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Buckling of sandwich wide columns

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Abstract

The paper deals with the theoretical prediction of buckling loads for sandwich columns with metallic and laminated facings and foam or honeycomb core. The loading is a uniform axial compression, applied statically (very slowly) and suddenly with constant magnitude and infinite duration (step loading). The effect of length and boundary conditions is assessed and results are presented for the following cases: for a cantilever column, a simply supported column and a clamped column, for several lengths. Several fiber materials are used in the laminated facings. Two types of core were examined: alloy-foam or hexagonal glass/phenolic honeycomb. The facings are Boron/Epoxy, Graphite/Epoxy and Kevlar/Epoxy laminates with 0° orientation with respect to the column axis and a metallic one made out of aluminum. These various materials are employed to provide comparative data that can be used in design. Results, for the static case are generated by computer codes as well as by the use of closed form theoretical solutions. For the dynamic case, results are generated by the DYNA3D code.

Keywords: Buckling; Sandwich; Column; Composites

1. Introduction

Lightweight sandwich construction is of great interest in the design and manufacture of spacecraft and marine vehicles because of high specific stiffness and strength. In addition, sandwich construction offers enhanced corrosion resistance, noise suppression and reduction in life-cycle costs. There are several issues and questions related to the use of sandwich construction that require attention and answers. One of the important issues is the prediction of buckling loads, under either static or sudden load application. For the static case, closed form solutions are derived for a symmetric sandwich column under axial compression and for various boundary conditions. In addition, the solution suggested by Bazant and Cedolin [1] is also used in the static analysis. Finally, static critical loads are predicted by a finite element computer code that includes transverse shear effects. The utilized code is DYNA3D [2], which is a dynamic three-dimensional code. The static critical load is obtained by applying the load very slowly and computing the maximum displacement (an extremely small transverse load is included in order to initiate bending). The critical load is the one for which the maximum transverse displacement becomes very large. In addition, results are generated by the computer code ABAQUS [3]. All predictions are computed and compared to each other in order to establish confidence in the results. For the dynamic case, the axial compression is applied suddenly and the Budiansky–Roth

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[4] criterion is applied in order to estimate the critical value. According to this criterion the equation of motion is solved and the maximum amplitude of vibrations is recorded for constant values of the sudden load. The load is step increased, starting with a small value, and the maximum amplitude of vibrations is plotted versus the sudden load. When the increase in amplitude is very large, we have a critical dynamic load. Note that in the case of sandwich columns, we do not have dynamic instability through escaping motion [5], but at certain value of the sudden load the maximum amplitude of vibrations starts to become unacceptably large. For this dynamic case, the FE computer code DYNA3D [2] is employed, and critical loads are computed for the same facing materials as in the static case. From the comparisons and computer simulation, several unexpected phenomena are observed, which are discussed in later sections. The effect of boundary conditions and length are assessed and several facing materials are employed. They are: Aluminum, Boron/Epoxy, Graphite/Epoxy and Kevlar/Epoxy with 0° orientation with respect to the column length. A sandwich column consists of two thin facings and a thick core made of foam or low strength honeycomb.

Research into structural behavior of sandwich structures commenced in the late 1940s [6–9] but intensified recently [10–13]. On the other hand interest in dynamic stability commenced in the 1950s and it continues to the present [5,14,15].

2. Structural geometries

Consider a column of length L, depth $c + 2h$ and width B. The column is of sandwich construction, symmetric about the midsurface and the depth of the facings is h, while the depth of the core is c. The boundary conditions are: (a) clamped-free-cantilever, (b) simply supported at both ends and (c) clamped at both ends. For the composite facings, all plies have 0° orientation with respect to the column axis. The material properties are given below in Table 1.

Closed form solutions have been developed and reported in Bazant and Cedolin [1] herein. According to Bazant and Cedolin, the derived expression is applicable to cantilever, simply supported and clamped (at both ends) columns. On the other hand the expression derived herein is applicable to all boundary conditions. In addition, in a few cases static critical conditions were obtained by DYNA3D [2] with a very slow application of the load, so as not to induce dynamic response. A slight load imperfection was introduced and a critical static load was approximated by observing the maximum amplitude (deflection).

A static critical load was established when the deflection was very large for small increment in the applied compression. Moreover, static critical conditions were also obtained by ABAQUS [3], for several configurations.

The two closed form expressions are presented below:

2.1. Reference 1 expression

The symbols used, herein, are not exactly those used in Bazant and Cedolin [1].

$$P_{\mathrm{cr}} = (\mathrm{EI})_\alpha k_{\mathrm{cr}}^2 \bigg/ \left[1 + \frac{k_{\mathrm{cr}}^2 (\mathrm{EI})_\mathrm{b}}{GA_1}\right], \tag{1}$$

where

$$k_{\mathrm{cr}}^2 = \begin{cases} (\pi/2L)^2 & \text{for cantilever columns,} \\ (\pi/L)^2 & \text{for simply supported columns,} \\ (2\pi/L)^2 & \text{for clamped columns.} \end{cases}$$

Moreover,

$$(\mathrm{EI})_\alpha = E_\mathrm{f}'(I_1 + I_\mathrm{f}) \tag{2}$$

and

$$(\mathrm{EI})_\mathrm{b} = E_\mathrm{f}' I_1,$$

where

$$E_\mathrm{f}' = E_\mathrm{f}/(1 - v_\mathrm{f}^2),$$

$$I_1 = \frac{h}{2}(c+h)^2 B, \quad I_\mathrm{f} = \frac{h^3}{12} B$$

and

$$v_\mathrm{f}^2 = \begin{cases} v^2 & \text{for metallic facings,} \\ v_{12} v_{21} & \text{for composite facings.} \end{cases}$$

Finally,

$$GA_1 = G_\mathrm{c}(h + c)B. \tag{3}$$

In the above expressions G_c is the core shear modulus, E_f the Youngs modulus of facing material, in the column direction (E_{11} for composite facings) and

Table 1
Material properties

	Designation	E_{11} (kPa)	E_{12} (kPa)	Density (kg/mm^3)	υ_{12}	υ_{21}	G_{13} (kPa)
AL	6061-T6	6.90E + 07	6.90E + 07	2.71E − 06	0.35	3.50E − 01	2.59E + 07
Boron/Ep	B(4)5505	2.21E + 08	2.07E + 07	2.02E − 06	0.23	2.16E − 02	5.79E + 06
Kevlar/Ep	49Epoxy	7.59E + 07	5.52E + 06	1.38E − 06	0.34	2.47E − 02	2.28E + 06
Graph/Ep	T300 5208	1.81E + 08	1.03E + 07	1.61E − 06	0.28	1.59E − 02	7.17E + 06
Alloy-foam		4.59E + 04	4.59E + 04	7.00E − 08	0.33	3.3E − 01	1.72E + 04
Honeycomb		3.90E + 05	3.20E + 04		0.25	2.05E − 02	4.80E + 4[a]

[a]The geometric values used herein are: $c = 25.4$ mm, $h = 2.54$ mm, and $L = 2032$ and 6096 mm. The width is $B = 76.2$ mm.

the geometric dimensions are defined in a previous section.

2.2. Present expression

In addition to the previous definitions, i.e., face sheets of thickness h and extensional modulus, E_{f}, and core of thickness c, extensional modulus, E_{c} and shear modulus G_{c}, we denote by G_{f} the shear moduli of the face sheets. The width is uniform, B, and the total cross-sectional area is denoted by $A = B(2h + c)$.

Since the section under consideration is symmetric, the neutral surface is at the middle surface, and the equivalent flexural rigidity of the sandwich section, $(\mathrm{EI})_{\mathrm{eq}}$, is

$$(\mathrm{EI})_{\mathrm{eq}} = B\left[E_{\mathrm{f}}\frac{h^3}{6} + 2E_{\mathrm{f}}h\left(\frac{h}{2}+\frac{c}{2}\right)^2 + E_{\mathrm{c}}\frac{c^3}{12}\right]. \quad (4)$$

Huang and Kardomateas [16] presented a solution for the buckling and initial postbuckling behavior of sandwich beams including transverse shear effects (for a general unsymmetric construction). The linearized differential equation for the beam is [16]

$$(\mathrm{EI})_{\mathrm{eq}}\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} + \left(\frac{\alpha P}{A\bar{G}} + 1\right)P\theta = 0, \quad (5a)$$

where α is the shear correction factor (its calculation is discussed later), and $\bar{G}$ is the "effective" shear modulus defined by:

$$\frac{2f + c}{\bar{G}} = \frac{2f}{G_f} + \frac{c}{G_c}. \quad (5b)$$

Then, following the usual procedure solving the critical load by using the general trigonometric solution of Eq. (5) and imposing the relevant boundary conditions (e.g. Simitses [17]) we can write the critical load as

$$P_{\mathrm{cr}} = \frac{-1 + \sqrt{1 + (4\alpha(\mathrm{EI})_{\mathrm{eq}}\lambda_{\mathrm{cr}}^2)/A\bar{G}}}{(2\alpha/A\bar{G})}, \quad (6)$$

where $\lambda_{\mathrm{cr}}L = 2\pi$ for a clamped–clamped and $\lambda_{\mathrm{cr}}L = \pi$ for a simply supported beam. For example, the critical load for a clamped–clamped sandwich beam can be written as

$$P_{\mathrm{cr}} = \frac{-1 + \sqrt{1 + (16\alpha(\mathrm{EI})_{\mathrm{eq}}\pi^2)/A\bar{G}L^2}}{(2\alpha/A\bar{G})}.$$

Now, regarding the shear correction factor, it is found from shear energy equivalency [16]. Then, if we define:

$$a = h + \frac{c}{2}; \quad b = \frac{c}{2}; \quad \bar{c} = \frac{h}{2} + \frac{c}{2},$$

the shear correction factor is found as [16]

$$\alpha = 2\bar{G}ABd, \quad (7)$$

where

$$d = \frac{E_{\mathrm{f}}^2}{4(\mathrm{EI})_{\mathrm{eq}}^2 G_{\mathrm{f}}}\left[a^4h - \frac{2}{3}a^2(a^3 - b^3) + \frac{1}{5}(a^5 - b^5)\right] + \frac{E_{\mathrm{f}}^2}{(\mathrm{EI})_{\mathrm{eq}}^2 G_{\mathrm{c}}}\left[h^2\bar{c}^2b + \frac{2}{15}\frac{E_{\mathrm{c}}^2}{E_{\mathrm{f}}^2}b^5 + \frac{2}{3}\frac{E_{\mathrm{c}}}{E_{\mathrm{f}}}h\bar{c}b^3\right]. \quad (8)$$

Notice that for a homogeneous part (i.e. same material for face sheets and core), it can be proved that this formula reduces to the simple and familiar value of $\alpha = 6/5$. Also, notice that the shear correction factor given in Ref. [16] is for a general unsymmetric construction (different properties of top and bottom face sheets).

3. Dynamic buckling

There exist several approaches in establishing critical conditions for suddenly loaded structures (see Simitses [5]). In the case of columns, the critical dynamic load is estimated by employing the Budiansky–Roth [4] criterion. Through this approach one applies suddenly (rapidly) a small (say one-tenth of the static critical load) load and through a finite element computer code (in this case DYNA3D) solves the equations of motion and records the maximum amplitude of vibrations. He then step increases the value of the suddenly applied load and repeats the procedure. He finally plots maximum amplitude versus values of the sudden load and when the amplitude of vibrations is becoming large, then the corresponding load represents the critical dynamic load for loads applied suddenly with constant magnitude and infinite duration (for details, the interested reader is referred to Simitses [5]).

4. Results and discussion

Static critical loads are computed for several facing materials, boundary conditions and column lengths. To this end, the two closed form expressions, Eqs. (1) and (6)–(8), are employed. In addition, DYNA3D was employed by applying the load very slowly to estimate the static critical load. This approach was employed for very few cases, because the required computer time is extremely large. Moreover, the classical critical value was computed by the following formula:

$$P_{\mathrm{cl}} = \frac{C\pi^2(\mathrm{EI})_{\mathrm{eq}}}{L^2}, \tag{9}$$

where $C = 1/4$, 1 and 4 for clamped-free, simply-supported at both ends and clamped at both ends, respectively. This is shown only for comparison purposes.

The DYNA3D study used hexahedral isoparametric elements (8 node brick elements) and there were 3750 elements, of which 1500 elements were for the two face sheets and 2250 elements for the core. Finally, results are also generated by employing ABAQUS for several geometries. For this study, 8 node brick elements were also used.

Table 2 presents buckling loads in Newtons, for the cantilever column and for both static and sudden application of loads. In this case the length of the column is 2032 mm. The materials are: (a) aluminum facings, (b) boron/epoxy facings, (c) graphite/epoxy facings and (d) kevlar/epoxy facings. The laminated facings consist of laminae with 0° orientation with respect to the column axis. Five sources are used: Ref. 1, present, DYNA3D-static, ABAQUS and DYNA3D-dynamic. Ref. 1 results differ from the ABAQUS results by $< 4\%$ whereas the present results differ by $< 2\%$. However, the DYNA3D results are higher by almost 10% versus the ABAQUS results for the Kevlar/Ep but are within 2% for all the other laminated facings. For the case of metallic facings, the ABAQUS, DYNA3D and Ref. 1 results are within 3% of each other but they are all higher than the Euler load (see Table 3) by about 6%, whereas the present results are the only ones that are lower than the Euler load (but differ by -12% from all the other results).

The dynamic critical loads seem reasonable and suggest that the present and the computer static critical loads are reasonable (with the possible exception of DYNA3D results for the Kevlar/Epoxy case). Note that for the case of sudden loads of constant magnitude and infinite duration, the dynamic critical load is smaller than the corresponding static critical load (see Simitses [5]).

Table 3 shows the effect of boundary conditions and it depicts static critical loads for $L = 2032$ mm and using various sources. One of the sources used to compute static critical loads is Eq. (9). This was done only for comparison purposes. It is expected that Eq. (9) overestimates the critical load because it does not account for transverse shear effects.

There are several observations that need to be pointed out. Since Ref. 1 and DYNA3D results for the cantilever column and metallic facings are larger than P_{cl}, Eq. (9), then the present results are more acceptable. Moreover, it is seen from the results of this table that the effect of transverse shear is more pronounced as we move from the cantilever column to the simply supported column to the clamped/clamped one. One reason for this is that the effective simply supported length decreases (EI remains constant) and transverse shear effects are more pronounced for shorter columns. Finally, it is seen that for the same core material and geometry and for the same thickness of the facings, the construction in going from the stronger (higher static critical load) configuration to the weaker one is: Boron/Epoxy, Graphite/Epoxy, to

Table 2
Buckling loads of sandwich columns for alloy-foam core in Newtons. Cantilever, $L = 2032$ mm

Unit Newton	AL 6061-T6	Boron/Ep B(4)-5505	Graph/Ep T300-5208	Kevlar/Ep 49Epoxy
Ref. 1	3192.05	7772.02	6618.52	3114.49
Present	2872.13	8183.58	6887.44	3139.48
DYNA3D (static)	3340.27	8230.00	6868.60	3495.02
ABAQUS (static)	3277.73	8086.45	6853.00	3197.38
DYNA3D (dynamic)	2868.96	7984.16	6560.80	3140.29

Table 3
Buckling loads of sandwich columns for alloy-foam core in Newtons, $L = 2032$ mm

Boundary conditions	Source	AL 6061-T6	Boron/Ep B(4)-5505	Graph/Ep T300-5208	Kevlar/Ep 49Epoxy
Cantilever	Ref. 1	3192.05	7772.02	6618.52	3114.49
	Present	2872.13	8183.58	6887.44	3139.48
	DYNA3D	3340.27	8230.00	6868.60	3495.02
	ABAQUS	3277.73	8086.45	6853.00	3197.38
	Euler (P_{cl})	3077.45	9850.58	8068.18	3384.92
S.S.	Ref. 1	10,107.70	18,946.00	17,126.90	9912.29
	Present	9880.09	24,482.50	21,144.40	10,692.60
	DYNA3D	Not available	Not available	Not available	Not available
	ABAQUS	10,543.29	20,215.10	18,189.46	10,340.02
	Euler (P_{cl})	12,309.80	39,402.30	32,272.70	13,539.70
C.C.	Ref. 1	22,051.50	29,576.90	28,399.60	21,816.90
	Present	28,715.30	61,982.10	54,677.30	30,695.50
	DYNA3D	23,700.01	25,166.65	24,999.98	21,066.66
	ABAQUS	22,863.74	37,311.03	36,523.38	25,291.58
	Euler (P_{cl})	49,239.30	157,609.00	129,091.00	54,158.70

Kevlar/Epoxy and Aluminum (virtually tied). If specific buckling strength is considered, Boron/Epoxy and Graphite/Epoxy are virtually tied. They are followed by Kevlar/Epoxy, which is better than Aluminum.

For the simply supported case, the present results differ from the ABAQUS results by a range of −6.3% for the metallic facings to +21% for Boron/Ep whereas the Ref. 1 results are always lower by −4.1% for the metallic facings and Kevlar/Ep to −6.2% for the Boron/Ep. The Euler load is higher than the ABAQUS results by amounts ranging from a factor of +16% for the metallic facings to +94% for the Boron/Ep.

For the clamped/clamped case (Table 3), the present results are higher than the ABAQUS results by a range of +21% for Kevlar/Ep to +66% for Boron/Ep whereas the Ref. 1 results are lower by a range of −3.5% for the metallic facings to −22% for Graph/Ep. At the same time, the DYNA3D results differ from the ABAQUS by amounts ranging from +3.6% for the metallic facings to −32% for Boron/Ep. Note that the Euler load is higher than the ABAQUS results by amounts ranging from a factor of 2.1 for the metallic facings to a factor of 4.2 for the Boron/Ep, i.e. by a very large amount.

Table 4 presents static buckling loads as computed from Ref. 1, present, ABAQUS and Eq. (9), for $L = 6096$ mm . It is clearly seen that for the laminated facings, Ref. 1 and present analysis yield approximately the same results for a cantilever or simply supported column.

For this length, the transverse shear effect is rather small for cantilever and increases as we move to the

Table 4
Buckling loads of sandwich columns for alloy-foam core in Newtons, $L = 6096$ mm

Boundary conditions	Source	AL 6061-T6	Boron/Ep B(4)-5505	Graph/Ep T300-5208	Kevlar/Ep 49Epoxy
Cantilever	Ref. 1	384.673	1065.97	877.25	374.56
	Present	339.8	1066.21	877.31	372.64
	ABAQUS	341.94	1084.24	892.27	378.82
	Euler (P_{cl})	340.69	1094.51	896.47	376.10
S.S.	Ref. 1	1491.39	3919.38	3272.28	1453.34
	Present	1324.12	3983.13	3312.71	1451.91
	ABAQUS	1344.18	4023.44	3353.95	1474.98
	Euler (P_{cl})	1367.76	4378.04	3585.86	1504.41
C.C.	Ref. 1	5312.25	11,848.30	10,307.80	5191.22
	Present	4878.61	13,185.00	11,213.60	5314.34
	ABAQUS	4904.90	12,498.59	10,820.34	5335.51
	Euler (P_{cl})	5471.03	17,512.10	14,343.40	6017.63

clamped/clamped case. For this length also, as in the case of $L = 2032$ mm, it is seen that for aluminum facings, the present analysis yields a more reasonable result than Ref. 1, since the Euler load, P_{cl}, must be larger than the loads computed by the closed form solutions.

For the cantilever case, in all cases the present results are within 2% of the ABAQUS results and always lower than the Euler load. However, the Ref. 1 results are 12% above the ABAQUS results and higher than the Euler load for the case of metallic facings but are almost identical to the present results in all other cases. Note also that the ABAQUS results are slightly higher than the Euler load (by $< 1\%$) for both the metallic and the Kevlar/Ep facings. But the Euler load is in general very close to the ABAQUS results (within 1%) in the cantilever case. Similar observations can be made for the simply supported case with the Ref. 1 results being again about 10% above the ABAQUS results and above the Euler load for the case of metallic facings, but now the ABAQUS results are always below the Euler load, and the present results are within 2% of the ABAQUS results. In the simply supported case, the Euler load is higher than the ABAQUS results by no more than 9%.

For the clamped/clamped case (Table 4), the present results are within about 5% of the ABAQUS results whereas the Ref. 1 results are higher than the ABAQUS results by 8% for the metallic facings case but lower within -5% for the laminated facings. In all cases the present results are closer to the ABAQUS results. The Euler load is higher than the ABAQUS results by a range of 11% (metallic facings) to 40% (Boron/Ep facings).

Tables 5 and 6 give the critical loads for the same geometry as in Tables 3 and 4 but with a honeycomb core instead of alloy foam. The results in Table 5 lead to similar observations as with those in Table 3. In the cantilever case, of concern is the fact that the ABAQUS results are above the Euler load for the metallic (by about 12%) facings. The Ref. 1 results are also higher than the Euler load for the metallic facings by about 9%. The present results are always below the Euler load and, for the laminated facings, they are the closest to the ABAQUS results. In the simply supported case, as opposed to the Table 3 results, the ABAQUS result is above the Euler load and the Ref. 1 result is close to the Euler load, for the metallic facings, whereas the present results are always below the Euler load. But for the laminated facings all results are within 5% of each other. For the clamped/clamped case, the present results are above the ABAQUS results, which seem to be in general almost half way between the Ref. 1 and the present results, but the Euler results are higher than the ABAQUS results by a range of between 38% (Kevlar/Ep) to 120% (Boron/Ep), i.e. by smaller amounts than the alloy foam case in Table 3.

Table 5
Buckling loads of sandwich columns for honeycomb core in Newtons, $L = 2032$ mm

Boundary conditions	Source	AL 6061-T6	Boron/Ep B(4)-5505	Graph/Ep T300-5208	Kevlar/Ep 49Epoxy
Cantilever	Ref. 1	3382.48	9006.61	7493.22	3295.51
	Present	3017.20	9127.89	7576.79	3308.11
	ABAQUS	3475.13	9256.00	7696.36	3385.95
	Euler (P_{cl})	3098.52	9871.65	8089.24	3405.98
S.S.	Ref. 1	12,300.60	28,454.00	24539.60	12,012.30
	Present	11,261.30	30,940.80	26,220.00	12,276.20
	ABAQUS	12,689.98	29,580.70	25,458.30	12,399.84
	Euler (P_{cl})	12,394.10	39,486.60	32,357.00	13,623.90
C.C.	Ref. 1	36,086.40	61,831.10	56,900.30	35,462.30
	Present	37,209.10	88,318.00	76,790.10	40,108.90
	ABAQUS	35,154.29	71,211.13	66,801.18	39,195.60
	Euler (P_{cl})	49,576.30	157,946.00	129,428.00	54,495.70

Table 6
Buckling loads of sandwich columns for honeycomb core in Newtons, $L = 6096$ mm

Boundary conditions	Source	AL 6061-T6	Boron/Ep B(4)-5505	Graph/Ep T300-5208	Kevlar/Ep 49Epoxy
Cantilever	Ref. 1	387.30	1086.40	891.03	377.05
	Present	343.23	1086.32	891.71	377.17
	ABAQUS	348.52	1104.88	906.95	383.39
	Euler (P_{cl})	344.28	1096.85	898.81	378.44
S.S.	Ref. 1	1531.68	4210.44	3472.71	1491.57
	Present	1360.58	4227.84	3486.70	1493.81
	ABAQUS	1381.71	4295.92	3544.12	1518.27
	Euler (P_{cl})	1377.12	4387.40	3595.22	1513.77
C.C.	Ref. 1	5861.42	14,978.30	12,598.10	5714.42
	Present	5261.22	15,425.50	12,896.30	5758.52
	ABAQUS	5336.79	15,475.53	13,004.55	5843.46
	Euler (P_{cl})	5508.47	17,549.60	14,380.90	6055.08

Regarding the results in the larger length case in Table 6, in the cantilever case they are all within 2% of each other except for the Ref. 1 result with metallic facings, which is 11% above the ABAQUS result and almost 14% above the Euler result. Of concern is again the fact that the ABAQUS results are above the Euler load in all cases by about 2%. But the present results are all below the Euler load.

In the simply supported case (Table 6), again of concern is the fact that the ABAQUS result is above the Euler load for both the metallic and the Kevlar/Ep facings, but in both cases by $< 2\%$, whereas the Ref. 1 result is above the Euler load by 12% for the metallic facings. The present results are below the Euler load in all cases. Again, as in Table 4, with the exception of the Ref. 1 result for metallic facings, all other results are within 2% of each other.

Finally, in the clamped/clamped case (Table 6), we find again (as in Table 4) that the present results are the closest to the ABAQUS results in all cases and they are always below the Euler load. For the case of metallic facings, Ref. 1 predicts a critical load above the Euler load by almost 7%. Other than that, the Ref. 1 results are close to the present results and the ABAQUS results, in general within 4%. The ABAQUS results are below the Euler load in all cases and the difference is $< 13\%$.

The effect of transverse shear is significant and can most easily be seen by comparing the Ref. 1 and the present results for the clamped–clamped case in the case of alloy foam core and boron/epoxy face sheets. In Table 3, for a column length of 2032 mm, the Ref. 1 result is only 18.7% of the Euler load and the present result is only 39.3% of the Euler load. For a shorter column, this effect would be even more pronounced. For example, for a column of 1270 mm long, the Ref. 1 result would be only 8.2% of the Euler load and the present result would be only 26.8% of the Euler load.

Since the Ref. 1 critical load formula is based on an "Engesser"-type [18] derivation, whereas the present critical load formula is based on a "Haringx"-type [19,20] derivation, it should be mentioned at this point that a study of column buckling for monolithic composites from three-dimensional elasticity which was done for transversely isotropic materials by Kardomateas [21] and a corresponding one for orthotropic columns [22], showed that the Engesser formula would predict in general smaller values for the critical load; therefore is expected to be the most conservative, but not the most accurate, and indeed the Haringx formula results were found to be in general closer to the elasticity results. The complexity of sandwich composites notwithstanding, this conclusion is not contrary to the general observations made in the present study (with the few exceptions noted in the detailed discussion of the Tables).

For the limited number of dynamic cases considered, the procedure is to compute the vibration response for various levels of the sudden load and plot the system response (Fig. 1). Then, from these curves, the maximum amplitude is plotted versus applied sudden load and the value of the load at which the maximum amplitude increases rapidly is called the critical dynamic load (Fig. 2). The limited dynamic results are shown on Table 2.

The dynamic critical load must always be smaller than the corresponding static critical load. This fact can be used in interpreting the accuracy of the computed static loads. For Boron/Ep, cantilever column and $L = 6096$ mm, the computed critical load (by DYNA3D) is 1045 N, which is approximately 3.4% smaller than the smallest static critical load (see Table 4).

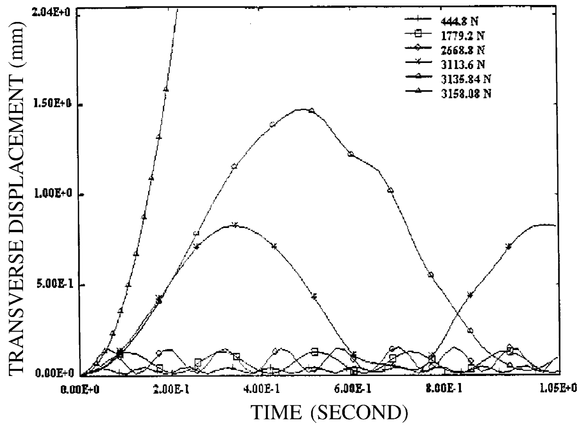

Fig. 1. Vibration response for cantilever column. Kevlar/Ep, $L = 2032$ mm. Transverse displacement versus time.

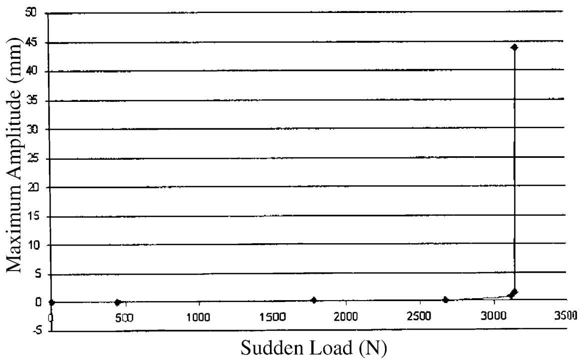

Fig. 2. Maximum amplitude versus sudden load. Kevlar/Ep, $L = 2032$ mm, clamped-free.

In closing, it is recommended that more work is needed in analytically predicting the static critical load. There is additional concern that the loads computed by the codes ABAQUS and DYNA3D in some cases differ by a substantial amount and that these calculations in some cases are above the Euler load.

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References

[1] Z.P. Bazant, L. Cedolin, Stability of Structures, Oxford University Press, New York, 1991.

[2] R.G. Whirley, B.E. Engelmann, DYNA3D, a nonlinear, explicit, three dimensional finite element code for solid and structural mechanics-user manual, LLNL Report UCRL-MA-107254-Rev-1, Lawrence Livermore National Laboratory, 1993.

[3] ABAQUS, Habbit, Karlsson and Sorensen, Inc., Providence, RI, 2001.

[4] B. Budiansky, R.S. Roth, Axisymmetric dynamic buckling of clamped shallow spherical shells, in: Collected Papers on Instability of Shell Structures, NASA TN D-1510, Washington, 1962.

[5] G.J. Simitses, Dynamic Stability of Suddenly Loaded Structures, Springer, New York, 1990.

[6] J.N. Goodier, C.S. Hsu, Nonsinusoidal buckling modes of sandwich plates, J. Aeronautical Sci. 21 (1954) 525–532.

[7] Anonymous, Symposium on Structural Sandwich Construction, ASTM Special Technical Publication No. 201, ASTM, Philadelphia, 1956.

[8] O. Michiharu, Antisymmetric and symmetric buckling of sandwich columns under compressive load, Trans. Jpn. Soc. Aeronaut. Space Sci. 19 (1976) 163–178.

[9] K.P. Chong, K.A. Wang, G.R. Griffith, Analysis of continuous sandwich panels in building systems, Build. Environ. 44 (1979) 125–130.

[10] G.J. Simitses (Ed.), Analysis and Design Issues for Modern Aerospace Vehicles—1997, ASME AD-Vol. 55, ASME, New York, 1997, pp. 271–400.

[11] B.V. Sankar (Ed.), Recent Advances in Mechanics of Aerospace Structures and Materials 1998, ASME AD-Vol. 56, ASME, New York, 1998, pp. 151–252.

[12] G. Newaz (Ed.), Advances in Aerospace Materials and Structures, ASME AD-Vol. 58, ASME, New York, 1999, pp. 1–18.

[13] Y. Frostig, M. Baruch, Buckling of simply supported sandwich beams with transversely flexible core—a high order theory, ASCE J. EM Division 119 (5) (1993) 955–972.

[14] A.N. Kounadis, Dynamic snap-through buckling of a Timoshenko two-bar frame under a suddenly applied load, Z. Angew. Math. Mech. 59 (1979) 523–531.

[15] N.J. Hoff, Dynamic Stability of Structures, in: G. Herrmann (Ed.), Pergamon, New York, 1967.

[16] H. Huang, G.A. Kardomateas, Buckling and initial postbuckling behavior of sandwich beams including transverse shear, AIAA J., 2000, in press.

[17] G.J. Simitses, An Introduction to the Elastic Stability of Structures, Krieger, New York, 1986.

[18] F. Engesser, Die Knickfestigheit gerader Stabe, Zentralb. Bauverwaltung 11 (1891) 483–486.

[19] J.A. Haringx, On Highly Compressible Helical Springs and Rubber Rods, and their Application for Vibration-Free Mountings, I. Phillips Research Reports, Eindhover, Holland, Vol. 3, 1948, pp. 401–449.

[20] J.A. Haringx, On Highly Compressible Helical Springs and Rubber Rods, and their Application for Vibration-Free Mountings, II. Phillips Research Reports, Eindhover, Holland, Vol. 4, 1949, pp. 49–80.

[21] G.A. Kardomateas, Three dimensional elasticity solution for the buckling of transversely isotropic rods: the Euler load revisited, J. Appl. Mech. ASME 62 (1995) 346–355.

[22] G.A. Kardomateas, D.S. Dancila, Buckling of moderately thick orthotropic columns: comparison of an elasticity solution with the Euler and Engesser/Haringx/Timoshenko formulas, Int. J. Solids Struct. 34 (3) (1997) 341–357.