

# Response of a sandwich panel subject to fire or elevated temperature on one of the surfaces

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## Abstract

The paper presents the analysis of deformations and stresses in a large aspect ratio sandwich panel subject to fire or another source of elevated temperature. The panel, assumed to bend into a cylindrical surface, is simply supported at the edges. The edges are also prevented from in-plane displacements providing an elastic restraint as the panel stretches due to bending. The solution is obtained in a closed form when the deformations are small and when geometrically nonlinear effects are incorporated into the analysis. The solution is open to modification with arbitrary temperature and property distributions through the thickness of the panel, enabling a designer to incorporate the results from a multitude of heat transfer scenarios so long as the structural problem can be treated as quasi-static.

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## 1. Introduction

The problems of durability, real-time and residual strength and stiffness of sandwich structures subject to fire or other sources of elevated temperature represent a major interest for designers. The complexity of the problem is related to a number of coupled phenomena, including the dynamic problem of heat transfer, property degradation due to an elevated temperature, resin decomposition in PMC facings, real-time strength and stiffness and residual strength of the structure after fire. In some situations, the issue is a predicted life of the structure subject to fire, as the stresses and deformation gradually build up to the instant when the structure collapses.

Although the temperature and properties of engineering materials are affected by the stress [1], it is usually possible to ignore this interaction. In this case, the problems listed above can be uncoupled, i.e. the dynamic problem of temperature distribution and material decomposition is analyzed first and subsequently, real-time maps of distribution of temperature and properties are applied to the stress and deformation analysis. In the case of a sandwich panel

subject to fire, the former problem has been addressed in a number of investigations [2,3]. While the heat transfer problem is dynamic, the structural problem can be formulated as a static case since the changes associated with fire are relatively slow.

The present paper is concerned with the response of a large aspect ratio sandwich panel subject to fire or another source of elevated temperature on one of the surfaces. The panel, bending into a cylindrical surface, is simply supported along the edges that are also prevented from in-plane displacements by adjacent structures. The solution is obtained in a closed form for small deformations and in a geometrically nonlinear formulation. Numerical results are obtained using a simplified quasi-static approach to a distribution of temperature through the thickness. In spite of this simplification, the results obtained from the solution are in a qualitative agreement with available experimental data. In the case where the thermal problem is solved by a more accurate approach, the corresponding adjustments to the properties and temperature can be incorporated in the present analysis without altering its validity.

## 2. Analysis

Consider a large aspect ratio sandwich panel with cross-ply facings and a foam core that represents a part of

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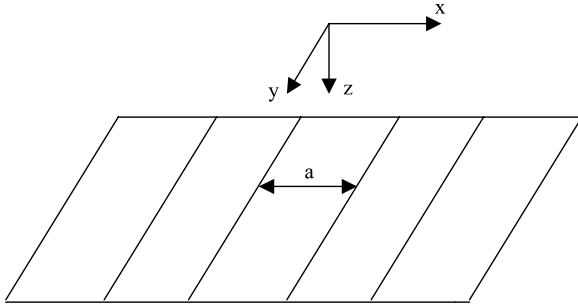


Fig. 1. Structure consisting of a number of identical sandwich panels supported by frames or bulkheads. Each panel represents a large aspect ratio plate. Long edges of such plate are constrained against displacements in the  $x$ -direction by adjacent plates.

the structure consisting of a number of identical panels supported by frames or bulkheads (Fig. 1). The adjacent panels severely limit in-plane extension and contraction of the panel in the  $x$ -direction. If the symmetry of load or geometry is violated, or if only one panel in the compartment is affected by an elevated temperature, the conservative approach is to assume simple support along the long edges.

In the problem considered in this paper, temperature is nonuniform through the thickness as may be the case if fire occurs on one side of the panel. The distribution of temperature through the thickness depends on whether thermal conductivities of the constituent materials are affected by temperature. The thickness of the facings is usually relatively small, so that temperature can be assumed constant in each facing [4]. The principal temperature gradient is the core. For example, if the conductivity of the core is a linear function of temperature, i.e.

$$k = k_0 + k_1 T \tag{1}$$

where  $k_0, k_1$  are constants and  $T$  is a change of temperature from the reference value, the variation of temperature in the core (without taking account of a decomposition of the material) is given by [4]

$$T = -\frac{k_0}{k_1} + \sqrt{S_1 z + S_0} \tag{2}$$

Constants of integration  $S_0, S_1$  are determined from the thermal boundary conditions at the facing–core interfaces.

If the effect of temperature on the thermal conductivity of the core material is negligible, temperature varies linearly from the heated facing to the cooler facing. In the subsequent discussion, it is assumed that the distribution of temperature and material properties through the thickness are known.

The analysis is conducted modeling the sandwich panel by a first-order shear deformable theory. As a result of a uniform over the surface temperature varying in the thickness direction the central part of the panel (at a sufficient distance from the short edges) deforms into

a cylindrical surface. Accordingly, all derivatives with respect to the  $y$ -coordinate as well as the rotation in the  $yz$ -plane and the displacement in the  $y$ -direction are equal to zero. Accordingly, the equations of equilibrium for the panel are obtained simplifying the general three-dimensional equations of a first-order theory for shear-deformable composite laminates. For convenience, a geometrically nonlinear formulation [5] is recalled here and presented in the form simplified for the present application.

The strain in the facings that are assumed to be in the state of plane stress is composed of the contributions of the strain of the middle plane of the panel and the change of its curvature (both of them in the  $xz$ -plane)

$$\varepsilon_x = \varepsilon'_x + z\psi \tag{3}$$

where

$$\varepsilon'_x = u'_{,x} + \frac{1}{2} w_{,x}^2 \quad \kappa_x = \psi_{,x} \tag{4}$$

The core works in transverse shear and the corresponding strain is

$$\gamma_{xz} = \psi + w_{,x} \tag{5}$$

In these equations,  $u^0$  is a displacement of the middle plane in the  $x$ -direction,  $w$  is a deflection of the panel,  $\psi$  is the rotation in the  $xz$ -plane, and  $(\dots)_{,x} = \frac{d(\dots)}{dx}$ .

The stresses in the  $i$ th layer of the cross-ply facings are given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \end{Bmatrix}_i = \begin{bmatrix} Q_{11}(T_i) & Q_{12}(T_i) \\ Q_{12}(T_i) & Q_{22}(T_i) \end{bmatrix} \begin{Bmatrix} \varepsilon_x - \alpha_x(T_i) \\ -\alpha_y(T_i) \end{Bmatrix} \tag{6}$$

where  $Q_{mn}(T_i)$  and  $\alpha_p(T_i)$  are transformed reduced stiffnesses and the coefficients of thermal expansion, respectively, evaluated at the temperature of the layer.

The stress in the isotropic core is given by

$$\tau_{xz} = G_{xz}(T(z))\gamma_{xz} \tag{7}$$

where the shear modulus  $G_{xz}(T(z))$  is affected by the local temperature.

The equations of equilibrium of a panel bent into the cylindrical surface are [5,6]

$$N_{x,x} = 0, \quad Q_{x,x} + N_x w_{,xx} = 0, \quad Q_x = M_{x,x} \tag{8}$$

where stress resultants and stress couple are given by

$$\begin{aligned} N_x &= A_{11}\varepsilon'_x + B_{11}\kappa_x - N_x^T, \\ M_x &= B_{11}\varepsilon'_x + D_{11}\kappa_x - M_x^T, \quad Q_x = A_{55}\gamma_{xz} \end{aligned} \tag{9}$$

In these equations,  $A_{11}, A_{55}, B_{11}, D_{11}$  are the extensional, coupling, and bending stiffnesses introduced according to the standard definition (though the engineering constants employed to evaluate these stiffnesses are affected by temperature). The thermally induced stress resultant and stress couple acting in the sandwich panel with cross-ply

facings can be evaluated from

$$\{N_x^T, M_x^T\} = \int_{-h/2}^{h/2} [Q_{11}(T(z))\alpha_x(T(z)) + Q_{12}(T(z))\alpha_y(T(z))]T(z)\{1, z\}dz \quad (10)$$

where  $h$  is the total thickness of the panel. The contribution of the core to the in-plane stress resultants and to the bending stress couples is usually neglected, i.e. the integration can be performed over the thickness of the facings only, excluding the core. The first part of the subsequent analysis (Section 2.1) presents the solution for the geometrically ‘pseudo-linear problem’ where bending deformations of the panel are small. Nevertheless, nonlinearity is present since the axial stress resultant is affected by stretching of the middle plane. The second part (Section 2.2) shows the stress analysis of the panel (this can be applied to both pseudo-linear and nonlinear problems). Finally, Section 2.3 illustrates the approach to the nonlinear analysis, accounting for moderately large deflections. The solution of the linear problem is exact. The nonlinear problem is reduced to a system of seven algebraic nonlinear equations for six constants of integration and the axial stress resultant. Exact solution of this system may be impossible, but the accuracy is limited only by the method of solution.

### 2.1. Geometrically pseudo-linear bending problem

In this formulation, the nonlinear term is neglected in the first relation in Eq. (4). The equations of equilibrium become

$$\begin{aligned} A_{11}u_{,xx}^0 + B_{11}\psi_{,xx} - N_{x,x}^T &= 0, \\ A_{55}(\psi_{,x} + w_{,xx}) + N_x w_{,xx} &= 0, \\ D_{11}\psi_{,xx} - A_{55}(\psi + w_{,x}) + B_{11}u_{,xx}^0 - M_{x,x}^T &= 0 \end{aligned} \quad (11)$$

Note that the thermally induced axial stress resultant and bending moment do not vary in the axial direction (temperature is uniform over the surface of the panel). Accordingly, the last terms in the first and last relations in Eqs. (11) disappear. Although Eqs. (11) seems linear, the nonlinearity is introduced through the axial restraint  $N_x$  that depends on the magnitude of deflections.

The solution of Eqs. (11) must satisfy the boundary conditions. If the panel is simply supported, these conditions are:

At  $x=0, x=a$ :

$$u^0 = w = 0, \quad M_x = B_{11}u_{,x}^0 + D_{11}\psi_{,x} - M_x^T = 0 \quad (12)$$

Omitting the last term in the first relation in (11) yields

$$u_{,xx}^0 = -\frac{B_{11}}{A_{11}}\psi_{,xx} \quad (13)$$

Substituting Eq. (13) into the last two Eqs. (11) gives the following result

$$w_{,x} = -\psi + \frac{1}{A_{55}} \left( D_{11} - \frac{B_{11}^2}{A_{11}} \right) \psi_{,xx} \quad (14)$$

and

$$\psi_{,xxx} + \lambda\psi_{,x} = 0 \quad (15)$$

where

$$\lambda = \frac{-N_x}{\left(1 + \frac{N_x}{A_{55}}\right) \left(D_{11} - \frac{B_{11}^2}{A_{11}}\right)} \quad (16)$$

The problem can be reduced to the classical result for a thin symmetrically laminated plate if  $\psi = -w_{,x}$ ,  $A_{55}=0$ ,  $B_{11}=0$ . Then if the plate is simply supported, it is easy to show that  $N_{x,cr} = -D_{11}(\pi/a)^2$ .

If thermally induced compressive stresses are large,  $N_x$  is negative. Consider the case where

$$1 + \frac{N_x}{A_{55}} < 0 \quad (17)$$

so that  $\lambda > 0$ . Accordingly, the solution of Eq. (15) is

$$\psi = C_1 + C_2 \sin\sqrt{\lambda}x + C_3 \cos\sqrt{\lambda}x \quad (18)$$

where  $C_i$  are constants of integration.

The solution of Eq. (14) becomes

$$w = C_4 - C_1x + f(x)C_2 \cos\sqrt{\lambda}x - f(x)C_3 \sin\sqrt{\lambda}x \quad (19)$$

where

$$f(x) = \frac{1}{\sqrt{\lambda}} + \frac{D_{11} - (B_{11}^2/A_{11})}{A_{55}} \sqrt{\lambda} \quad (20)$$

Note that the integration of Eq. (14) added an additional constant of integration.

Finally, integrating Eq. (13) yields

$$u^0 = C_5 + C_6x - \frac{B_{11}}{A_{11}}(C_1 + C_2 \sin\sqrt{\lambda}x + C_3 \cos\sqrt{\lambda}x) \quad (21)$$

Six constants of integration in Eqs. (18), (19) and (21) can be determined from six boundary conditions (12). Therefore, given the prescribed value of the thermally induced stress couple, one can determine  $u^0$ ,  $w$ ,  $\psi$  as functions of the stress resultant  $N_x$ . However, this solution is not sufficient to predict thermal bending in terms of applied temperature. This is because  $N_x$  accounts for two effects: (i) the thermally induced stress resultant  $N_x^T$  based on the solution of the heat-transfer problem and Eq. (10) and (ii) deflections of the panel that result in stretching of its middle plane.

Integrating the first Eq. (9) between  $x=0$  and  $x=a$ , one obtains the relationship between the axial stress resultant

and deformations  $u^0$  and  $\psi$ :

$$N_x = N_x^T - \frac{1}{a} \left( A_{11} \int_0^a u_{,x}^0 dx + B_{11} \int_0^a \psi_{,x} dx \right) \quad (22)$$

Now, given a distribution of temperature and stiffness through the thickness of the panel, it is possible to determine six constants of integration and  $N_x$  from seven linear Eqs. (12) and (22).

In the case where compression is small, the inequality (17) is not satisfied,  $\lambda < 0$ , and the solution of Eq. (15) becomes

$$\psi = C_1 + C_2 \sinh \sqrt{|\lambda|x} + C_3 \cosh \sqrt{|\lambda|x} \quad (23)$$

The approach to the analysis, similar to that described above for the case where  $\lambda > 0$ , is omitted for brevity. Of course, the sign of  $\lambda$  is unknown in advance, and the solution may have to be repeated if this sign was not guessed correctly. Note that Thornton considered the problem of bending of an isotropic beam subject to a nonuniform temperature using a similar approach [7].

### 2.2. Stress analysis

The thermally induced strains in the facings assumed in the state of plane stress are given by (inequality (17) is satisfied)

$$\begin{aligned} \epsilon_x = u_{,x}^0 + z' \psi_{,x} = C_6 - \frac{B_{11} \sqrt{\lambda}}{A_{11}} \left( C_2 \cos \sqrt{\lambda}x - C_3 \sin \sqrt{\lambda}x \right) \\ + z' \sqrt{\lambda} \left( C_2 \cos \sqrt{\lambda}x - C_3 \sin \sqrt{\lambda}x \right) \end{aligned} \quad (24)$$

where  $z'$  is a coordinate of the point where the strain is evaluated. Now the stresses can be calculated in each layer of the facings by Eq. (6).

The transverse shear strains in the core are

$$\gamma_{xz} = C_2 \left( 1 - f(x) \sqrt{\lambda} \right) \sin \sqrt{\lambda}x + C_3 \left( 1 - f(x) \sqrt{\lambda} \right) \cos \sqrt{\lambda}x \quad (25)$$

The core carries only shear stresses available from Eq. (7). Note that contrary to thin facing layers, where it is possible to use the average-through-the thickness value of temperature, the variations of temperature through the thickness of the core are significant. Accordingly, the shear modulus is a function of the  $z$ -coordinate.

### 2.3. Geometrically nonlinear problem

The first equilibrium relation in Eq. (8), written to account for the nonlinear strain–displacement Eq. (4), yields the axial displacement at the middle plane. It is a nonlinear function of deflections:

$$u_{,xx}^0 = -\frac{B_{11}}{A_{11}} \psi_{,xx} - w_{,x} w_{,xx} \quad (26)$$

The second Eq. (8) is not explicitly affected by nonlinear effects. It follows that

$$\psi_{,x} = -\left( 1 + \frac{N_x}{A_{55}} \right) w_{,xx} \quad (27)$$

Substituting Eq. (27) into Eq. (26) results in the expression for  $u^0$  as a function of  $w$ :

$$u_{,xx}^0 = \frac{B_{11}}{A_{11}} \left( 1 + \frac{N_x}{A_{55}} \right) w_{,xxx} - w_{,x} w_{,xx} \quad (28)$$

Eq. (27) can be integrated yielding the expression for the rotation

$$\psi = -\left( 1 + \frac{N_x}{A_{55}} \right) w_{,x} + C'_4 \quad (29)$$

where  $C'_4$  is a constant of integration.

The nonlinear version of the third equilibrium Eq. (8) is

$$D_{11} \psi_{,xx} + B_{11} (u_{,xx}^0 + w_{,x} w_{,xx}) - A_{55} (\psi + w_{,x}) - M_{xx}^T = 0 \quad (30)$$

After the substitution of Eqs. (28) and (29) and transformations, this equation assumes the form

$$w_{,xxx} - \lambda w_{,x} - \eta C'_4 = 0 \quad (31)$$

where  $\lambda$  is defined by Eq. (16) and

$$\eta = \frac{A_{55}}{(D_{11} - (B_{11}^2/A_{55})) \left( 1 + \frac{N_x}{A_{55}} \right)} \quad (32)$$

Nonlinear terms cancel out in Eq. (31), so that the solution can be obtained in the closed form.

The integration of Eq. (31) yields (for the case where  $\lambda > 0$ )

$$w = C'_1 + C'_2 \sinh \sqrt{\lambda}x + C'_3 \cosh \sqrt{\lambda}x - \frac{\eta C'_4}{\lambda} x \quad (33)$$

in which we have three additional constants of integration. Note that a deviation of the nonlinear solution from its linear counterpart is reflected in a difference between hyperbolic and trigonometric functions that becomes essential only at large values of the argument.

Finally, integrating Eq. (28) twice results in the solution for the axial displacement

$$\begin{aligned} u^0 = C'_5 + C'_6 x + \frac{B_{11}}{A_{11}} \left( 1 + \frac{N_x}{A_{55}} \right) \left( C'_2 \sqrt{\lambda} \cosh \sqrt{\lambda}x \right. \\ \left. + C'_3 \sqrt{\lambda} \sinh \sqrt{\lambda}x - \frac{\eta}{\lambda} C'_4 \right) - F(C'_2, C'_3, C'_4) \end{aligned} \quad (34)$$

where  $F(C'_2, C'_3, C'_4)$  is a nonlinear function that is easily evaluated. Two additional constants of integration in Eq. (34) bring the total number of constants that have to be specified to six (as in the linear case).

Six constant of integration have to be determined from the boundary conditions that do not differ from those for

the linear case, except for the expression for the bending moment:

At  $x=0, x=a$ :

$$u^0 = w = 0, \quad M_x = B_{11}(u_{,x}^0 + \frac{1}{2}w_{,x}^2) + D_{11}\psi_{,x} - M_x^T = 0 \tag{35}$$

The solution procedure is similar to the linear case, though it may be tedious due to nonlinearities. The axial stress resultant is related to the deformations by the nonlinear version of Eq. (22):

$$N_x = N_x^T - \frac{1}{a} \left( A_{11} \int_0^a (u_{,x}^0 + \frac{1}{2}w_{,x}^2) dx + B_{11} \int_0^a \psi_{,x} dx \right) \tag{36}$$

### 3. Numerical results

The solution of the nonlinear problem requires us to determine seven unknowns ( $C'_1, C'_2, C'_3, C'_4, C'_5, C'_6, \lambda$ ) from six boundary conditions and Eq. (36). This nonlinear problem was solved by the Newton–Raphson iteration method.

The room temperature properties of the layers of cross-ply facings considered in the examples corresponded to a typical graphite/epoxy:

$$E_1 = 120.87 \text{ GPa}, \quad E_2 = 18.58 \text{ GPa}, \quad \mu_{12} = 0.276,$$

$$\alpha_1 = 1.977 \text{ } \mu\text{m/m per } ^\circ\text{C}, \quad \alpha_2 = 3.2881 \text{ } \mu\text{m/m per } ^\circ\text{C}$$

The variations of the stiffness of the facings with temperature are reflected in Table 1, obtained by assuming that the effect of temperature is similar to that for glass/epoxy [8]. The room-temperature properties of two grades of Divinycell core considered in the examples are presented in Table 2.

The variation of the shear modulus of the core with temperature is assumed in the form  $G_c(T) = G_c[1 - G_1(T/T_{rf})]$ , where  $G_c$  is the value at the room temperature  $T_{rf}$  and  $G_1$  is a nonlinear function of  $T/T_{rf}$  given by

$$G_1 \left( \frac{T}{T_{rf}} \right) = 0.0068 \left( \frac{T}{T_{rf}} \right)^3 - 0.070 \left( \frac{T}{T_{rf}} \right)^2 + 0.27 \frac{T}{T_{rf}} - 0.29 \tag{37}$$

Eq. (37) is applicable if the ratio  $T/T_{rf} > 3$ . Note that Eq. (37) yields variations of the shear modulus of the core with temperature that are in a qualitative agreement with those reported for a typical thermoplastic foam [9].

Table 1  
Variations of the stiffness of the facing material with temperature

$T$ ( $^\circ\text{C}$ )	20.0	50.0	75.0	100.0	125.0
$E1$ (GPa)	120.87	97.20	87.30	76.06	67.73
$E2$ (GPa)	18.58	14.94	13.42	11.69	10.41

Table 2  
Stiffness of the core materials at room temperature

Material	H45	H60
$E$ (MPa)	40.0	60.0
$G$ (MPa)	18.0	22.0

The length of the short edges of the sandwich panel considered in the examples was  $a = 101.6$  mm, the core was 20 mm thick and two thickness of the facings was either  $h_f = 2.5$  or 5.0 mm. As was shown in Ref. [4] for the case of quasi-isotropic glass vinyl ester facings and a polymeric 20 mm thick core, temperature remains practically uniform in the facings and its variation is mostly limited to the core. An example of such analysis presented in Table 3 confirms the validity of this statement. In this table,  $T_0, T_1, T_2$  and  $T_3$  are temperatures of the surface of the heated facing, heated facing–core interface, colder facing–core interface and the surface temperature of the colder facing, respectively. Accordingly, in the following examples, the temperature of the heated facing was assumed constant and equal to  $T_0$ , while the temperature of the opposite facing was found from the static heat transfer problem, assuming that the air outside this facing is at 20  $^\circ\text{C}$ .

The axial stress resultant is shown as a function of the temperature of the heated facing in Fig. 2 and Table 4. The axial force increases from zero to a maximum value but once temperature reaches a certain level, the tendency is reversed, i.e. the axial force begins to decrease (the increasing force at temperatures below 50  $^\circ\text{C}$  is not shown). Physically, such decrease is due to tensile reactive stresses at the immovable in the  $x$ -direction edges attributed to stretching produced by bending (similar to the reaction of immovable supports of a beam subject to large bending deformations). The thickness of the facings has little effect on the magnitude of the force.

The shape and magnitude of deflections along the span of the panel are shown in Fig. 3 (a similar result obtained for a panel with thicker facings is omitted for brevity). As follows from this figure, deflections increase until temperature reaches the value of  $T_0 = 100$   $^\circ\text{C}$  and begin to decrease at higher temperatures. Deflections of the panels with various facing thickness and core materials are also shown as a function of temperature in Fig. 4. As is clearly observed in this figure, deflections experience a rapid buildup as temperature varies between the room value and about 60  $^\circ\text{C}$ . At higher temperatures, the increase of deflections

Table 3  
Temperature distribution along the facing–core interfaces and on the surfaces of a sandwich panel ( $h_f = 5.0$  mm)

	50.0	75.0	100.0	125.0
$T_0$	50.0	75.0	100.0	125.0
$T_1$	49.7	74.5	99.3	124.0
$T_2$	25.4	29.8	34.3	38.7
$T_3$	25.1	29.3	33.5	37.8

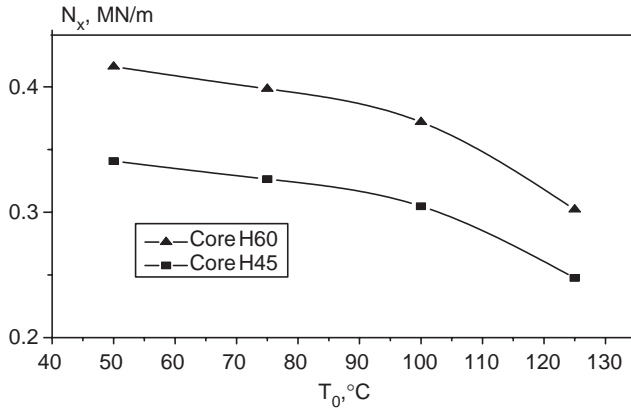


Fig. 2. Absolute value of the axial edge restraint stress resultant ( $MN \times m$ ) as a function of the exposed surface temperature  $T_0$ . The facings are 2.5 mm thick.

Table 4  
Absolute value of the axial edge restraint stress resultant ( $MN \times m$ ) as a function of the exposed surface temperature  $T_0$

$T$ (°C)	50.0	75.0	100.0	125.0
H45 ( $h_f = 2.5$ mm)	0.340791	0.326334	0.304722	0.247510
H60 ( $h_f = 2.5$ mm)	0.416041	0.398344	0.371917	0.302148
H45 ( $h_f = 5.0$ mm)	0.341365	0.327056	0.305511	0.248027
H60 ( $h_f = 5.0$ mm)	0.417263	0.399664	0.373288	0.303093

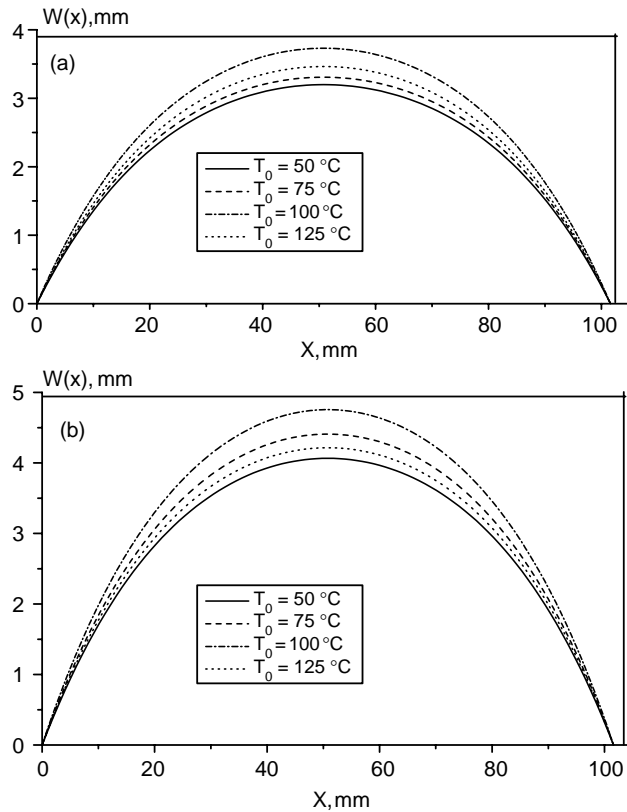


Fig. 3. Maximum deflections of the panel as a function of the temperature of the heated surface  $T_0$ . The thickness of the facings is  $h_f = 2.5$  mm. Case (a): core H45; Case (b): core H60.

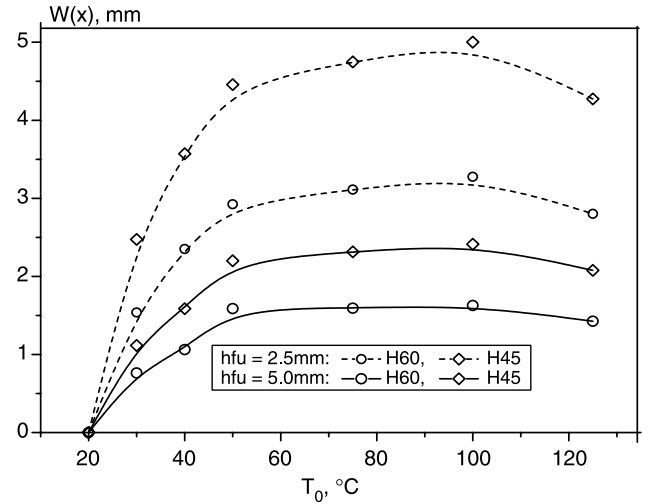


Fig. 4. Maximum deflections of the panel as a function of the temperature of the heated surface  $T_0$ , the thickness of the facings and the material of the core.

slows down and eventually reverses itself. Predictably, a thicker core resulted in a decrease in deflections.

Note that the phenomena of the reversal of deflections with a higher temperature have been reported in literature. In particular, Meyers and Hyer observed such a reversal of deflections of a composite panel subject to a linearly distributed through thickness temperature [10]. A recent paper of Lattimer et al. [11] on deformations of sandwich panels subjected to fire also supports the observations in the present paper.

A distribution of the maximum transverse shear stresses in the core throughout the span of the panel is depicted in Fig. 5. The variations of these stresses at the supports where they reach the extreme values are shown in Fig. 6 as a function of temperature. The observed

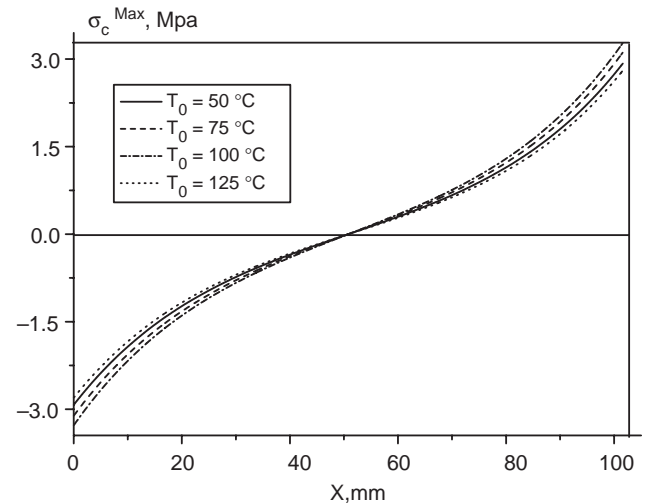


Fig. 5. A distribution of the maximum shear stresses along the span of the panel as a function of the temperature of the heated surface  $T_0$ . The core is H45, the thickness of the facings is  $h_f = 2.5$  mm.



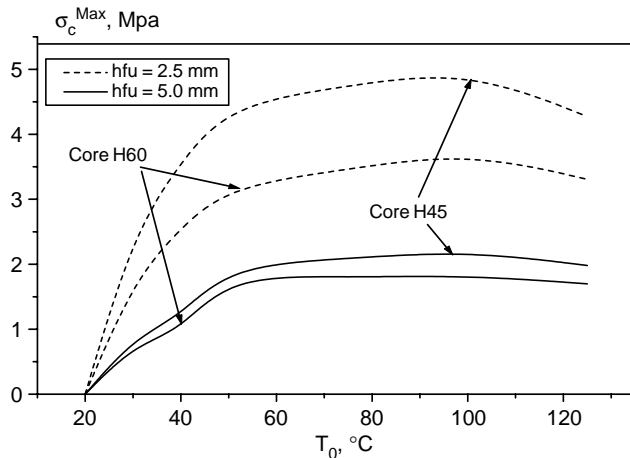


Fig. 6. Maximum transverse shear stress as a function of the temperature of the heated surface  $T_0$ , the thickness of the facings and the material of the core.

tendency in variations of deflections with an increasing temperature is mirrored by the similar trend for the transverse shear stress. The magnitude of shear stress was reduced in the panels with a thicker core, reflecting a decrease in deflections in such panels. Note that the magnitudes of the stresses at the supports shown in Fig. 6 indicate that the loss of shear strength may become a possible mode of failure in the problem considered in the paper.

#### 4. Conclusions

The problem of deformations and stresses in a large aspect ratio sandwich panel subject to an elevated temperature on one of the surfaces that results in cylindrical bending is considered in the paper. The panel is simply supported and the long edges are prevented from in-plane displacements. The formulation enables us to account for a deterioration of the properties of the constituent phases, i.e. the matrix of the facings and the core material. Both geometrically nonlinear as well as transverse shear deformations are taken into account. While a simultaneous effect of geometrically nonlinear and transverse shear deformations is seldom encountered in practical problems, in the presence of an elevated temperature it may become essential due to a degradation of the material properties combined with resin decomposition.

As follows from the analysis of deformations and stresses of representative panels, they experience a significant buildup as temperature of the heated surface increases from the room value to about 60 °C. At higher temperatures, the rate of this buildup slows and eventually, at the value of the surface temperature exceeding about 100 °C, deflections and stresses begin to decline. Such behavior is in agreement

with theoretical and experimental results reported in literature.

An increase in the thickness of the facings does not substantially change the distribution of temperature through the thickness as the temperature of each facing practically does not vary between the surface of the facing and its interface with the core. Such relatively little effect of the thickness of the facings on a distribution of temperature was explained by a mismatch between thermal conductivities of typical facing and core materials [4]. Thicker facings resulted in smaller deflections of the panel and reduced stresses.

As follows from numerical examples dealing with the maximum transverse shear stress in the core that occurs along the edges of the panel, this stress may reach dangerous levels, even if temperature changes are relatively modest. In the panels with thin facings experiencing significant deformations, the failure of the core may become a possibility.

It is emphasized that the stress analysis of sandwich panels operating in high temperature environments should account for variations in the strengths of facings and core associated with the instantaneous temperature values at the point of interest. In addition, it is necessary to account for the process of resin decomposition that may significantly affect the strength.

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