31**, 721–727.

Hutchinson, J. W. and Suo, Z. (1992). Mixed mode cracking in layered materials. In *Advances in Applied Mechanics, Vol.* 28, Academic Press, New York.

Kardomateas, G. A. (1990). Postbuckling characteristics in delaminated Kevlar/Epoxy laminates: an experimental study. *J. Compos. Technol. Res. (ASTM)* **12**, 85–90 (1990).

Kardomateas, G. A. (1993). The initial post-buckling and growth behavior of internal delaminations in composite plates. *ASME J. Appl. Mech.* **60**, 903–910.

Kardomateas, G. A. and Pelegri, A. A. (1994). The stability of delamination growth in compressively loaded composite plates. *Int. J. Fracture* **65**, 261–276.

Kardomateas, G. A., Pelegri, A. A. and Malik, B. (1995). Growth of internal delaminations under cyclic compression in composite plates. *J. Mech. Phys. Solids* **43**, 847–868.

Kriz, R. D. and Stinchcomb, W. W. (1979). Elastic moduli of transversely isotropic graphite fibers and their composites. *J. Exp. Mech.* 41–49.

Law, G. E. (1984). A mixed mode fracture analysis of $(\pm 25/90n)_s$ Graphite/Epoxy composite laminates. In *Effects of Defects in Composite Materials, ASTM-STP* 836, American Society for Testing and Materials, pp. 160–173.

O'Brien, T. K. (1982). Characterization of delamination onset and growth in a composite laminate. *Damage in Composite Materials, ASTM STP* 775 (ed. K. L. Reifsnider), American Society for Testing and Materials, pp. 140–167.

O'Brien, T. K. (1984). Mixed mode strain-energy release rate effects on edge delamination of composites. In *Effects and Defects in Composite Materials, ASTM-STP* 836, American Society for Testing and Materials, pp. 125–172.

Rice, J. R. (1968). A path independent integral and the approximate analysis of strain concentration by notches and cracks. *ASME J. Appl. Mech.* **35**, 379–386.

Sallam, S. and Simitses, G. J. (1985). Delamination buckling and growth of flat cross-ply laminates. *Comp. Structures* 4, 361–381.

Sheinman, I., Bass, M. and Ishai, O. (1989). Effect of delamination on stability of laminated composites strip. *Comp. Structures* **11**, 227–242.

Sheinman, I. (1989). Cylindrical buckling load of laminated columns. *J. Engng Mech. ASCE* **115**, 659–661.

Sheinman, I. and Soffer, M. (1991). Post-buckling analysis of composite delaminated beams. *Int. J. Solids Structures* **27**, 636–646.

Sih, G. C., Paris, P. C. and Irwin, G. R. (1965). On crack in rectilinearly anisotropic bodies. *Int. J. Fracture Mech.* **1**, 189–203.

Simitses, G. J., Sallam, S. and Yin, W. L. (1985). Effect of delamination of axially loaded homogeneous laminated plates. *AIAA* **23**, 1437–1445.

Suo, Z. and Hutchinson, J. W. (1989). Steady state cracking in brittle substrates beneath adherent films. *Int. J. Solids Structures* **25**, 1337–1353.

Suo, Z. (1990). Delamination specimens for orthotropic materials. *ASME J. Appl. Mech.* **57**, 627–634.

Suo, Z. and Hutchinson, J. W. (1990). Interface crack between two elastic layers. *Int. J. Fracture* **43**, 1–18.

Suo, Z., Bao, G., Fan, B. and Wang, T. C. (1991). Orthotropy rescaling and implications for fracture in composite. *Int. J. Solids Structures* **28**, 235–248.

Whitcomb, J. D. (1984). *Strain-energy Release Rate Defects in Composite Materials, ASTM-STP* 836, American Society for Testing and Material, pp. 175–193.

Yin, W. L. and Wang, J. T. S. (1984). The energy-release rate in the growth of a one dimensional delamination. *ASME J. Appl. Mech.* **51**, 939–941.