Statistical Considerations in the Analysis of Data From Fatigue Tests on Delaminated Cross-Ply Graphite/Epoxy Composites

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The objective of this paper is to analyze the results of compressive fatigue experiments performed on a set of delaminated Graphite/Epoxy cross-ply composites. Crack branching, the failure mode we are interested in, occurred during the tests. Due to scatter, it is somewhat difficult to draw conclusions on the values of the branching angles, the key parameter of the problem, unless the tools of statistical and exploratory data analysis (EDA) are used. Here, a brief discussion on some of these techniques is presented, and their application to the set of obtained test data is carried out. The results seem to indicate that the crack grows faster when it is not self-similar, with a higher rate of growth for cracks that branch with a greater angle out of the interface.
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1 Introduction

Crack branching in composites is a phenomenon still not fully understood. The limited body of experimental data does not allow a suitable correlation with analyses and/or predictions. A group of data under static and fatigue loading has been obtained by the authors and is described in a previous work [1]. New data has been acquired, and used in the present study. The objective of the paper is to analyze a set of this fatigue data (namely, the branching angles and the total crack growth observed at a given cycle), with the aid of statistics and exploratory data analyses. The purpose is gaining some insight that will assist in future research work. The analysis is not ideal and is not meant to be a complete statistical assessment of data but rather a first attempt to include these important statistical components into the interpretation of experimental results. Future work will target a more efficient design of experiments and use of the tools described in the paper, toward a better understanding of the crack branching phenomenon.

2 Test Specimens

Twelve specimens of T7G145/F1914 Graphite/Epoxy (donated by Hexcel Corporation, CA) have been tested under fatigue loading in compression. The specimens' lay-up is $[(0/90)_7,0]$, with average dimensions of the laminates as shown in Fig. 1. The original delamination was created by an unetched and unperforated Teflon film inserted in each plate during manual lay-up. It was located between the 4th and the 5th plies, in this 15-ply configuration. The motivation and the details of the fatigue tests for the first eight specimens have been described in [1], as well as some of the experimental problems. The last four tests are new, and will be illustrated in this paper.

The specimens were directly clamped in the flat-faced grips of an Instron 8501 dynamic testing machine and subjected to static loading. Buckling was reached, leading to the opening of the delamination. Fatigue tests were then performed at a frequency of 5 Hz, with a sine wave of amplitude 0.0381 mm, in displacement control. The buckling configuration was symmetric (Fig. 2). Crack growth could occur on either side of the delamination (indicated in Fig. 2 as *upper crack* and *lower crack*). The expression *marker* indicates the fact that a marker line was drawn on both sides of the original delamination before the testing.

Figures 3–6 show branching (or kinking), that occurs when a crack, growing along a given interface, moves to an adjacent interface, in the same ply or in an adjacent ply. The initial crack that grows from the delamination starter is identified as ''primary delamination.'' A ''secondary delamination'' is created as the crack branches and turns in the direction parallel to the direction of the starter. Figure 3 shows the first branching for specimen 10, described later in the paper (in Table 1). The secondary delamination is closer to the edge of the specimen. Figure 4 shows subsequent branching for the same specimen, with the interface moving away from the edge. In Figs. 3 and 4, the plies are not visible as the specimen was coated with white paint to improve visualization of the crack during the test. The same surface has been cleaned after the test and prepared for a different microscope. The crack and the layers in specimen 10 are visible in Figs. 5 and 6.

The branching (or kink) angle is measured with respect to the direction of the original delamination, and is positive if counterclockwise. All measurements have been taken when the specimens were in the testing machine.

Each of the twelve specimens is characterized by what will be

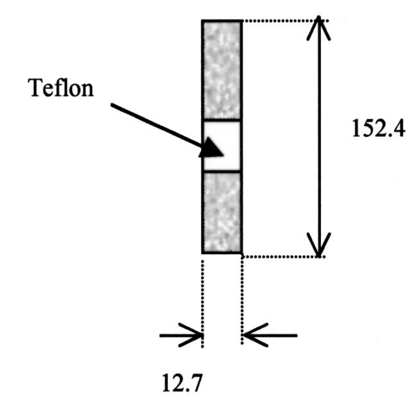

Fig. 1 Specimen's geometry (dimensions are in mm)

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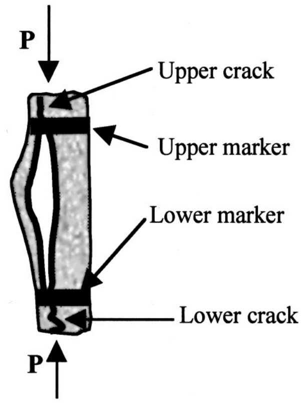

Fig. 2 Buckling of the specimen under loading

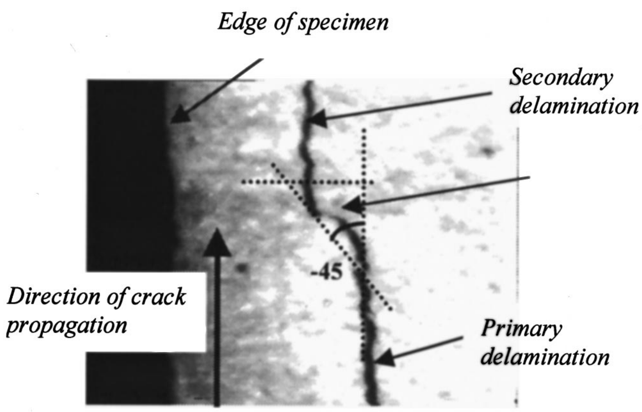

Fig. 3 First branching in specimen 10 (Table 1)

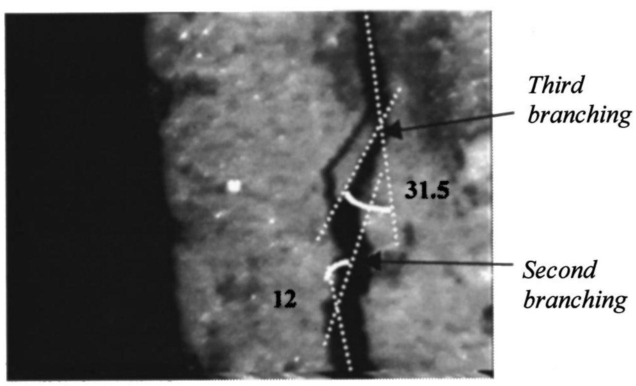

Fig. 4 Second and third branching in specimen 10 (Table 1)

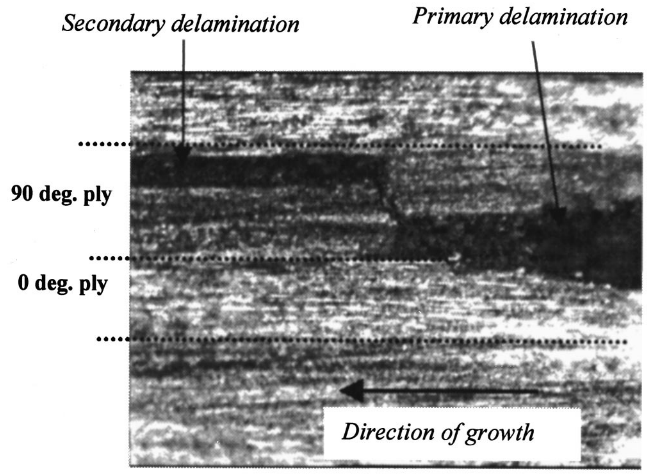

Fig. 5 First branching in specimen 10

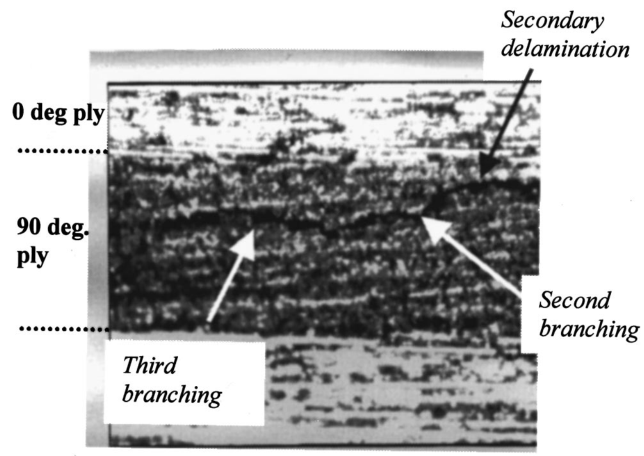

Fig. 6 Second and third branching in specimen 10

defined here for convenience *length ratio*: the ratio of the original delamination a_0 to the free length of the plate L (the length of the specimen outside the grips).

In Table 1, the following data are given:

(a) length ratio a_0/L (length ratio);
(b) kink angle for each side of the delamination (*kink angle (side 1), kink angle (side 2)*);
(c) number of cycles at which branching was observed (*cycles kink*);
(d) total number of cycles (*total cycles*).

In this paper, there will be no consideration for whether growth occurred on the upper part or on the lower part of the original delamination. Therefore *Side 1* and *Side 2* (Table 1) are left unspecified.

Branching occurs generally in the first 100,000 cycles. The absolute value of the branching angles varies between 12 deg and 90 deg. Branching occurs on one side of the initial delamination, or on both, or on neither. Out of 12 experiments, only 5 presented self-similar crack growth, with no kinking.

Table 2 reports the total crack growth a, normalized with respect to the initial crack length a_0. The length a is measured as projection with respect to the direction of the delamination starter. The growth is reported at a given number of cycles, indicated in the ''*Cycles*'' column. To be able to draw conclusions from different data, it has been decided to refer to the crack growth around 100,000 cycles for each specimen. In case this was not possible (due to an earlier end of the test or to the way data have been collected), the values mentioned are for a later number of cycles. The absolute values of the branching angles are reported as well, in bold; the numbers not in bold are the normalized crack lengths. Finally, the column ''percent *change final* a/a_0'' shows the increase in relative crack length at the end of the test, with respect to the data around 100,000 cycles. For example, referring to Table 2, changes of ''14 (side 1)'' and ''6.7 (side 2)'' for specimen number 4 indicate that the final crack lengths (normalized) were respectively 14 percent greater than *0.06091* for side 1 and 6.6 percent greater than *0.06331* for side 2.

Table 1 Results of the fatigue experiments of the sample of 12 specimens. Specimens 1–8 are described in [1]; specimens 9–12 have been tested later on.

Specimen #	Total cycles	Length ratio, a_0/L	Kink angle (side 1)	Kink angle (side 2)	Cycles kink
1	169,000	0.416	*No kink*	*No kink*	*No kink*
2	48,483[a]	0.419		38.7	12,483
			No kink	53.1	27,483
3	110,000	0.422	*No kink*	*No kink*	*No kink*
4	142,858	0.430	−25.2		36,201
				29.3	84,070
5	100,000	0.436	51.2		11,200
				−50.9	83,100
6	110,000	0.484	*No kink*	63.4	31,411
7	100,266	0.5	33.7	*No kink*	80,266
8	25,267[b]	0.569	−59.3,45,−21.2,39.8	90	25,007
				46.5,36	25,267
9	250,000	0.5948	*No kink*	*No kink*	*No kink*
10	200,000	0.4921	−45		16,062
			12,31.5	*No kink*	70,600
11	200,000	0.4655	*No kink*	*No kink*	*No kink*
12	200,000	0.4922	*No kink*	*No kink*	*No kink*

[a]After 48,483 cycles, cycling was stopped to take some replicas. The crack grew under static loading and the experiment ended earlier. The data mentioned in the table refer to the last values of crack growth at the end of the fatigue loading, before application of the replicas.
[b]Crack growth was unstable and the experiment ended earlier.

Table 2 Report of (a) crack growth, normalized with respect to the initial delamination, and calculated as projection with respect to the direction of the delamination starter; (b) number of cycles at which the crack growth is observed; (c) change of relative crack growth by the end of the test. The absolute values of the branching angles are in bold.

Specimen #	Side 1 (deg, a/a_0)	Side 2 (deg, a/a_0)	Cycles	Percent change final a/a_0
1	**No kink**, 0.04361	**No kink**, no growth	107,900	0.
2	**No kink**, no growth	**38.7, 53.1**, 0.4965	48,483	0.
3	**No kink**, no growth	**No kink**, 0.1063	110,000	0.
4	**25.2**, 0.06091	**29.3**, 0.06331	109,400	14 (side 1), 6.7 (side 2)
5	**51.2**, 0.06252	**50.9**, 0.1499	100,000	0.
6	**No kink**, no growth	**63.4**, 0.1210	110,000	0.
7	**33.7**, 0.0422	**No kink**, no growth	100,266	0.
8	**59.3, 45, 21.2, 39.8** 0.3800	**90, 46.5, 36** 0.1925	25,267	0.
9	**No kink**, 0.04410	**No kink**, no growth	111,500	0.
10	**45, 12, 31.5**, 0.1232	**No kink**, 7.819e-3	100,000	2.2 (side 1), 71 (side 2)
11	**No kink**, 0.04373	**No kink**, no growth	121,404	0.
12	**No kink**, 0.03830	**No kink**, no growth	147,881	16.5

On the average, crack growth slows down, for cracks that branched in the first 100,000 cycles. A self-similar crack seems to grow at a higher rate after the first 100,000 cycles, with respect to a non-self-similar crack.

Analysis of the data in Tables 1 and 2 will be next performed using statistics. First, there will be an assessment about whether the data can be represented by a normal distribution, by using the *Chi-square Goodness-of-Fit* test. Second, we will look for understanding the trend of the branching angles with respect to crack lengths, by means of the tools of Exploratory Data Analysis.

3 Statistical Considerations

Any experimenter will eventually have to face the problem of scatter of data. Scatter is even more likely to appear in the case of testing on composites, as the local properties can distinctly affect the macroscopic behavior, and in fatigue tests more than in static tests. Statistics can be used to analyze data subject to random errors. Systematic errors are more difficult to detect and statistics does not deal with them. A review of some basic concepts follows.

A random variable is the outcome of a random event. Three parameters, used to describe a sample of random variables $\underline{X_i}$, are the *mean*, $\bar{X}=1/n\Sigma_{i=1}^{n}X_i$, *the variance*, $S^2=(\Sigma_{i=1}^{n}(X-\bar{X}))/(n-1)$, and the *standard deviation*, *S*.

Normal distributions (also called *Gaussian*) are always referred to when it is necessary to analyze random variables. This is due to the fundamental Central Limit Theorem [2–4]: in most cases, a set of n random variables tends to a normal distribution (or to a lognormal[1] distribution) as n increases. Therefore, we can say that we can ''adopt'' the normal distribution as ''parent distribution'' for a sample of n random variables. The mean, variance, and standard deviation of a normal distribution are indicated in the literature with Greek letters, as μ, σ^2, σ. The distribution function $F(x)$ is given by

$$F(x)=P(X\leqslant x)=\int_{-\infty}^{x}\frac{1}{\sigma\sqrt{2\pi}}e^{-(t-\mu)^2/2\sigma^2}dt \qquad (1)$$

where $P(X\leqslant x)$ is the probability for the variable X to be less than a given value x. The expression (1) is solved numerically through a transformation of variables and the so-called *cumulative normal distribution* function, Φ, whose values are given in tables (e.g., in [3]) or stored in subroutines (like the ones in MATLAB Statistics toolbox)

$$F(x)=P(X\leqslant x)=P\left(Z\leqslant\frac{x-\mu}{\sigma}\right)=\Phi(z) \qquad (2)$$

In particular,

$$P(X_1\leqslant X\leqslant X_2)=P(Z_1\leqslant Z\leqslant Z_2)=\Phi(Z_2)-\Phi(Z_1) \qquad (3)$$

[1]The Naperian logarithm of the random variable follows the normal distribution.

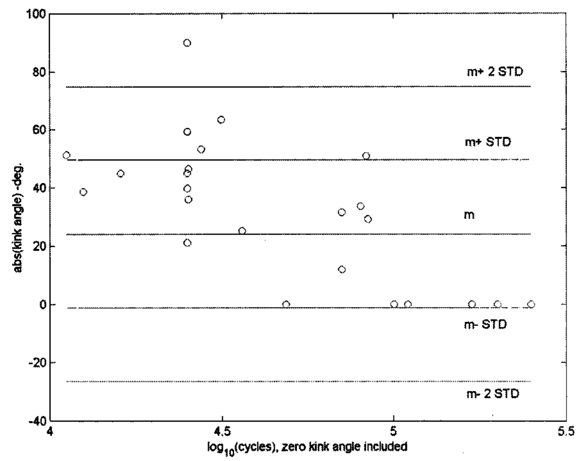

Fig. 7 Absolute value of the kink angle (in deg) versus the $\log_{10}$ of the number of cycles. Zero branching angle is included. m=mean STD=standard deviation.

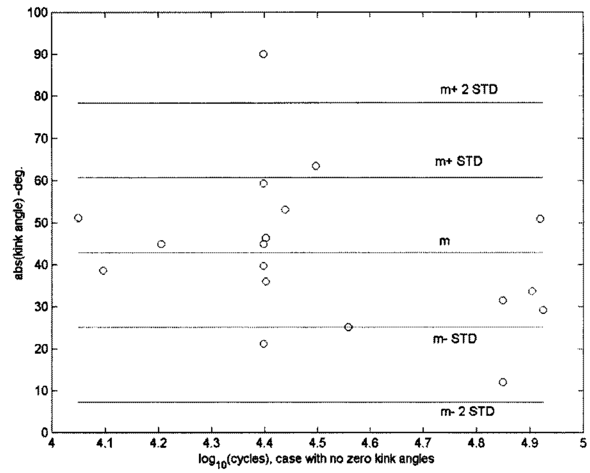

Fig. 8 Absolute value of the kink angle (in deg) versus the $\log_{10}$ of the number of cycles. Zero branching is not included in the calculations. m=mean, STD=standard deviation.

A normal distribution is such that [4]

(a) the probability for a data point to fall within one standard deviation of the mean is 68 percent;

(b) the probability for a data point to fall within two standard deviations is 95 percent;

(c) the probability for a data point to be farther away from the mean than three standard deviations is 0.3 percent.

Figure 7 refers to the data in Table 1. The logarithm in base 10 has been typically chosen for representing the number of cycles, on the *x*-axis. On the *y*-axis, the absolute value of the branching angle has been plotted. The absolute value has been chosen as we are interested in the magnitude of the branching angle, not in how it has been measured with respect to a reference line. The ''*0*'' value indicates no branching. These data have been plotted in correspondence of the total number of cycles for the given specimen, according to the idea that a zero branching angle was observed at that count. Also, there is no distinction between Side 1 and Side 2 in the plot. What matters here are the frequency of a given angle and the number of cycles at which the event is noticed. The mean $\bar{X}$ of the sample has been calculated considering the absolute values of all the angles, and is equal to *24.12* deg. The standard deviation of the sample is *25.31* deg.

As it can be seen, only one data point, that corresponds to an angle of *90* deg has a distance from the mean greater than two standard deviations. The overall distribution of the data points seems well behaved except for the magnitude of the standard deviation.

Figure 8 gives the distribution of data in the case in which the zero branching angles are not included in the calculation: the mean is *42.88* deg and the standard deviation is *17.78* deg, much smaller than in the case shown in Fig. 7. The angle of *90* deg is again the outlier data in the distribution.

Still, we wonder about how close this sample of specimens is to the normal distribution, even if we can rely on the Central Limit Theorem. The answer is given by the so-called *Chi-square Goodness-of-Fit* test. The test is based on measuring the difference between the frequency of an event of the given sample and the frequency that the event would have if it belonged to the parent distribution we want to ''adopt'' (in this case, the normal distribution).

The *n* data are subdivided in *k* classes. The quantity calculated for the test is

$$\chi_0^2=\sum_{i=1}^{k}\frac{(O_i-E_i)^2}{E_i} \tag{4}$$

O_i is the frequency of a given event in the *i*th class. E_i is the expected frequency of the event in the *i*th class if it belongs to the parent distribution. Once (4) is calculated, it is compared with the values of the *Chi-square* distribution related to a given confidence, $(1-\alpha)\times 100$ percent, and to *k-p-1* degrees of freedom[2] $\chi^2_{\alpha,k-p-1}$. If $\chi_0^2<\chi^2_{\alpha,k-p-1}$, we can be $(1-\alpha)\times 100$ percent confident that the given sample conforms to that parent distribution. The values of $\chi^2_{\alpha,k-p-1}$ are generally tabulated (e.g. [2,3]). In our case, seven intervals are considered for the absolute value of the branching angles: [*0,20*], [*21,30*], [*31,40*], [*41,50*], [*51,60*], [*61,70*], [*71,90*] deg. The calculated frequency O_i for our sample is given by the ratio of the number of times that a given angle is in an interval, and the total number of events. For example, the case of ''no branching'' (i.e., *0* degree) is observed *14* times over a total number of *32* times, so the calculated frequency is *14/32*. For the calculation of the expected frequency E_i, we considered a normal distribution with mean equal to *24.12* deg and standard deviation *25.31* deg corresponding to the case in which zero branching angles are included in the calculations (that is also the distribution with greater standard deviation). The frequencies were obtained by expression (3), the tables of the cumulative normal distribution function in [3] and the Statistics Toolbox in MATLAB.

The results are given in Table 3.

By expression (4), we obtain $\chi_0^2=0.2332$. The value of $\chi^2_{\alpha,k-p-1}$ for $k=7$ intervals, $p=2$ parameters assigned to the distribution (the mean and the standard deviation) and a 95 percent confidence is $\chi_{0.05,4}=9.49$. Since $0.2332<9.49$, we are 95 percent confident that our distribution conforms to a normal distribution (for the case with a greater standard deviation, the most conser-

[2]For more details, refer to [2–4] or other books of statistics. ''*p*'' is the number of parameters assigned to the parent distribution.

Table 3 Frequencies for the Chi-Square Goodness-of-Fit test

abs (angle range)	O_i (n/total)	E_i
0-20	15/32=0.4688	0.2651
21-30	3/32=0.0938	0.1409
31-40	5/32=0.1563	0.1277
41-50	3/32=0.0938	0.0991
51-60	4/32=0.1250	0.0660
61-70	1/32=0.0313	0.0376
71-90	1/32=0.0313	0.0274

vative situation). Therefore, the trend of absolute values of branching angles occurring in the fatigue tests described here can be considered as a normal distribution.

We are left with the problem of interpreting the trend of branching angles with respect to the crack growth. This task is definitely more challenging.

4 Use of Exploratory Data Analysis

Exploratory data analysis (EDA) provides a set of techniques that allows interpretation of data beyond statistics. It transforms a picture in a way that ''forces us to notice what we never expected to see,'' as J. Tukey said in a groundbreaking book on the subject [5].

One of the techniques described in [5] will be utilized, with the following warning: the number of data points available does not allow extreme confidence regarding the final results. The technique of *smoothing by running medians of 3*, explained later in the paragraph, is better used with more points available than what we have at this state of the research. We are going to apply the tool described in [5], in search of more insights in the trend than what are offered in Table 1. The outcome will be thoroughly subject to verification in future work.

Another problem is given by the format of our data: we have *12* specimens and *32* corresponding angle data. Since the smoothing technique is based on medians, it makes sense to consider medians from the beginning. If we have an ordered sample of n points, say y_i, with $y_i < y_{i+1}$, the *median* is the point that divides the sample into two equal halves. If n is even, the median is given by $(y_{n/2}+y_{(n/2)+1})/2$. Table 4 shows the normalized crack growth and the absolute value of the branching angles for each specimen (in bold), together with the medians of these two parameters. The specimens have been sorted so to have ascending median angles.

It can be seen that there are five values of crack growths corresponding to zero branching angles. These five values can be ordered in 5! possible ways. Two combinations will be selected for these five data: the first has the crack lengths in ascending order, the second has the crack lengths in descending order.

The *smoothing by running median of 3* is a technique for smoothing the data sample. By smoothing, we try to get rid of *spikes*, outliers in the sample. The resulting curve is called *smoother*. The smoother is *resistant* if outliers have little weight on it [6]. The equation to keep in mind is: ''data=smooth +rough.''

We want to make the rough as small as possible. The smooth is obtained in the following way: the set of data is organized in such a way that each point slides in sets of *3* points at the time. There will be two starting sequences overall, as mentioned above:

Table 4 Report of normalized crack growth and absolute values of branching angles (deg), in bold, with the corresponding medians

Specimen #	Side 1 (deg, a/a_0)	Side 2 (deg, a/a_0)	Median a/a_0	Median angle
1	**No kink**, 0.04361	**No kink**, no growth	0.02180	**0.**
9	**No kink**, 0.04410	**No kink**, no growth	0.02205	**0.**
3	**No kink**, no growth	**No kink**, 0.1063	0.05315	**0.**
11	**No kink**, 0.04373	**No kink**, no growth	0.02186	**0.**
12	**No kink**, 0.03830	**No kink**, no growth	0.01915	**0.**
7	**33.7**, 0.0422	**No kink**, no growth	0.02110	**16.85**
4	**25.2**, 0.06091	**29.3**, 0.06331	0.06211	**27.25**
10	**45, 12, 31.5**, 0.1232	**No kink**, 7.819e-3	0.06551	**31.5**
6	**No kink**, no growth	**63.4**, 0.1210	0.06050	**31.7**
2	**No kink**, no growth	**38.7, 53.1**, 0.4965	0.2482	**38.7**
8	**59.3, 45, 21.2, 39.8** 0.3800	**90, 46.5, 36** 0.1925	0.2862	**45**
5	**51.2**, 0.06252	**50.9**, 0.1499	0.1062	**51.05**

Table 5 Step 1 of smoothing by running medians of 3

Sets of 3			Medians
0.01915	0.02180	0.02186	0.02180
0.02180	0.02186	0.02205	0.02186
0.02186	0.02205	0.05315	0.02205
0.02205	0.05315	0.02110	0.02205
0.05315	0.02110	0.06211	0.05315
0.02110	0.06211	0.06551	0.06211
0.06211	0.06551	0.0605	0.06211
0.06551	0.0605	0.2482	0.06551
0.0605	0.2482	0.2862	0.2482
0.2482	0.2862	0.1062	0.2482

Table 6 First smoothing

Given	Smooth	Rough
0.01915	0.01915 (copy-on)	0.
0.02180	0.02180	0.
0.02186	0.02186	0.
0.02205	0.02205	0.
0.05315	0.02205	0.0311
0.02110	0.05315	−0.03205
0.06211	0.06211	0.
0.06551	0.06211	3.4e-3
0.0605	0.06551	−5.01e-3
0.2482	0.2482	0.
0.2862	0.2482	0.038
0.1062	0.1062 (copy-on)	0.

Table 7 Final smoothing for Sequence 1

Given	Smooth	Rough
0.01915	0.01915 (copy-on)	0.
0.02180	0.02180	0.
0.02186	0.02186	0.
0.02205	0.02205	0.
0.02205	0.02205	0.
0.05315	0.05315	0.
0.06211	0.06211	0.
0.06211	0.06211	0.
0.06551	0.06551	0.
0.2482	0.2482	0.
0.2482	0.2482	0.
0.1062	0.2482 (end smoothing)	−0.1420[a]

[a]This number is not negligible. However, if a given condition (p. 221 [5]) is verified, then the end smoothed value calculated is the correct choice. The condition is verified in this case, so the value of *0.2482* for the smoother is accepted.

	Median a/a_0
Sequence 1	0.01915, 0.02180, 0.02186, 0.02205, 0.05315, 0.02110, 0.06211, 0.06511, 0.0605, 0.2482, 0.2862, 0.1062
Sequence 2	0.05315, 0.02205, 0.02186, 0.02180, 0.01915, 0.02110, 0.06211, 0.06511, 0.0605, 0.2482, 0.2862, 0.1062

Sequence 1 will be used to show the steps of the analysis. The calculations are very similar for Sequence 2.

The sequence is written down and subdivided in sets of 3 data at a time, and the median is calculated for each set (Table 5):

The medians are compared with the given data. Each data is associated to the median of the set in which the data is the midpoint: for example, the smooth of *0.02186* is the median of the set (*0.02180*, *0.02186*, *0.02205*), that is equal to *0.02186*. By doing this, we obtain Table 6. The rough is equal to the difference of the data itself and its smooth.

The expression *copy-on* indicates the fact that the end-values have been just copied on. At the end of the smoothing process, the end values will be reconsidered. As it can be seen in Table 6, the

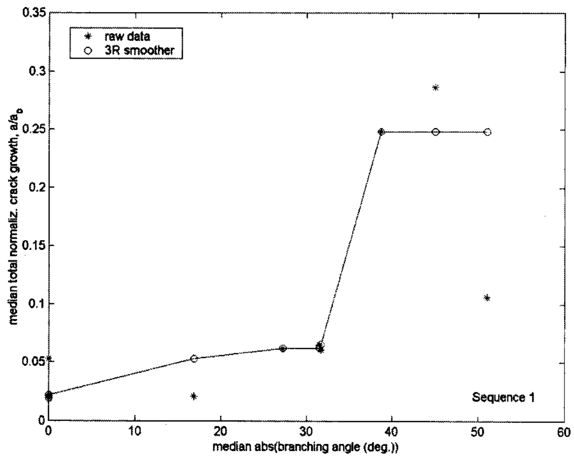

Fig. 9 Median of normalized crack length versus the median of the absolute value of branching angles, for Sequence 1. ○ =3*R* smoother, *=raw data.

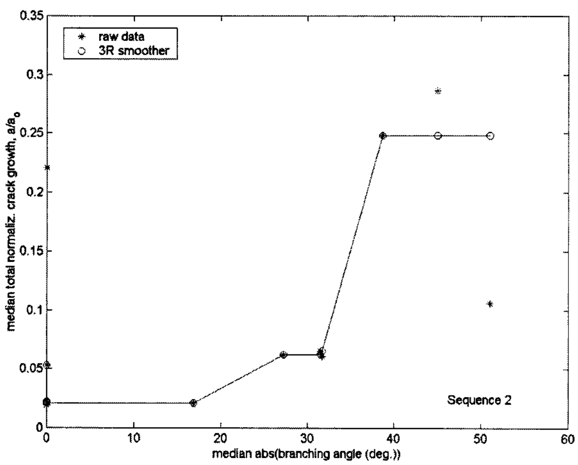

Fig. 10 Median of normalized crack length versus the median of the absolute value of branching angles, for Sequence 2. ○ =3*R* smoother, *=raw data.

terms in the column of the rough are still pretty high. Another smoothing is done, using the same technique. The overall procedure is defined as $3R$, that is, *smoothing by running median of 3 repeated* until the results of a step are equal to the results of the previous step and the rough is negligible. This technique does not allow to interpret the local behavior of a data set, but can give a good idea of what is the general trend. This is what we are looking for in our case. Advances on the tool are offered in [5,6]. The final smoother is obtained after one extra iteration, and is given in Table 7.

Due to the format of the first few data in the sequence (values corresponding to the same angle 0) it was not possible to perform a smoothing on the first number, *0.01915*. Therefore, this value has been just copied-on in the final smoother. On the other hand, smoothing of the last number, *0.1062*, has been done according to [5]: it is the median of three values obtained from the last data points of the smoother.

It is apparent from Table 7 that there is an increasing trend. To make sure that this is not due to the first five values of the sequence, chosen in ascending order, the same calculations have been done for Sequence 2, that has those five values in descending order. The resulting $3R$ smoothers and raw data for Sequences 1 and 2 are plotted, respectively, in Figs. 9 and 10, that show how the normalized median crack length varies with respect to the absolute values of the median branching angles.

Both plots show that the crack growth increases with the branching angle. It seems that the smoothers are not much affected by the choice of the initial set of data corresponding to the zero branching angles. Considering the fact that the data are reported for a similar number of cycles, we may interpret this trend as acceleration in crack growth due to the branching phenomenon.

Overall, we may conclude that branching for the specimens selected occurs in average in the first 100,000 cycles. The greater the branching angle, the faster the growth. Afterward, crack growth slows down, but the total growth happens at a faster rate than for a crack that does not kink.

5 Conclusions

Analysis of experimental data from fatigue tests on delaminated cross-ply laminates of Graphite/Epoxy has been discussed, with the aim of determining trends for the magnitude of the branching angles formed during the fatigue tests. Several statistical and exploratory data analysis techniques have been surveyed, and some have been applied to the data set. The results are:

(a) by the *Chi-square Goodness-of-Fit* test, it is shown that the magnitude of the branching angle follows a normal distribution when plotted versus the number of cycles;
(b) by the *smoothing by running median of 3 repeated* technique, it is shown that the crack growth increases with the magnitude of the branching angle. Comparing data corresponding to similar number of cycles, it seems that the crack grows faster when it is not self-similar.

Future research work will concentrate on determining with more precision the role of the branching angle magnitude on crack growth instability. This goal will be pursued not only by analytical modeling but also by designing the upcoming experiments and collecting information that will be investigated by more sophisticated exploratory data analysis techniques.

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