Available online at www.sciencedirect.com
SCIENCE DIRECT®
Composites: Part A 37 (2006) 972–980
composites
Part A: applied science and manufacturing
www.elsevier.com/locate/compositesa

Thermal buckling of a heat-exposed, axially restrained composite column

L. Liu[a], G.A. Kardomateas[a,*], V. Birman[b], J.W. Holmes[a], G.J. Simitses[a]

[a]School of Aerospace Engineering, Georgia Institute of Technology, North Avenue and Lucky Street, Room 308, Atlanta, GA 30332-0150, USA
[b]Department of Mechanical Engineering, University of Missouri-Rolla, Rolla, MO, USA

Received 25 June 2004; revised 29 March 2005; accepted 12 April 2005

Abstract

The response of composite columns under axial compressive loading, and in which a non-uniform temperature distribution through the thickness exists, is investigated. This non-uniform temperature distribution can develop when one side of the structures is exposed to heat flux. In this paper, we assume that this distribution is linear, which corresponds to a steady state temperature profile due to heat conduction. The degradation of the elastic properties with temperature (especially near the glass transition temperature of the matrix) is accounted for, by using experimental data for the elastic moduli. Furthermore, the formulation includes transverse shear and it is done first for the general non-linear case and subsequently linearized. Due to the non-uniform stiffness and the effect of the ensuing thermal moment, the structure behaves like an imperfect column, and responds by bending rather than buckling in the classical Euler (bifurcation) sense. Another important effect of the non-uniform temperature is that the neutral axis moves away from the centroid of the cross-section, resulting in another moment due to eccentric loading, which would tend to bend the structure away from the heat source. Simple equations for the response of the column are derived and results are presented for the variation of the deflection with the heat flux, as well as for the combined effects of the applied load and heat flux. It is found that the thermal moment would tend to bend the structure away from the heat source for small temperatures (small heat fluxes) but towards the heat source for large temperatures. On the contrary, the moment induced due to the eccentric loading would always tend to bend the structure away from the heat source. Results indicate the combined influences of these moments and that of axial constraint.
© 2005 Elsevier Ltd. All rights reserved.

Keywords: D. Thermal analysis; B. Buckling; C. Analytical modelling

1. Introduction

Fiber-reinforced polymeric composites are used extensively in aerospace, marine, infrastructure and chemical processing applications. In these applications, events creating a heat flux (e.g. due to fire), and their resulting effects on the structural integrity, are of considerable concern. In addition to the implications for design, quantitative information regarding the nature of the strength loss is required to make decisions regarding, for example, the seaworthiness of a ship that has sustained fire damage.

Many of the thermal properties of composites related to fire have been thoroughly studied and are well understood, including ignition times, heat release rates, smoke production rates and gas emissions [1–7]. Also, some recent work into the post-fire residual properties has been conducted, for example, a preliminary investigation into the effect of fire damage on the edgewise compression properties and failure mechanisms of sandwich composites [7] showed large reductions to the edgewise compression properties of phenolic-based sandwich composites despite having good flame resistance. However, one important gap in the understanding of composites is their response and structural integrity due to the combined effect of mechanical loading and thermal loading due to fire. This paper addresses this issue as far as compressive loading, which in an otherwise purely mechanical loading (no fire) would lead to bifurcational (Euler) buckling.

One important characteristic of fiber-reinforced polymeric composites is that increases in temperature cause a gradual softening of the polymer matrix material with a significant effect near the glass transition temperature, T_g.

* Corresponding author. Tel.: +1 404 894 8198; fax: +1 404 894 9313.
E-mail address: george.kardomateas@aerospace.gatech.edu (G.A. Kardomateas).

1359-835X/$ - see front matter © 2005 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compositesa.2005.04.006

Recent experiments on E-glass vinyl ester composites, conducted by Kulkarni and Gibson [8] can be used as a basis for including the resulting non-uniform stiffness distribution.

When heat flux is applied on one side of a column/plate, a non-uniform temperature develops through the thickness. Since the modulus of elasticity of polymeric composites depends on temperature, this non-uniform temperature results in a non-uniform distribution of stiffness through the thickness. In addition, a thermal moment is developed, which causes bending of the column from the very start of heat exposure when only the slightest change of temperature occurs. An additional 'eccentricity moment' is created since the non-uniform temperature induces a shift of the neutral axis away from the centroid of the cross-section. Thus, the column bends like an imperfect beam (even if it is initially straight) and cannot buckle in the classical Euler (bifurcation) sense. In this paper, we investigate the general bending response of such a column that is pinned at both ends, with an applied axial force. In simple terms, the column is subjected to both (1) an axial force that can cause buckling at the Euler load if it is large enough; (2) a thermal moment that causes bending immediately; and (3) an 'eccentricity moment' induced by the neutral axis moving away from the centroid of the cross-section, resulting in eccentric loading. The details of the formulation are outlined in Section 2.

2. Formulation

Let us assume a symmetric cross-section of thickness h. First, we derive the temperature distribution through the thickness. Assuming steady state, the temperature, T, satisfies the heat conduction equation

$$\frac{d^2T}{dy^2} = 0; \quad -\frac{h}{2} < y < \frac{h}{2}. \tag{1a}$$

At the side of the fire, $y = -h/2$, a constant heat flux, Q, is supplied by the fire:

$$-K\frac{dT}{dy} = Q \quad \text{at} \quad y = -h/2, \tag{1b}$$

and at the other side, $y = h/2$, we have a 'radiation' boundary condition to the air at room temperature, T_0:

$$-K\frac{dT}{dy} = H(T - T_0) \quad \text{at} \quad y = h/2. \tag{1c}$$

In these equations, K is the thermal conductivity, and H is the surface conductivity from the composite surface to the air. Of course, the temperature could be regarded as a function of time and in this case the heat conduction equation can be solved with the help of Laplace transforms [9], but we shall not address this problem in this paper.

It is easy to see that the steady temperature distribution, which satisfies (1a)–(1c), is linear and given by

$$T - T_0 = \Delta T = -\frac{Q}{K}y + \frac{Q}{KH}\left(K + H\frac{h}{2}\right) = \frac{Q}{K}(-y + c), \tag{2a}$$

where

$$c = \frac{K}{H} + \frac{h}{2}. \tag{2b}$$

This is the temperature distribution, which will be used in the following; we shall also denote by T_Q the surface temperature on the side of the impending heat flux.

Regarding the stiffness, E, it is well known that the modulus of polymers depends strongly on the temperature and especially on how close the temperature is to its glass transition temperature, T_g. Around T_g, the modulus drops by a factor of 100 or more [10]. For composites with polymeric matrices there is not much data in the literature yet, but it is logical to expect a noteworthy dependence on temperature, T not as strong as with pure polymers. In fact, a recent paper by Kulkarni and Gibson [8] studied the effects of temperature on the elastic modulus of E-glass/vinyl-ester composites. By using vibrating beam samples and the impulse–frequency response method, they provided measurements of temperature dependence of the elastic modulus of the composite in the range of 20–140 °C. The glass transition temperature of the matrix was $T_g = 130$ °C. Near this temperature the Young's modulus shows a significant variation but below T_g the variation is small. If we denote by E_0 the modulus at room temperature, $T_0 = 20$ °C, then we have reduced the data in Kulkarni and Gibson [8] to the following equation for the modulus as a function of temperature, $E(T)$:

$$\begin{aligned}\frac{E}{E_0} &= 1 - a_1\left(\frac{T - T_0}{T_g - T_0}\right) + a_2\left(\frac{T - T_0}{T_g - T_0}\right)^2 - a_3\left(\frac{T - T_0}{T_g - T_0}\right)^3 \\ &= 1 - a_1\left(\frac{\Delta T}{\Delta T_g}\right) + a_2\left(\frac{\Delta T}{\Delta T_g}\right)^2 - a_3\left(\frac{\Delta T}{\Delta T_g}\right)^3.\end{aligned} \tag{3}$$

For the present E-glass/vinyl-ester, $E_0 = 3 \times 10^8$ psi and $a_1 = 0.348$, $a_2 = 0.715$ and $a_3 = 0.843$. The composite studied had a fiber volume fraction of 0.516 and consisted of four sub-layers with the orientation of each sub-layer [0/90/+45/−45/Random].

The above equation captures the physics of the nonlinear dependence of the composite on the glass transition temperature of the matrix, T_g, and is expected to be able to represent such behavior for other composites as well (with proper adjustment of the coefficients). Around T_g of the matrix, the modulus of this composites drops to about half its value at room temperature.

It is assumed that the composite remains in its original condition and there is no charring. Next, let us define an 'average' modulus, E_{av}, and a 'first and second moment' of the modulus with respect to the mid-surface x-axis, E_{m1} and E_{m2}, respectively, by:

$$E_{av}A = \int_A E\,\mathrm{d}A; \quad E_{m1}hA = \int_A Ey\,\mathrm{d}A; \quad E_{m2}I = \int_A Ey^2\,\mathrm{d}A. \tag{4a}$$

Then by using (2) and (3) we obtain the 'average' modulus

$$E_{av} = E_0\left[1 - a_1\frac{Q}{K\,\Delta T_g}c + a_2\left(\frac{Q}{K\,\Delta T_g}\right)^2\left(c^2 + \frac{I}{A}\right) - a_3\left(\frac{Q}{K\,\Delta T_g}\right)^3\left(c^3 + 3c\frac{I}{A}\right)\right], \tag{4b}$$

the 'first moment' of the modulus

$$E_{m1} = E_0\left(\frac{Q}{K\,\Delta T_g}\right)\frac{I}{Ah}\left[a_1 - a_2\left(\frac{Q}{K\,\Delta T_g}\right)2c + a_3\left(\frac{Q}{K\,\Delta T_g}\right)^2\left(3c^2 + \frac{I^*}{I}\right)\right], \tag{4c}$$

and the 'second moment' of the modulus:

$$E_{m2} = E_0\left[1 - a_1\left(\frac{Q}{K\,\Delta T_g}\right)c + a_2\left(\frac{Q}{K\,\Delta T_g}\right)^2\left(c^2 + \frac{I^*}{I}\right) - a_3\left(\frac{Q}{K\,\Delta T_g}\right)^3\left(c^3 + 3c\frac{I^*}{I}\right)\right], \tag{4d}$$

In the above relations, A is the cross-sectional area, I is the moment of inertia ($I = \int_A y^2\,\mathrm{d}A$) and the I^* is an area property defined as:

$$I^* = \int_A y^4\,\mathrm{d}A. \tag{4e}$$

Also, in deriving (4b)–(d), we have made use of the fact that, due to the symmetric section,

$$\int_A y\,\mathrm{d}A = \int_A y^3\,\mathrm{d}A = \int_A y^5\,\mathrm{d}A = 0.$$

For a rectangular cross-section of width b and height h, $A = bh$, $I = bh^3/12$ and $I^* = bh^5/80$.

Due to the non-uniform modulus, the neutral axis of the column is not at the mid-surface. The distance, e, of the neutral axis from the mid-surface axis, x, is determined from:

$$e\int_A E(y)\mathrm{d}A = \int_A E(y)y\mathrm{d}A, \tag{4f}$$

which, by use of (4a) leads to

$$e = E_{m1}h/E_{av}, \tag{4g}$$

where E_{av} and E_{m1} are defined in (4b) and (4c).

Assuming a thermal expansion coefficient, α, independent of temperature, the thermal force is:

$$N_x^T = \int_A E\alpha\,\Delta T\,\mathrm{d}A, \tag{5a}$$

which, by use of (2) and (3), results in:

$$N_x^T = E_0\frac{Q\alpha}{K}\left[cA - a_1\frac{Q}{K\,\Delta T_g}(c^2A + I) + a_2\left(\frac{Q}{K\,\Delta T_g}\right)^2 \times (c^3A + 3cI) - a_3\left(\frac{Q}{K\,\Delta T_g}\right)^3(c^4A + 6c^2I + I^*)\right], \tag{5b}$$

if the thermal force can be developed due to the constraints at the ends of the beam, it cause the bar to buckle; however, the problem is not a bifurcation because a thermal moment is also developed due to the non-uniform stiffness distribution.

The thermal moment (with respect to the neutral axis of the beam) is:

$$M_x^T = \int_A E\alpha\,\Delta T(y - e)\mathrm{d}A, \tag{5c}$$

Again, by use of (2) and (3), and with the definitions of (4f) and (4g), the integral in (5c) is found to be:

$$M_x^T = -E_0\frac{Q\alpha}{K}\left[I - a_1\frac{Q}{K\,\Delta T_g}2cI + a_2\left(\frac{Q}{K\,\Delta T_g}\right)^2 \times (3c^2I + I^*) - a_3\left(\frac{Q}{K\,\Delta T_g}\right)^3 4c(c^2I + I^*)\right] - eN_x^T, \tag{5d}$$

This thermal moment would cause bending of the column. In deriving (5b) and (5d), we have made use again of the fact that, due to the symmetric section,

$$\int_A y\,\mathrm{d}A = \int_A y^3\,\mathrm{d}A = \int_A y^5\,\mathrm{d}A = 0.$$

The problem now is to determine the response of the column under the influence of both N_x^T and M_x^T, which as has already been noted, changes the character of the problem from bifurcation buckling to a bending or bowing problem. We shall consider two cases: (a) immovable ends (constrained), in which case an axial reaction develops at the supports, and (b) ends free to move axially and an axial compressive load is applied.

First of all, we assume immovable conditions at both pinned ends. In this case, an axial reaction force, P, develops as the heat flux is applied. The axial force N_x does not vary with axial position x [11]. Thus, it can be seen N_x equal to $-P$, the support force at the ends, due to the axial equilibrium. However, unlike the case of a uniformly heated column, the force P is less than N_x^T because the transverse deflection, $v(x)$, of the bar during heating decreases the axial force at the supports. That is, the column bends away from its original straight configuration and relieves some of the force at the immovable ends. Note that the known N_x^T is greater than the unknown P, which we must find. Thus, P is a derived quantity, not a controlled quantity. The controlled quantity is the thermal loading due to the fire, and the response quantity is the mid-span transverse deflection of the bar.

Let us denote by u_0 and v_0 the x and y displacements at the neutral axis and by θ the rotation of the cross-section due to bending. The non-linear strain at the neutral axis, $y=e$, is:

$$\epsilon_0(x) = u_{0,x} + \tfrac{1}{2}\theta^2. \tag{6a}$$

In the following we account for transverse shear following the procedure in [12]. In particular, we can set:

$$\frac{\mathrm{d}v_0}{\mathrm{d}x} = \sin(\theta + \gamma_{\mathrm{eq}}), \tag{6b}$$

where γ_{eq} is the equivalent shear angle, i.e. the difference between the slope of the deflected beam axis and the rotation θ of the cross-section due to bending.

It is reasonable to assume that the shear modulus, G, will change with temperature in the same manner as the extensional modulus, E. Therefore, taking into account (2) and (3), we can write:

$$G(y) = G_0\left[1 - a_1\frac{Q}{K\,\Delta T_{\mathrm{g}}}(-y+c) + a_2\left(\frac{Q}{K\,\Delta T_{\mathrm{g}}}\right)^2 \times(-y+c)^2 - a_3\left(\frac{Q}{K\,\Delta T_{\mathrm{g}}}\right)^3(-y+c)^3\right]. \tag{6c}$$

An effective shear modulus, $\bar{G}$ is now defined based on the shear compliance as [12]:

$$h\bar{G} = \int_{-h/2}^{h/2}\frac{\mathrm{d}y}{G(y)}. \tag{6d}$$

This integral is evaluated numerically as a simple closed form expression cannot be obtained.

The equivalent shear angle, γ_{eq}, is then defined as [12]

$$\gamma_{\mathrm{eq}} = \frac{\beta P\sin\theta}{\bar{G}A}, \tag{6e}$$

where β is the shear correction factor which accounts for the non-uniform distribution of shear stresses throughout the cross-section.

Then, the strain at an arbitrary point, $\bar{\epsilon}(x,y)$, can be represented by:

$$\bar{\varepsilon}(x,y) = \varepsilon_0(x) - (y-e)\frac{\mathrm{d}(\theta+\gamma_{\mathrm{eq}})}{\mathrm{d}x}. \tag{6f}$$

When the resulting stress from (6f) is integrated through the section, the resultant should equal $-P+N_x^T$, i.e.:

$$\int_A E(y)\bar{\varepsilon}(x,y)\mathrm{d}A = -P + N_x^T. \tag{6g}$$

Then, by use of (4a), (6a) and (6e), becomes:

$$E_{\mathrm{av}}A\left(u_{0,x} + \frac{1}{2}\theta^2\right) + (E_{\mathrm{av}}e - E_{m1}h)A\left(1 + \frac{\beta P\cos\theta}{\bar{G}A}\right)\theta_{,x} = N_x^T - P, \tag{6h}$$

which, by use of (4g), results in:

$$u_{0,x} = \frac{N_x^T - P}{E_{\mathrm{av}}A} - \frac{1}{2}\theta^2, \tag{6i}$$

which we can integrate over the length of the column subject to the boundary conditions that the ends are restrained in the axial direction, i.e. $u_0(0)=0$ and $u_0(L)=0$.

Therefore, we obtain the following

$$(N_x^T - P)\frac{L}{E_{\mathrm{av}}A} - \frac{1}{2}\int_0^L \theta^2\,\mathrm{d}x = 0, \tag{6j}$$

which is applicable for the entire loading range of the column and is a ‘constraint equation’ expressing the condition that the overall change in displacement between the end supports must be zero because the two ends of the beam are immovable and there is a support load P.

Now, the bending rigidity, $(EI)_{\mathrm{eq}}$ of the column, is likewise influenced by the non-uniform stiffness and is defined by:

$$(EI)_{\mathrm{eq}} = \int_A E(y)(y-e)^2\,dA, \tag{7a}$$

By use of (4a) this results in:

$$(EI)_{\mathrm{eq}} = E_{m2}I - \frac{E_{m1}^2h^2A}{E_{\mathrm{av}}}. \tag{7b}$$

where E_{av}, E_{m1} and E_{m2} is defined in (4b)–(4d).

We will modify the beam equation to include the thermal loading and moderately large deflections. Transverse shear will also be included. In doing so, we shall properly modify the equations developed in [12]. The moment including the thermal effect is given by:

$$M = -(EI)_{\mathrm{eq}}\frac{\mathrm{d}\theta}{\mathrm{d}x} - M_x^T. \tag{7c}$$

From equilibrium, taking into account the (compressive) reaction force, P, the moment at any position is given by

$$M = Pv + M_0 + Pe, \tag{7d}$$

where M_0 is the moment at $x=0$.

Differentiating (7c) and (7d) with respect to x and using (6b) and (6e) with the additional assumption that the shear angle is small, so that $\sin\gamma_{\mathrm{eq}} \simeq \gamma_{\mathrm{eq}}$ and $\cos\gamma_{\mathrm{eq}}=1$, results in:

$$(EI)_{\mathrm{eq}}\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2} + P\left(\frac{\beta P}{2A\bar{G}}\sin 2\theta + \sin\theta\right) + \frac{\mathrm{d}M_x^T}{\mathrm{d}x} = 0. \tag{7e}$$

At the ends (simple supports), we have the moment boundary conditions of

$$-(EI)_{\rm eq}\frac{{\rm d}\theta}{{\rm d}x}(0)-M_x^T=Pe;\qquad -(EI)_{\rm eq}\frac{{\rm d}\theta}{{\rm d}x}(L)-M_x^T=Pe. \tag{7f}$$

2.1. Linear analysis

Taking into account the fact that the thermal moment, M_x^T, is independent of x, and linearizing, $\sin\theta\simeq\theta$, results in the differential equation

$$(EI)_{\rm eq}\frac{{\rm d}^2\theta}{{\rm d}x^2}+P\left(\frac{\beta P}{A\bar{G}}+1\right)\theta=0. \tag{8a}$$

with the boundary conditions (7f).

If we set

$$\lambda^2=\frac{P}{(EI)_{\rm eq}}+\frac{\beta P^2}{(EI)_{\rm eq}A\bar{G}}, \tag{8b}$$

then the solution is:

$$\theta(x)=\frac{(M_x^T+Pe)}{\lambda(EI)_{\rm eq}}\left[\frac{(1-\cos\lambda L)}{\sin\lambda L}\cos\lambda x-\sin\lambda x\right]. \tag{8c}$$

Notice that the symmetry condition $\theta(L/2)=0$ is satisfied automatically in (8c).

And the constraint equation (6j), again linearizing, $\cos\theta\simeq 1$, becomes:

$$(N_x^T-P)\frac{L}{E_{\rm av}A}-\frac{(M_x^T+Pe)^2}{2[(EI)_{\rm eq}\lambda]^2}\frac{(1-\cos\lambda L)}{\sin\lambda L}\times\left(\frac{L}{\sin\lambda L}-\frac{1}{\lambda}\right)=0. \tag{8d}$$

The vertical deflection of the beam is obtained for the linear problem by using (6b) and (6e) and integrating:

$$v(x)=\left(1+\frac{\beta P}{\bar{G}A}\right)\int_0^x\theta(\xi)\,{\rm d}\xi$$

Substituting (8c) gives:

$$v(x)=\frac{(M_x^T+Pe)}{(EI)_{\rm eq}\lambda^2}\left(1+\frac{\beta P}{\bar{G}A}\right)\left[\frac{(1-\cos\lambda L)}{\sin\lambda L}\sin\lambda x+(\cos\lambda x-1)\right]. \tag{8e}$$

Notice that from (8d), the deflections at the ends are zero (as they should), $v(0)=v(L)=0$, and that the mid-point deflection, $v(L/2)=v_m$, is:

$$v_m=\frac{(M_x^T+Pe)}{(EI)_{\rm eq}\lambda^2}\left(1+\frac{\beta P}{\bar{G}A}\right)\left[\frac{1}{\cos(\lambda L/2)}-1\right]. \tag{8f}$$

and tends to infinity for $\lambda L=\pi$ (the Euler load of the column).

If the thermal loading is prescribed via the heat influx, Q, then N_x^T and M_x^T can be determined and the only unknown in Eq. (8e) is P (or λ from (8b)). Then we can solve the transcendental equation (8d) for P and thus obtain the relationship between the heat loading Q and the transverse deflection v. This relationship is obtained for constrained beams only, which means the support force P is large enough to prevent the beam from expanding along the axial direction under the effects of the thermal force N_x^T and the thermal moment M_x^T. In this case, P, which is obtained by Eq. (8d), is the support reaction; if, on the other hand, the 'constraint' condition of immovable supports is released, then P is the applied load and the relationship between the mid-point deflection v_m and $P_{\rm appl}=P$ can be obtained from Eq. (8f). Note also that for zero M_x^T, the constraint equation (8d) reduces to $N_x^T=P$, i.e. the solution for a uniformly heated column.

2.2. Results and discussion

Let us consider a composite column made of E-glass/vinyl-ester (Fig. 1). The thermal conductivity is taken to be $K=0.50$ W/m K and the thermal expansion coefficient $\alpha=18.0\times10^{-6}$ (1/°C). The surface conductivity from the composite surface to the air is taken to be

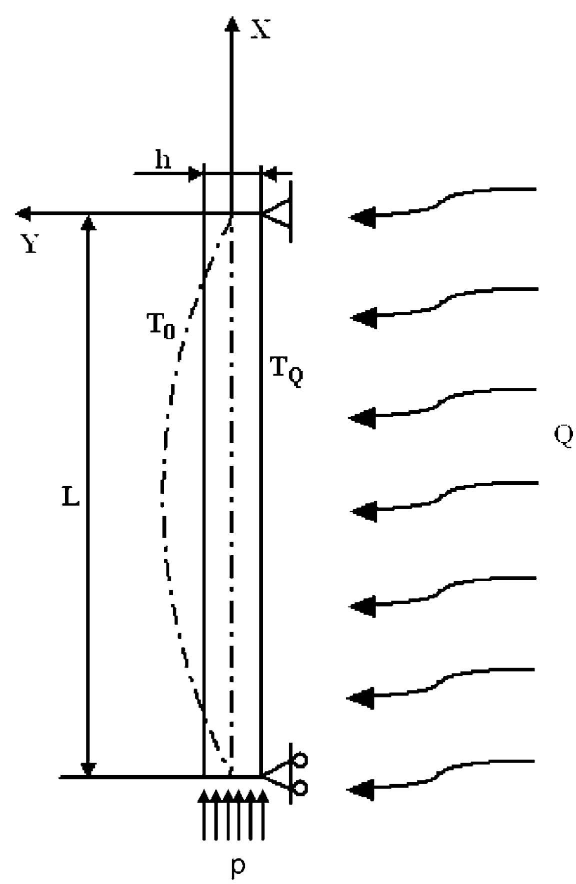

Fig. 1. Definition of the geometry for a composite column under axial compression and heat flux.

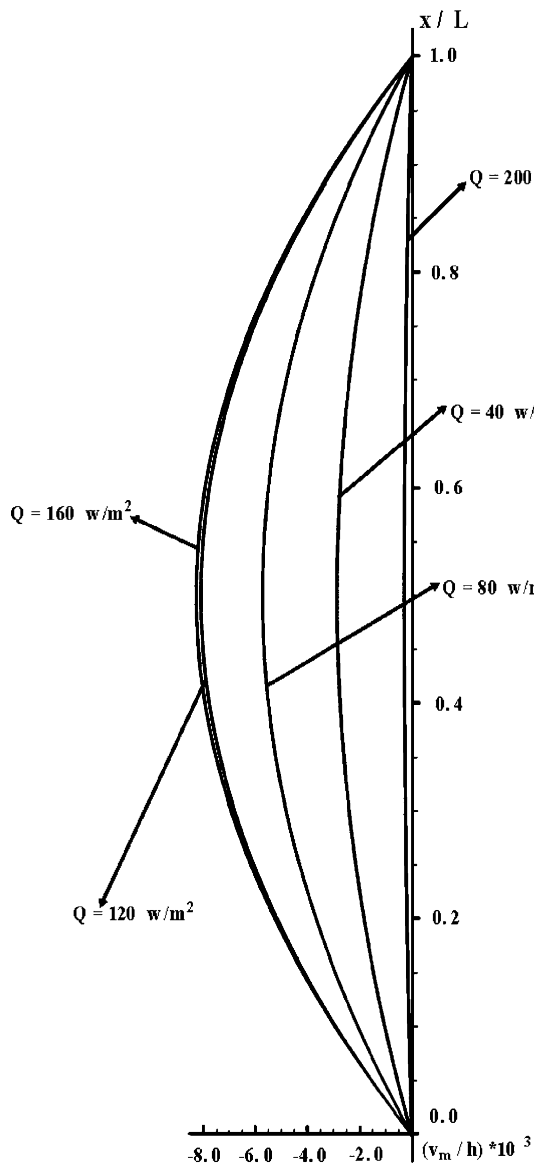

Fig. 2. Deformation of the beam with different surface temperature (immovable ends).

$H = 6.0\ \mathrm{W/m^2\ K}$. The room temperature modulus is $E_0 = 20.6$ GPa and the shear modulus, $G_0 = 2.1$ GPa. Let us assume a column of length, $L = 2$ m, thickness, $h = 0.25$ m and width $b = 0.5$ m.

In Fig. 2 we show the deformation of the beam, which is pinned at both ends (immovable) and subjected to an external heat flux. It is obvious that the negative deformation increases with the external heat flux Q (negative values are towards the heat source), which increased from 40 to 160 $\mathrm{W/m^2}$. Actually, the difference of the deformation of the column between $Q = 120$ and 160 $\mathrm{W/m^2}$ is very small; But for $Q = 200\ \mathrm{W/m^2}$, the deformation decreases and is even smaller than the deformation under $Q = 40\ \mathrm{W/m^2}$. The reason is that the transverse deformation of the column is due to the bending moments applied. There are two bending moments, one is the thermal bending moment, as shown in Eq. (5d), the other is the moment due to the eccentric loading, as shown in Eq. (7d). Since the material properties degrade with temperature, the neutral axis of the column moves away from the heat source, therefore the bending moment due to the eccentric loading always makes the column bend away from the heat source; but for the thermal moment, there are two possibilities, one is to make the column bend towards the heat source, anther is to make the column bend away from the heat source, which depends on the variations of the temperature and material properties. When the heat flux increased from $Q = 40$–160 $\mathrm{W/m^2}$, the deformation was governed by the thermal loading, which increased with the heat flux, so the deformation increased towards the heat source with the heat flux Q; but as Q increased further, the influence of the material properties degradation took effect which lead to opposite transverse deformation (i.e. away from the heat source). As a result, under the overall bending moments, the deformation decreased.

The corresponding support force, P_{con} can also be calculated for a given external heat flux. P_{con} is normalized with the Euler critical load of the column for room temperature, which is: $P_{\mathrm{Euler}} = \pi^2 EI/L^2$. Fig. 3 shows the support force, P_{con} as a function of the heat flux, Q, from 10 to 200 $\mathrm{W/m^2}$. It can be seen that P_{con} increases with the heat flux when $Q < 180\ \mathrm{W/m^2}$. And as the heat flux increases further and $Q > 180\ \mathrm{W/m^2}$, the support force P_{con} decreases. The variation is non-linear which is due to the material properties, which decrease with the larger heat flux non-linearly, as well as due to the fact that the ends are restrained, therefore, beyond a certain level of deformation, the structure starts to 'pull' from the ends rather than 'push' against the ends.

Based on the axial support force P_{con} obtained, the mid-point deflection v_m is calculated, which is normalized by the thickness of the column h. In Fig. 4, we show the absolute value of mid-point deflection v_m (negative values are towards the heat source). It is clear that as $Q \leq 140\ \mathrm{W/m^2}$,

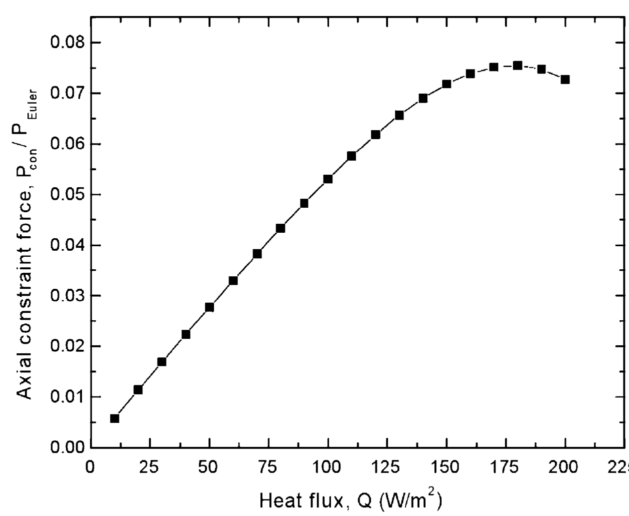

Fig. 3. Axial reaction force vs heat flux for a pinned beam with external heat flux (immovable ends).

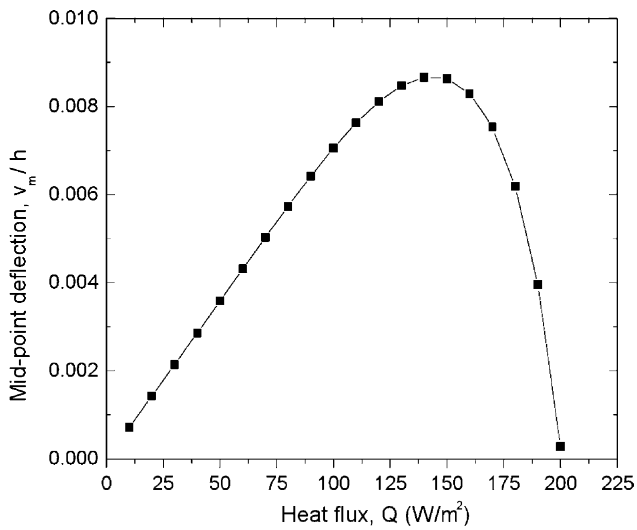

Fig. 4. Mid-point deflection vs heat flux for a pinned beam under with external heat flux (immovable ends).

v_m increases with the larger heat flux almost linearly, which means non-linearity of the material properties due to temperature have little influence on the deflection of the column; however, as $Q > 140\ \mathrm{W/m^2}$, the mid-point transverse deflection decreases since the influence of the eccentrcity moment becomes significant and the overall moment variation with the heat flux is highly non-linear. There is another fact we should note: the normalized mid-point transverse deflection is rather small, therefore the linear analysis is reasonable.

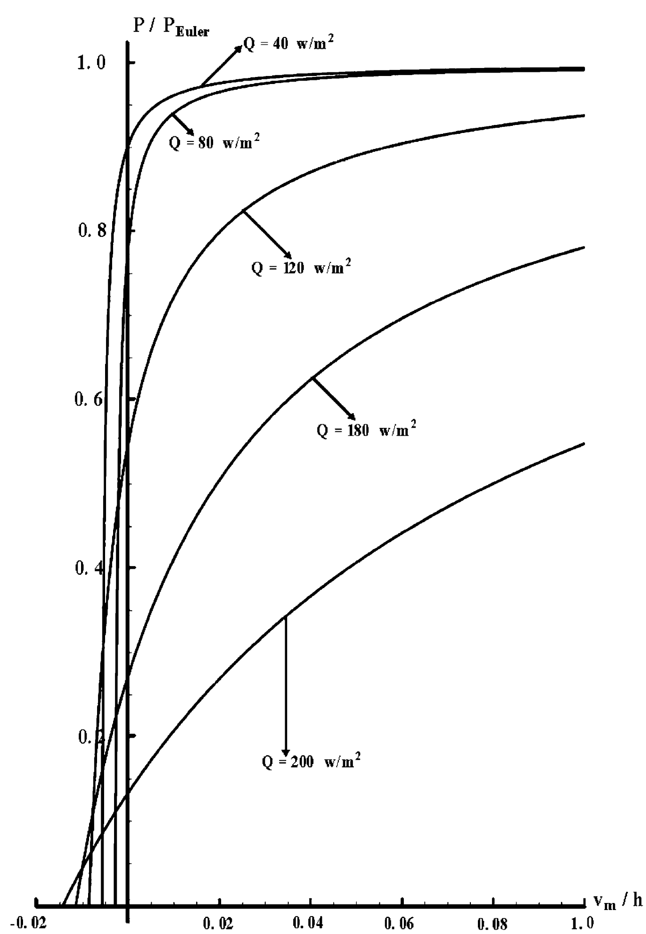

Fig. 5. Axial applied force vs mid-point deflection for a pinned beam with external heat flux (ends free to move axially).

Fig. 5 presents a plot of the axial support force P vs. the mid-point deflection v_m for various values of heat flux Q. In each case, the mid-point deflection v_m was calculated from the linear analysis for the pinned beam under the external heat flux and the end of the column is free to move, so the constraint condition in Eq. (8d) is removed. The solution of the problem is Eq. (8f) and the variation between the axial force P and the mid-point deflection v_m can be obtained from Eq. (8e). The figure shows that, for smaller heat fluxes Q, the axial force P increases initially with only a small bending deflection. But as P approaches the Euler load P_{cr}, which is obtained for $\lambda L = \pi$, the transverse deflection increases rapidly, with P becoming asymptotic to P_{cr}. For the larger heat flux values of Q, the load–deflection curve 'bends over' much earlier in the response, and the beam behaves much like an 'imperfect' column. Eventually, in all cases, the axial support force approaches P_{cr} as the mid-span deflection becomes large. The temperature change through the thickness has effectively an analogous role as that of an imperfection on a mechanically loaded bar. That is, both a temperature change difference through the thickness and an initial imperfection would cause a moment that bends the bar from the instant any load, whether thermal or mechanical, is applied. One other observation from Fig. 5 is that initially the deflections are negative, i.e. the beam bends towards the heat source (e.g. fire) and more so with the higher heat fluxes. But very quickly it turns around and bends away from the heat source (positive deflections), eventually reaching large values of deflections away from the heat source as the applied load approaches the Euler load.

A few final remarks. In this paper we have assumed a simple (linear) temperature profile because the focus of the paper is on providing a solution for the structural response characteristics under combined compression and non-uniform temperature, and not on the solution of the fire equations, which has been successfully done by other researchers [13]. Actually, if the heat flux from a fire is not very severe (such as 1 or 2 $\mathrm{kW/m^2}$), the temperature profile could be approximated by a linear steady-state one, as assumed in this paper.

Furthermore, the solution methodology is actually given for any temperature distribution and therefore the approach does not rely on the linearity of temperature profile. We made such a simple assumption just to show the mechanical response under a non-uniform temperature distribution, and because with this assumption some of the final expressions turn out to be in closed form. In future work, we plan to use the Laplace transformation method to solve for the transient

Fig. 6. The failed fiber glass reinforced composite column under combined constant heat flux $Q = 25\ \mathrm{kW/m^2}$ and compressive load.

temperature distribution under heat flux conditions, and, furthermore, in an even more accurate approach, to use the temperature profile provided by other researchers who have already solved for the temperature distribution resulting from fire [13], and then couple these more complex temperature profiles with the present thermal buckling analysis.

It should also be noted that resin decomposition can certainly happen under a high intensity fire, which can be a very important factor. But if the fire intensity is small, the resin decomposition will not occur before the composite beam collapses, which had been observed in our composite column tests under combined loading. Recently, our experiments on glass fiber-reinforced composite columns under combined heat flux due to fire and compressive loading show that the column failed due to the bending moments which are induced by the non-uniform temperature distribution and the eccentric loading. A failed column from our experiments is shown in Fig. 6. From the figure, we can see that under the constant heat flux, $25\ \mathrm{kW/m^2}$, resin decomposition is not apparent. The composite column failed since the material properties degraded with the temperature increasing and the structure collapsed due to the thermal bending and the eccentric loading. All these influences, which lead to failure of the column are considered in this paper. In future work, the material decomposition and charred layer formation will be considered as well. And in these future studies with more complex temperature profiles, the approach outlined in this paper can be used to analyze the buckling response.

3. Conclusions

The thermal buckling of a composite column under external heat flux and compressive load is investigated based on a linear temperature distribution. We considered two cases: one is a constrained pinned column (immovable ends) under an external heat flux and the other is the same pinned column, but with the ends of the column free to move axially and subjected to both an applied compressive load and an external heat flux. For the constrained column, the variations of the axial constraint force P_{con} and the mid-point transverse deflection v_m with the external heat fluxes are derived shown; for the unconstrained column, the variation of the applied axial forces P with the mid-point transverse deflection v_m is derived and shown. All parameters are normalized appropriately. Based on the results obtained, several conclusions can be made:

1. Due to the non-uniform temperature distribution, a thermal moment is created.
2. Since the properties of polymers depend strongly on the temperature, and especially on how close the temperature is to the resin glass transition temperature, the neutral axis moves away from the centroid of the column cross-section which lead to an additional, 'eccentric moment', due to the eccentric loading.
3. The overall moment applied to the column comes from (a) the thermal moment, the value and direction of which depend on the temperature and material properties variation; in fact it would tend to bend the structure away from the heat source for small temperatures (small heat fluxes) but towards the heat source for large temperatures and (b) the 'eccentric moment', which always makes the column bend away the heat source. Based on the present analysis, the direction of bending for specific conditions of applied heat flux and material properties can be predicted.
4. The column behaves much like an 'imperfect' column and the deformation of the column strongly depends on the thermal and eccentric moments. Actually, the overall moment varies with the heat flux non-linearly.
5. For the composite column made of E-glass/vinyl-ester, as an example, the transverse deflection is very small compared with the thickness of the column, therefore the linearization of the solution is reasonable.

Acknowledgements

The financial support of the Office of Naval Research, Grant N00014-03-1-0189, and the interest and encouragement of the Grant Monitors, Dr Patrick C. Potter and Dr Luise Couchman is gratefully acknowledged.

References

[1] Sorathia U, Rollhauser CM, Hughes WA. Improved fire safety of composites for naval applications. Fire Mater 1992;16:119–25.
[2] Sorathia U, Lyon R, Gann R, Gritzo L. Materials and fire threat. SAMPE J 1996;32(3):8–15.
[3] Egglestone GT, Turley DM. Flammability of GRP for use in ship superstructures. Fire Mater 1994;18:255–60.

[4] Scudamor MJ. Fire performance studies on glass-reinforced plastic laminates. Fire Mater 1994;18:313–25.
[5] Gibson AG, Hume J. Fire performance of composite panels for large marine structures. Plast Rubber Compos Process Appl 1995;23: 175–83.
[6] Brown JR, Mathys Z. Reinforcement and matrix effects on the combustion properties of glass reinforced polymer composites. Composites 1997;28A:675–81.
[7] Mouritz AP, Gardiner CP. Compression properties of fire-damaged polymer sandwich composites. Composites Part A 2002;33:609–20.
[8] Kulkarni AP, Gibson RF. Nondestructive characterization of effects of temperature and moisture on elastic moduli of vinyl ester resin and E-glass/vinylester composite. Proceedings of the American society of composites 18th annual technical conference, October 20–22, 2003, University of Florida, Gainesville, FL; 2003.
[9] Carlsaw HS, Jaeger JC. Conduction of heat in solids. Oxford: Clarendon Press; 1959.
[11] Simitses GJ. An introduction to the elastic stability of structures. New York: Krieger; 1986.
[12] Huang H, Kardomateas GA. Buckling and initial postbuckling behavior of sandwich beams including transverse shear. AIAA J 2002;40(11):2331–5.
[13] Gibson AG, Wu YS, Chandler HW, Wilcox JAD, Bettess P. A model for the thermal performance of thick composite laminates in hydrocarbon fires. Composite materials in the petroleum industry, Revue de l' Institut Francais du Petrole. vol. 50 1995 p. 69–74.