Post-buckling Analysis of Multiply Delaminated Composite Plates

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This paper presents an elastic post-buckling analysis of an axially loaded beam-plate with two central across-the-width delaminations located at arbitrary depths. The analysis is based on the nonlinear beam equations, combined with the appropriate kinematic continuity and equilibrium conditions. A perturbation technique is employed, which transforms the nonlinear equations into a sequence of linear equations. An asymptotic solution of the post-buckling behavior of the plate is thus obtained. It is shown that with two delaminations, both the maximum deflection and the internal load of the first buckled (top) subplate increase as the external load increases. Of particular interest is the redistribution of load among subplates, which keeps the increase rate of internal load of the top buckled subplate much less than that of the external load. In other words, the load of the buckled subplate is close to the critical value even though the externally applied load is much larger than the critical load. In addition to the two-delamination configuration, a single delamination case is studied based on the present approach in order to verify the accuracy of the method. Also, a comparison with available finite element results is performed.

Introduction

Due to the weak interlaminar toughness of composite materials, delaminations can be easily introduced into the interface region by impact loading, fatigue, or poor manufacture. In this event, the critical load of a delaminated composite plate will be less than the corresponding pure plate without delaminations; however, buckling of the plate does not necessarily indicate its failure. In order to determine the ultimate load capacity of a delaminated composite material, the elastic post-buckling analysis of the delaminated plate is called for.

Numerous papers have been published on the prediction of the critical load of composite plates with one or multiple delaminations. However, because the nonlinearity introduced by large deformation makes the description of the post-buckling behavior extremely complex, the post-buckling behavior that ultimately governs the performance characteristics of the composite structure is not yet as well studied and understood. Kardomateas (1989, 1993) analyzed the post-buckling of composite plates with one delamination as well as the possible delamination growth behavior. An elastic buckling and post-buckling analysis of an axially loaded column with an across-the-width delamination symmetrically located at an arbitrary depth was presented by Roorda and Sakthykanapathy (1989). Also, Yin et al. (1986) investigated the ultimate axial load capacity of a plate with a single delamination. The shear buckling and post-buckling behavior of delaminated composite plates were also analyzed and the effects of delamination on shear properties were studied by Suemasu (1991). For the more complex configuration of multiple delaminations, Kutlu and Chang (1992) did some extensive investigations on the compression responses of laminate composites containing multiple delaminations, both analytically and experimentally. The compression behavior of composite panels with through-width, equal size, equally spaced multiple delaminations were also investigated analytically by Suemasu (1993) by applying the Rayleigh-Ritz approximation method. Sheinman and Soffer (1991) and Sheinman et al. (1993) developed a system composed of partial nonlinear differential equations in terms of the transverse and axial displacements to analyze the post-buckling behavior of multiply delaminated beams or long plates under cylindrical bending. A finite element method based on layerwise laminated composite plate theory was presented by Lee et al. (1995) to formulate and solve the post-buckling problem of laminated composites with delaminations.

Unlike the previously published papers, which were mostly based on finite element analysis, this paper presents an asymptotic solution for the post-buckling analysis of composite plates containing two central delaminations. It should be noted that the approach is also applicable to the case of a single delamination. Because of the nonlinearity of the post-buckling behavior, the perturbation method is introduced to transform the nonlinear problem into a linear problem. It should also be noted that good agreements were obtained between the calculation results from the present method and previously published data, both for two-delamination problems and one-delamination problems. Some interesting conclusions are drawn. The interaction among subplates is also studied and it is found that the redistribution of load among the subplates keeps the buckled subplate close to the critical load.

Formulation

The model of a delaminated plate employed in the analysis is shown in Fig. 1. A plate of half-length L, thickness T, and width W has two pre-existing central delaminations. It is assumed that the delaminations will not propagate under axial compressive load. Both ends of the plate are clamped and the external load is symmetrically applied. Because of the symmetry of the structure, only the left half can be considered. The half-plate is divided into five subplates, each having a size parameter pair (L_i, t_i), L_i and t_i being the length and thickness of the ith plate, respectively. The coordinate systems for each subplate are also shown in Fig. 1. Points A and B are the crack tips. The first, shallow (top) delamination will be denoted as delamina-

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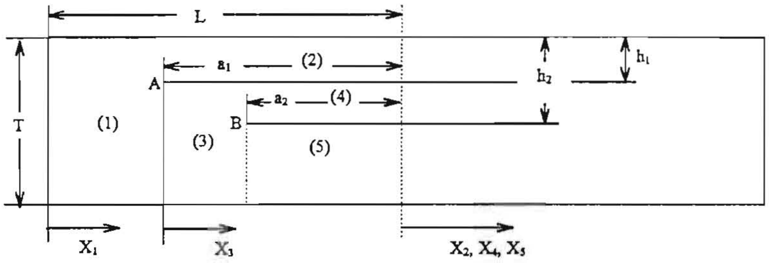

Fig. 1 Definition of the geometry for the multiple delamination problem

tion I and the second deep one as delamination II. The sizes and locations of these two delaminations are defined by their lengths a_i and depths h_i. Notice that $t_1 = T$, $t_2 = h_1$, $t_3 = T - h_1$, $t_4 = h_2 - h_1$, $t_5 = T - h_2$, and $L_1 = L - a_1$, $L_2 = a_1$, $L_3 = a_1 - a_2$ and $L_4 = L_5 = a_2$.

According to the nonlinear beam theory, the differential equations for subplates 1–5 are (e.g., Dym, 1974):

$$D_i \frac{d^2\theta_i}{dx_i^2} + P_i \sin\theta_i = 0, \qquad (1a)$$

with boundary conditions

$$\theta_i(0) = 0, \quad i = 1, 2, 4, 5. \qquad (2)$$

Notice that for $i = 2, 4, 5$, this is due to symmetry, whereas $\theta_1(0) = 0$ is due to the clamped end where D_i is the bending stiffness of the plate, $D_i = E_{11}I/(1 - \nu_{12}\nu_{21})$ for orthotropic materials.

Referring to Fig. 2, the equilibrium conditions are as follows: At the section of point A,

$$M_1 - M_2 - M_3 - P_2\frac{t_3}{2} + P_3\frac{t_2}{2} = 0, \qquad (3a)$$

$$P_2 + P_3 = P_1. \qquad (3b)$$

At the section of point B,

$$M_3 - M_4 - M_5 - P_4\frac{t_5}{2} + P_5\frac{t_4}{2} = 0, \qquad (3c)$$

$$P_4 + P_5 = P_3. \qquad (3d)$$

The kinematic continuity conditions are as follows: At point A,

$$\theta_1(L_1) = \theta_2(-L_2) = \theta_3(0) = \theta_A, \qquad (3e)$$

$$u_2^A = u_3^A. \qquad (3f)$$

At point B,

$$\theta_3(L_3) = \theta_4(-L_4) = \theta_5(-L_5) = \theta_B, \qquad (3g)$$

$$u_4^B = u_5^B. \qquad (3h)$$

In these relations $M_i = D_i d\theta_i/dx_i$, and u is the displacement along the x direction. The displacements at points A and B are due to the rotation of the cross section about the midplane and the shortening of the midplane under compression and bending, so we can write

$$u_2^A = \frac{1}{2}\int_{-L_2}^{0} \theta_2^2 dx_2 + \frac{P_2L_2}{WEt_2} + \theta_A\frac{t_2}{2}, \qquad (3i)$$

$$u_3^A = \frac{1}{2}\int_{-L_5}^{0} \theta_5^2 dx_5 + \frac{1}{2}\int_{0}^{L_3} \theta_3^2 dx_3 + \frac{P_3L_3}{WEt_3} + \frac{P_5L_5}{WEt_5} - \theta_B\frac{t_4}{2} - \theta_A\frac{t_3}{2}, \qquad (3j)$$

$$u_4^B = \frac{1}{2}\int_{-L_4}^{0} \theta_4^2 dx_4 + \frac{P_4L_4}{WEt_4} + \theta_B\frac{t_4}{2}, \qquad (3k)$$

$$u_5^B = \frac{1}{2}\int_{-L_5}^{0} \theta_5^2 dx_5 + \frac{P_5L_5}{WEt_5} + \theta_B\frac{t_5}{2}, \qquad (3l)$$

where W is the width of the plate.

Equation (1a) looks very simple; however, it is actually very difficult to solve because of the nonlinearity of the sine function. The perturbation method is a very powerful technique for solving nonlinear differential equations. In order to take advantage of the perturbation method, it is neccessary to modify Eq. (1a) into a form such that the perturbation expansion of its solution will result in a series of linear equations that can be solved iteratively. Using Taylor's formula to expand the sine function, i.e.,

$$\sin\theta = \theta - \frac{\theta^3}{6} + O(\theta^5),$$

and substituting into (1a) gives differential equations for each subplate 1–5 in the form

$$D_i \frac{d^2\theta_i}{dx^2} + P_i\left(\theta_i - \frac{\theta_i^3}{6}\right) = 0. \qquad (1b)$$

To implement the perturbation technique, a straightforward expansions of the solutions θ_i and P_i are constructed first (e.g., Bush, 1990):

$$\theta_i = \theta_i^{(0)} + \epsilon^{0.5}\theta_i^{(1)} + \epsilon^{1.5}\theta_i^{(2)} + O(\epsilon^{2.5}), \qquad (4a)$$

$$P_i = P_i^{(0)} + \epsilon^{0.5}P_i^{(1)} + \epsilon^{1.5}P_i^{(2)} + O(\epsilon^{2.5}), \qquad (4b)$$

where ϵ is a small perturbation parameter. The asymptotic sequence $\{\epsilon^{0.5}, \epsilon^{1.5}, \epsilon^{2.5}, \ldots\}$ was chosen as opposed to the more

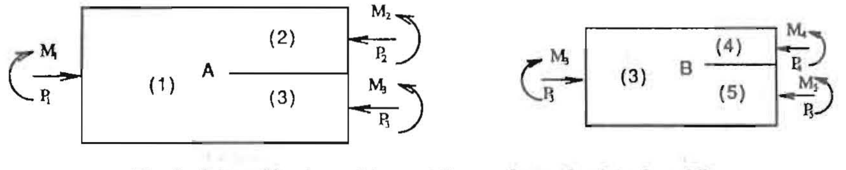

Fig. 2 Internal load conditions at the sections of points A and B

usually used $\{\epsilon^0, \epsilon^1, \epsilon^2, \ldots\}$ because it will generate the linear differential equations such that the second-order equation would include the first-order term and thus can be solved iteratively.

Now the initial position is the prebuckling state of uniform compression, so $\theta_i^{(0)} = 0$ and $P_i^{(0)} = \bar{P}t_i/T$, where $\bar{P}$ is the applied external load.

Substituting (4) into (1b) and dividing by $\epsilon^{0.5}$, yields

$$D_i \frac{d^2\theta_i^{(1)}}{dx_i^2} + (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\theta_i^{(1)} + \epsilon\Bigg[D_i \frac{d^2\theta_i^{(2)}}{dx_i^2} + (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\theta_i^{(2)} - (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\frac{\theta_i^{(1)2}}{6} + \epsilon^{0.5}P_i^{(2)}\theta_i^{(1)}\Bigg] + O(\epsilon^2) = 0. \quad (5)$$

The boundary conditions and compatibility conditions are also processed in a similar fashion. Next, two groups of linear equations are obtained by a proper grouping of the terms of powers of ϵ, based on the following.

Since ϵ is a very small number, $\{1, \epsilon^{0.5}\}$ are very large compared to $\{\epsilon, \epsilon^{1.5}\}$. In turn, $\{\epsilon, \epsilon^{1.5}\}$ are very large compared to $\{\epsilon^2, \epsilon^{2.5}\}$, etc. Therefore, starting from Eq. (5), the first-order terms are selected to be both $\{O(1)$ *and* $O(\epsilon^{0.5})\}$, and the second-order terms are both $\{O(\epsilon)$ and $O(\epsilon^{1.5})\}$, i.e., unlike the conventional asymptotic expansions, the term "first-order terms" means to include terms *up to* $\epsilon^{0.5}$ [which, again, means both $O(\epsilon^0)$ and $O(\epsilon^{0.5})$] and the term "second order terms" means to include terms up to $\epsilon^{1.5}$ (which, again, means both $O(\epsilon)$ and $O(\epsilon^{1.5})$). This methodology for constructing the asymptotic solution is essentially a matter of proper grouping of the terms, or, in other words, proper selection of what is retained, and it is necessary to achieve the solution since the traditional approach would not be successful.

Thus, the following two groups of linear equations are obtained:

First-Order, i.e., up to $O(\epsilon^{0.5})$.

$$D_i \frac{d^2\theta_i^{(1)}}{dx_i^2} + (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\,\theta_i^{(1)} = 0, \quad (6a)$$

$$\theta_i^{(1)}(0) = 0 \quad \text{for} \quad i = 1, 2, 4, 5. \quad (6b)$$

At the section of point A, by using (3e),

$$\theta_1^{(1)}(L_1) = \theta_2^{(1)}(-L_2) = \theta_3^{(1)}(0), \quad (6c)$$

and by applying (3f) and (3i, j),

$$\frac{1}{2}\int_{-L_5}^{0} \theta_5^{(1)2}dx_5 + \frac{1}{2}\int_{0}^{L_3} \theta_3^{(1)2}dx_3 + \epsilon^{-0.5}\frac{P_5^{(1)}L_5}{WEt_5} + \epsilon^{-0.5}\frac{P_3^{(1)}L_3}{WEt_3} = \frac{1}{2}\int_{-L_2}^{0} \theta_2^{(1)2}dx_2 + \epsilon^{-0.5}\frac{P_2^{(1)}L_2}{WEt_2} + \epsilon^{-0.5}\theta_5^{(1)}(-L_5)\frac{t_5}{2} + \epsilon^{-0.5}\theta_3^{(1)}(0)\frac{T}{2}, \quad (6d)$$

and by applying (3a, b),

$$P_2^{(1)} + P_3^{(1)} = 0, \quad (6e)$$

$$-D_1\frac{d\theta_1^{(1)}}{dx_1} + D_2\frac{d\theta_2^{(1)}}{dx_2} + D_3\frac{d\theta_3^{(1)}}{dx_3} = P_3^{(1)}\frac{t_2}{2} - P_2^{(1)}\frac{t_3}{2}. \quad (6f)$$

At the section of point B, by applying (3g),

$$\theta_3^{(1)}(L_3) = \theta_4^{(1)}(-L_4) = \theta_5^{(1)}(-L_5), \quad (6g)$$

and now by applying (3h) and using (3k, l),

$$\frac{1}{2}\int_{-L_5}^{0} \theta_5^{(1)2}dx_5 + \epsilon^{-0.5}\frac{P_5^{(1)}L_5}{WEt_5} = \frac{1}{2}\int_{-L_4}^{0} \theta_4^{(1)2}dx_4 + \epsilon^{-0.5}\frac{P_4^{(1)}L_4}{WEt_4} + \epsilon^{-0.5}\theta_5^{(1)}(-L_5)\frac{t_4+t_5}{2}, \quad (6h)$$

and by applying (3c, d),

$$P_4^{(1)} + P_5^{(1)} = P_3^{(1)}, \quad (6i)$$

$$-D_3\frac{d\theta_3^{(1)}}{dx_3} + D_4\frac{d\theta_4^{(1)}}{dx_4} + D_5\frac{d\theta_5^{(1)}}{dx_5} = P_5^{(1)}\frac{t_4}{2} - P_4^{(1)}\frac{t_5}{2}. \quad (6j)$$

Second order, i.e. up to $O(\epsilon^{1.5})$. Substituting (4) into (1b) gives

$$D_i \frac{d^2\theta_i^{(2)}}{dx_i^2} + (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\theta_i^{(2)} = (P_i^{(0)} + \epsilon^{0.5}P_i^{(1)})\frac{\theta_i^{(1)3}}{6} - \epsilon^{0.5}P_i^{(2)}\theta_i^{(1)}, \quad (7a)$$

$$\theta_i^{(2)}(0) = 0 \quad \text{for} \quad i = 2, 4, 5. \quad (7b)$$

At the section of point A, by applying (3e),

$$\theta_1^{(2)}(L_1) = \theta_2^{(2)}(-L_2) = \theta_3^{(2)}(0), \quad (7c)$$

and now by applying (3f) and using (3i, j),

$$\int_{-L_5}^{0} \theta_5^{(1)}\theta_5^{(2)}dx_5 + \int_{0}^{L_3} \theta_3^{(1)}\theta_3^{(2)}dx_3 + \epsilon^{-0.5}\frac{P_5^{(2)}L_5}{WEt_5} + \epsilon^{-0.5}\frac{P_3^{(2)}L_3}{WEt_3} = \int_{-L_2}^{0} \theta_2^{(1)}\theta_2^{(2)}dx_2 + \epsilon^{-0.5}\frac{P_2^{(2)}L_2}{WEt_2} + \epsilon^{-0.5}\theta_5^{(2)}(-L_5)\frac{t_4}{2} + \epsilon^{-0.5}\theta_3^{(2)}(0)\frac{T}{2}, \quad (7d)$$

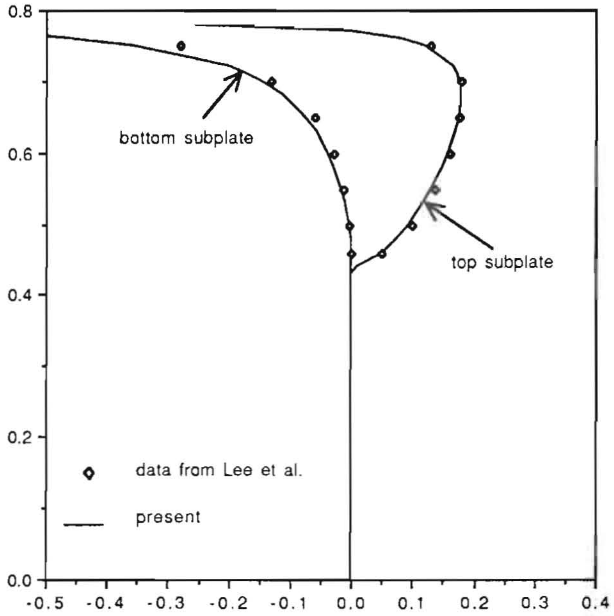

Fig. 3 Deflection-load curve of a plate with a single delamination: $\bar{h}$ = 0.2 and $\bar{a}$ = 0.3. Horizontal axis is w_i and vertical axis is $\bar{P}$.

Table 1 Post-buckling behavior of a plate with two equal-length delaminations

$\bar{P}$		0.2	0.3	0.4	0.5	0.6	0.7
Lee et al. (1995)	$\bar{W}_2$	0.06	0.13	0.18	0.20	0.22	0.21
Lee et al. (1995)	$\bar{W}_5$	0.0	−0.01	−0.02	−0.05	−0.095	−0.2
Present	$\bar{W}_2$	0.0635	0.1399	0.1838	0.2138	0.2298	0.2187
Present	$\bar{W}_5$	−0.0025	−0.0125	−0.0293	−0.0561	−0.1046	−0.2095
Present	$\bar{P}_2$	1.0088	1.0218	1.0334	1.0474	1.0679	1.1028

and by applying ($3a$, b),

$$P_2^{(2)} + P_3^{(2)} = 0, \tag{7e}$$

$$-D_1 \frac{d\theta_1^{(2)}}{dx_1} + D_2 \frac{d\theta_2^{(2)}}{dx_2} + D_3 \frac{d\theta_3^{(2)}}{dx_3} = P_3^{(2)} \frac{t_2}{2} - P_2^{(2)} \frac{t_3}{2}. \tag{7f}$$

At the section of point B, by applying ($3g$),

$$\theta_3^{(2)}(L_3) = \theta_4^{(2)}(-L_4) = \theta_5^{(2)}(-L_5), \tag{7g}$$

and now by applying ($3h$) and using ($3k$, l),

$$\int_{-L_5}^{0} \theta_5^{(1)}\theta_5^{(2)} dx_5 + \epsilon^{-0.5} \frac{P_5^{(2)} L_5}{WEt_5} = \int_{-L_4}^{0} \theta_4^{(1)}\theta_4^{(2)} dx_4 + \epsilon^{-0.5} \frac{P_4^{(2)} L_4}{WEt_4} + \epsilon^{-0.5}\theta_5^{(2)}(-L_5)\frac{t_4 + t_5}{2}, \tag{7h}$$

and, finally, by applying ($3c$, d),

$$P_4^{(2)} + P_5^{(2)} = P_3^{(2)}, \tag{7i}$$

$$-D_3 \frac{d\theta_3^{(2)}}{dx_3} + D_4 \frac{d\theta_4^{(2)}}{dx_4} + D_5 \frac{d\theta_5^{(2)}}{dx_5} = P_5^{(2)} \frac{t_4}{2} - P_4^{(2)} \frac{t_5}{2}. \tag{7j}$$

The general solution for the first-order differential Eq. ($6a$) is

$$\theta_i^{(1)} = A_i^{(1)} \sin \lambda_i x_i + B_i^{(1)} \cos \lambda_i x_i, \tag{8a}$$

with

$$\lambda_i = \sqrt{(P_i^{(0)} + \epsilon^{0.5} P_i^{(1)})/D_i}, \tag{8b}$$

where $A_i^{(1)}$, $B_i^{(1)}$, $i = 1, 2, 3, 4, 5$ and $P_i^{(1)}$, $i = 2, 3, 4, 5$ are unknowns; i.e., 14 unknowns. $P_1^{(1)} = 0$ because $P_1 = \bar{P}$ is a constant. Substituting (8) into boundary condition ($6b$), the number of unknowns reduces to ten. Solving the ten compatibility condition equations ($6c-j$) by the Newton's continuation method, allows determining $\theta_i^{(1)}$.

To solve the second-order system of equations, substitute $\theta_i^{(1)}$ from ($8a$) into Eq. ($7a$), then the general solutions for Eq. ($7a$) are

$$\theta_i^{(2)} = A_i^{(2)} \sin \lambda_i x_i + B_i^{(2)} \cos \lambda_i x_i - \frac{C_i^{(1)}}{2\lambda_i} x_i \cos \lambda_i x_i + \frac{C_i^{(2)}}{2\lambda_i} x_i \sin \lambda_i x_i - \frac{C_i^{(3)}}{8\lambda_i^2} \sin 3\lambda_i x_i - \frac{C_i^{(4)}}{8\lambda_i^2} \cos 3\lambda_i x_i \tag{9a}$$

where

$$C_i^{(1)} = \frac{\lambda_i^2 A_i^{(1)}}{8} (A_i^{(1)2} + B_i^{(1)2}) - \epsilon^{0.5} A_i^{(1)} \frac{P_i^{(2)}}{D_i}, \tag{9b}$$

$$C_i^{(2)} = \frac{\lambda_i^2 B_i^{(1)}}{8} (A_i^{(1)2} + B_i^{(1)2}) - \epsilon^{0.5} B_i^{(1)} \frac{P_i^{(2)}}{D_i}, \tag{9c}$$

and

$$C_i^{(3)} = \frac{\lambda_i^2 A_i^{(1)}}{24} (3B_i^{(1)2} - A_i^{(1)2}), \tag{9d}$$

$$C_i^{(4)} = \frac{\lambda_i^2 B_i^{(1)}}{24} (B_i^{(1)2} - 3A_i^{(1)2}). \tag{9e}$$

So again we have 14 unknowns. Introducing Eq. ($9a$) into ($7b$) and ($7c-j$), the 14 unknowns are easy to calculate because all the equations are linear. Then the asymptotic solutions for the post-buckling deformation quantities are expressed as

$$\theta_i = \epsilon^{0.5}\theta_i^{(1)} + \epsilon^{1.5}\theta_i^{(2)} + O(\epsilon^{2.5}). \tag{10}$$

Although $\theta_i^{(1)}$ and $\theta_i^{(2)}$ are functions of ϵ as $\epsilon^{0.5}$ appears in both the first and second-order equations, θ_i is not a function of ϵ as long as ϵ is chosen to be a small value.

Finally, the deflection $W(x)$ can be obtained by integrating Eq. (10) and using boundary conditions and compatibility conditions, i.e.,

$$W_1(0) = 0; \quad W_2(-L_2) = W_3(0) = W_1(L_1), \tag{11a}$$

$$W_4(-L_4) = W_5(-L_5) = W_3(L_3). \tag{11b}$$

Results

A simple computer code was written according to the preceding theory. Delaminations of different sizes and locations were investigated. The calculation results are presented both in tables and graphs. The parameters are nondimensionalized as

$$\bar{h}_i = h_i/T; \quad \bar{a}_i = a_i/L; \quad \bar{P} = P/P_{cr}^0; \quad \bar{W}_i = W_i/T.$$

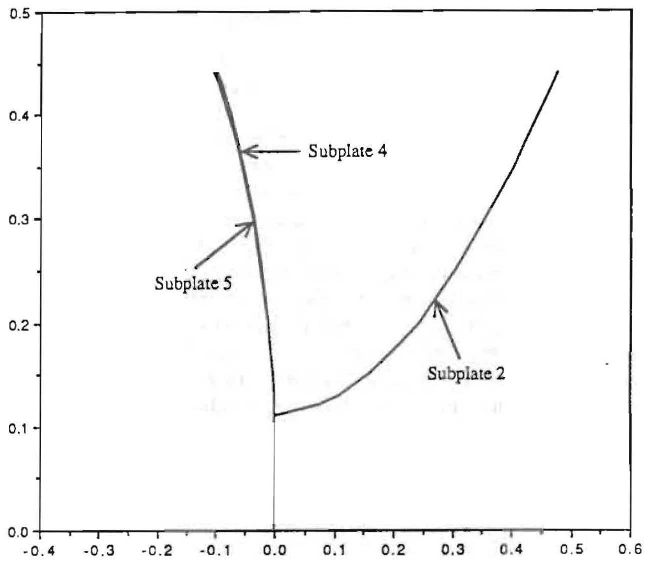

Fig. 4 Load-deflection curve for a two-delamination configuration, in which delamination I is long

Notice that P_{cr}^0 is the critical load of the plate without delamination and W_i represents the maximum deflection of the ith subplate with the upward direction as its positive direction. In Figs. 3, 4, and 5, the horizontal axis is $\bar{w}_i$ and the vertical axis is $\bar{P}$.

Single Delamination. To verify the accuracy of the present method, an example employed by Lee et al. (1995) is revisited. The material being used is T300/5208 graphite/epoxy with typical material parameters E_{11} = 181 Gpa, E_{22} = 10.3 Gpa, $G_{12} = G_{13}$ = 7.17 Gpa, and $\nu_{12} = \nu_{13}$ = 0.28. A plate containing a single through-the-width delamination at its center with $\bar{l}$ = 0.2 and $\bar{a}$ = 0.3 is considered. The thickness-to-span ratio is assumed to be T/L = 1/400. Figure 3 presents the calculation results as well as the plotted data from finite element study in Fig. 3 (Lee et al., 1995), which shows that they are in good agreement.

Two Delaminations. To further verify this method, a two-delamination example by Lee et al. (1995) is reconsidered. The example considers two delaminations symmetrically located with respect to midplane and both delaminations are assumed to have the same length. The material properties of the composite material in the study are as follows: E_{11} = 10.9 Gpa, E_{22} = 7.58 Gpa, $G_{12} = G_{13}$ = 2.48 Gpa, ν_{21} = 0.31, ν_{13} = 0.22. The post-buckling analysis results for $\bar{h}_1$ = 0.125, $\bar{h}_2$ = 0.75 and $\bar{a}_1 = \bar{a}_2$ = 0.3 are listed in Table 1, as well as data extracted from a figure presented by Lee et al. (1995). In this example, the second delamination remains closed. The composite plate behaves as if it contains the upper delamination only. Another interesting observation is the load redistribution among subplates. $\bar{P}_2$ in Table 1 is the ratio between the current axial load in subplate 2 and its critical load. It is clearly shown in Table 1 that the increase of $\bar{P}_2$ is not proportional to that of $\bar{P}$. This is because the stiffness of subplate 2 will decrease when it buckles and the load is distributed through the thickness of the plate in relation to the thickness and stiffness properties of each subplate. As a result, the load redistribution keeps the load of the buckled subplate at a level close to the critical load.

A delaminated plate has a variety of buckling modes, including the global mode, which does not involve lamina separation, the local mode, which involves buckling of the surface subplate only (thin film mode), and the general mode, which is the coupled mode of the global mode and the local mode. It has been mentioned by other researchers that a plate with long delaminations behaves differently from that with short delaminations. To investigate the effect of delamination size on the post-buckling behavior of plates, two configurations involving two delaminations are considered: (1) long delamination: $\bar{h}_1$ = 0.2, $\bar{h}_2$ = 0.5, $\bar{a}_1$ = 0.6 and $\bar{a}_2$ = 0.3; (2) short delamination: $\bar{h}_1$ = 0.2, $\bar{h}_2$ = 0.5, $\bar{a}_1$ = 0.3, and $\bar{a}_2$ = 0.3. Figures 4 and 5 are the load-deflection curves of these two configurations, respectively. For the long delamination case, subplate 2 buckles first when the external load $\bar{P}$ is larger than the critical load $\bar{P}_{cr}$ = 0.1103. As the external load $\bar{P}$ increases, the maximum deflection of subplate 2 increases, too. However, the maximum deflections of subplate 4 and 5 are kept at a small level. In fact, the whole structure stays in local buckling mode even when the external load reaches the second eigenvalue. In contrast, the plate with short delamination behaves differently. At the initial post-buckling state, the plate behaves in a local buckling mode. However, the maximum deflections of subplates (4) and (5) become large as $\bar{P}$ increases further, which indicates a switch to a global buckling mode. When $\bar{P}$ reaches 0.71, the global

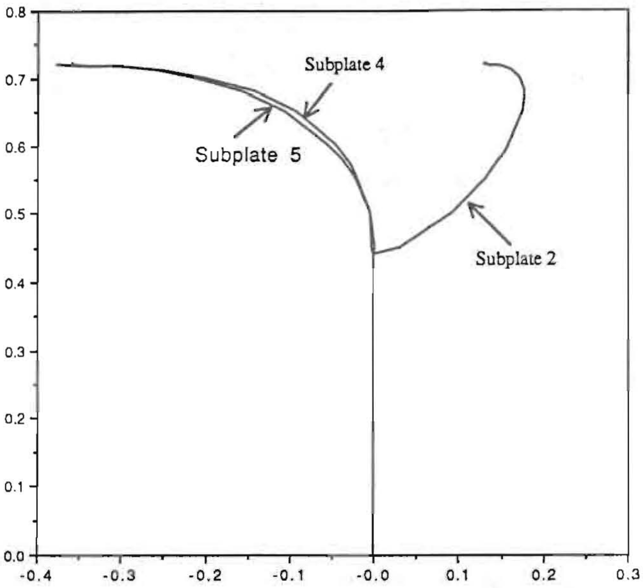

Fig. 5 Load-deflection curve for a two-delamination configuration, in which delamination I is short

buckling mode becomes dominant. For a small increment of external load, that is $\bar{P}$ going from 0.71 to 0.72, the deflection increases abruptly and the whole structure collapses.

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