The Mode III Interface Crack in Piezo-Electro-Magneto-Elastic Dissimilar Bimaterials

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The mode III interface crack problem is investigated for dissimilar piezo-electro-magneto-elastic bimaterial media, taking the electro-magnetic field inside the crack into account. Closed form solutions are derived for impermeable and permeable cracks. The conventional singularity of $r^{-1/2}$ is found for the fields at the distance r ahead of the interface crack tip. Expressions for extended crack tip stress fields and crack opening displacements (ECODs) are derived explicitly, and so are some fracture parameters, such as extended stress intensity factors (ESIFs) and energy release rate (G) for dissimilar bimaterials. An approach called the "energy method," finding the stationary point of the saddle surface of energy release rate with respect to the electro-magnetic field inside the crack, is proposed. By this method, the components of the induced electro-magnetic field inside the crack are determined, and the results are in exact agreement with those in the literature if the two constituents of the bimaterial media are identical. The effects from mechanical and electro-magnetic property mismatches, such as differences in the stiffness, electric permittivity and magnetic permeability, between the two constituents of the bimedia on the mode III interface crack propagation are illustrated by numerical simulations. The results show that the applied electric and magnetic loading usually retard the growth of the interface crack and the directions of the combined mechanical, electric, and magnetic loading have a significant influence on the mode III interface crack propagation. [DOI: 10.1115/1.2073328]

1 Introduction

One class of contemporary materials, widely used in engineering in devices (in sensor, transducer, actuator components, etc.), are the piezoelectric and piezomagnetic composite materials. Due to their exceptional functions, such as flat frequency response [1–4] and transformation of energy from one form to the other (mechanical, electric, and magnetic energy, or thermal energy) [5,6], this type of composite exhibiting piezoelectric and piezomagnetic properties has found increasing applications in microwave electronic, optoelectronic, and electronic instruments. Like in conventional composites, defects or flaws may usually be introduced during the manufacturing process or during service by impact loading. These defects would often deteriorate the performance of the devices being made of piezo-electro-magneto-elastic media.

Recently, more and more attention has been directed towards the problems of cracks in the electro-magneto-elastic solids [7–12]. Dissimilar bimaterials or layered composites are often incorporated into a variety of components, such as smart structure sensors, actuators, and broadband magnetic probes. Having been recognized as one of the common failure modes of general dissimilar bimaterial media, the interface cracks could also be developed in the piezo-electro-magneto-elastic structures and thus affect the features of the electro-magneto-elastic apparatus. Though these interface cracks may severely diminish the performance of this type of structure, one may see that little attention has been given to the study of the magneto-electric coupling effects on the interface crack propagation behavior in piezo-electro-magneto-elastic bimaterials.

The magneto-electric coupling effect of piezoelectric and piezomagnetic fields usually has a significant influence on the behavior of piezo-electro-magneto-elastic bimaterials or layered structures [3,4,13]. This coupling among the magnetic, electric, and elastic fields is also expected to have an influence on the propagation behavior of interface crack/delaminations when piezoelectric, piezomagnetic, and magneto-electric effects, or any two of these effects, are present simultaneously [1]. These coupling effects usually complicate this interface crack problem. In order to get insight into the interface crack problems of dissimilar piezo-electro-magneto-elastic bimaterial composites, the mode III interface crack is investigated in this paper by using Stroh's formulism [14] and the complex variable method. Two types of mode III interface cracks are analyzed. One is called permeable interface crack for which the magneto-electric field inside the interface crack is considered. The other type is called impermeable.

This paper is organized as follows: (1) In Sec. 2 is a summary of some basic equations for piezo-electro-magneto-elasticity in Strohs formalism. (2) A compact form solution to the interface crack is formulated in Sec. 3. The expressions for the ECOD, ESIF, and the energy release rate are derived in closed form. The "energy method" is also proposed in this section and used to obtain the solution to the magneto-elastic field inside the interface crack. One may interestedly find that this method could be extended to more complicated problems in piezo-electro-magnetic elastic solids. (3) The numerical results in Sec. 4 show the influence of the property mismatches between the two constituents on the interface crack propagation. An interesting result one may find is that the applied external electric-magnetic field may slow the growth of mode III interface cracks in piezo-electro-magneto-elastic bimaterial solids. Since all the formulas in this paper are obtained in explicit expressions, and are thus easily trackable, this study may serve as a benchmark for further investigations in piezo-electro-magneto-elastic media

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2 Basic Equations

In a fixed Cartesian coordinate system (x_1, x_2, x_3), the generalized Hookes law for an elastic material with both piezoelectric and piezomagnetic fields is of the following form [1]:

$$\sigma_{ij} = c_{ijkl}u_{k,l} + e_{lij}\varphi^E_{,l} + \varrho_{lij}\varphi^H_{,l},$$
$$D_i = e_{ikl}u_{k,l} - \varepsilon_{il}\varphi^E_{,l} - \alpha_{il}\varphi^H_{,l}, \tag{1}$$
$$B_i = \varrho_{ikl}u_{k,l} - \alpha_{li}\varphi^E_{,l} - \mu_{il}\varphi^H_{,l}$$

where i, j, k, l range in {1, 2, 3} and the repeated indices imply summation, the comma stands for differentiation with respect to corresponding coordinate variables; σ_{ij} is the elastic stress, u_k is the elastic displacement, and c_{ijkl} is the elastic moduli tensor; D_i is the electric displacement, φ^E is the electrostatic potential, and ε_{il} is the electric permittivity; B_i is the magnetic induction (magnetic fluxes), φ^H is the magnetic scalar potential, and μ_{il} the magnetic permeability; e_{ikl}, ϱ_{ikl}, and α_{li} are the piezoelectric, piezomagnetic, and magnetoelectric coefficients, respectively. For the material constants, the following relationships hold:

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij}, \quad e_{ikl} = e_{ilk}, \quad \varrho_{ikl} = \varrho_{ilk},$$
$$\alpha_{il} = \alpha_{li}, \quad \varepsilon_{il} = \varepsilon_{li}, \quad \mu_{il} = \mu_{li} \tag{2}$$

The equilibrium equations in the absence of body forces read

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad B_{i,i} = 0 \tag{3}$$

For two-dimensional antiplane deformation of a transversely isotropic solid, we have

$$u_1 = 0, \quad u_2 = 0, \quad u_3 = u_3(x_1, x_2),$$
$$\varphi^E = \varphi^E(x_1, x_2), \quad \varphi^H = \varphi^H(x_1, x_2) \tag{4}$$

One may define the extended displacement as

$$\mathbf{u} = [u_3, \varphi^E, \varphi^H]^T \tag{5}$$

For a plane system, a nontrivial solution to Eq. (3) may then take the following form:

$$\mathbf{u} = A\ f(z_\alpha) + \bar{A}\bar{f}(\bar{z}_\alpha), \quad \psi = B\ f(z_\alpha) + \bar{B}\bar{f}(\bar{z}_\alpha), \quad z_\alpha = x_1 + p_\alpha x_2 \tag{6}$$

where ψ is the stress function vector and $f(z_\alpha)$ are functions to be determined by boundary conditions.

If one defines the extended stress fields as

$$\mathbf{t} = [\sigma_{32}, D_2, B_2]^T, \quad \mathbf{s} = [\sigma_{31}, D_1, B_1]^T \tag{7}$$

then these stresses can be written in terms of the stress functions as

$$\mathbf{s} = \left(-\frac{\partial\psi_i}{\partial x_2}\right)^T, \quad \mathbf{t} = \left(\frac{\partial\psi_i}{\partial x_1}\right)^T = \psi' \tag{8}$$

Substituting Eq. (6) back into the equation (3), one readily obtains

$$A = I = \mathrm{diag}[1,1,1], \quad B = i\begin{pmatrix} c_{44} & e_{15} & \varrho_{15} \\ e_{15} & -\varepsilon_{11} & -\alpha_{11} \\ \varrho_{15} & -\alpha_{11} & -\mu_{11} \end{pmatrix}, \quad p_\alpha = i \tag{9}$$

where $i^2 = -1$.

If we define a matrix M as

$$M = iAB^{-1}, \tag{10}$$

then

$$M = \begin{pmatrix} \varepsilon_{11}\mu_{11} - \alpha_{11}^2 & e_{15}\mu_{11} - \alpha_{11}\varrho_{15} & \varepsilon_{11}\varrho_{15} - \alpha_{11}e_{15} \\ e_{15}\mu_{11} - \alpha_{11}\varrho_{15} & -\varrho_{15}^2 - c_{44}\mu_{11} & c_{44}\alpha_{11} + e_{15}\varrho_{15} \\ \varepsilon_{11}\varrho_{15} - \alpha_{11}e_{15} & c_{44}\alpha_{11} + e_{15}\varrho_{15} & -e_{15}^2 - c_{44}\alpha_{11}^2 \end{pmatrix} \Bigg/ \Delta \tag{11}$$

where

$$\Delta = c_{44}\varepsilon_{11}\mu_{11} + e_{15}^2\mu_{11} + \varepsilon_{11}\varrho_{15}^2 - 2\alpha_{11}e_{15}\varrho_{15} - c_{44}\alpha_{11}^2 \tag{12}$$

The matrix M is real and symmetric.

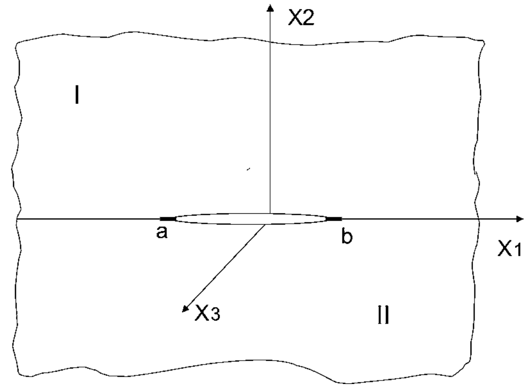

Fig. 1 An interface crack between dissimilar anisotropic bimedia

3 A Solution to Mode III Interface Crack

Let the medium "I" occupy the upper half-space (donated by L) and medium "II" be in the lower half-space (donated by R) (Fig. 1). Then from Eqs. (6) and (9), one has the following expression for this bimedia:

$$\mathbf{u}^I = \phi_I(z) + \bar{\phi}_I(\bar{z}), \quad \psi^I = B_I\phi_I(z) + \bar{B}_I\bar{\phi}_I(\bar{z}) \tag{13}$$

where, $\mathbf{u}^I, \psi^I$ are displacement and stress functions for $z \in L$, and

$$\mathbf{u}^{II} = \phi_{II}(z) + \bar{\phi}_{II}(\bar{z}), \quad \psi^{II} = B_{II}\phi_{II}(z) + \bar{B}_{II}\bar{\phi}_{II}(\bar{z}) \tag{14}$$

where $\mathbf{u}^{II}, \psi^{II}$ are displacement and stress functions for $z \in R$.

For convenience, the symbols "I" and "II," denoting the quantities for medium "L" and "R," respectively, may be put as subscripts or subscripts.

Let the interface crack be located in the region $a < x_1 < b, -\infty < x_3 < \infty$ of the plane $x_2 = 0$. The $p_0^\infty = [\sigma_{i2}^\infty]^T = [\sigma_{32}^\infty, D_2^\infty, B_2^\infty]^T$ is applied at infinity (Fig. 1). Inside the crack often is air or vacuum, and the electro-magnetic field usually is considered constant under uniform remote applied load [11,12, etc.]. These unknown components for the electro-magnetic field are denoted as D_α^0, B_α^0, E_α^0, and H_α^0, which, respectively, observe the relationships

$$E_\alpha^0 = \frac{D_\alpha^0}{\varepsilon_0}, \quad H_\alpha^0 = \frac{B_\alpha^0}{\varepsilon_0}, \quad \alpha = \{1,2\} \tag{15}$$

Employing the superposition principle leads the original boundary value problem to an equivalent problem with the loading

$$p_0 = [\sigma_{32}^{\infty}, \Delta D_2^0, \Delta B_2^0]^T \tag{16}$$

being applied on the two surfaces of the interface crack, where, in Eq. (16),

$$\Delta D_2^0 = D_2^{\infty} - D_2^0, \quad \Delta B_2^0 = B_2^{\infty} - B_2^0. \tag{17}$$

The displacement continuity along the bonded interface gives

$$\phi_{I+}(x_1) + \bar{\phi}_{I-}(x_1) = \phi_{II-}(x_1) + \bar{\phi}_{II+}(x_1)$$

or

$$\phi_{I+}(x_1) - \bar{\phi}_{II+}(x_1) = \phi_{II-}(x_1) - \bar{\phi}_{I-}(x_1) \tag{18}$$

A function can be defined being analytical on the whole plane, except the cut along the interface crack, as follows:

$$\Phi(z) = \begin{cases} \phi_I(z) - \bar{\phi}_{II}(z), & z \in L \\ \phi_{II}(z) - \bar{\phi}_I(z), & z \in R \end{cases} \tag{19}$$

Then, this function automatically satisfies the condition (18). Here, a convention

$$\phi(z) = \phi_{\pm}(x_1), \quad x_2 \to 0^{\pm} \tag{20}$$

is employed and will be used in the following sections.

Differentiation of Eq. (19) with respect to z yields

$$\Phi'(z) = \begin{cases} \phi_I'(z) - \bar{\phi}_{II}'(z), & z \in L \\ \phi_{II}'(z) - \bar{\phi}_I'(z), & z \in R \end{cases} \tag{21}$$

The stress continuity on the bonded interface leads to

$$B_I \phi_{I+}'(x_1) + \bar{B}_I \bar{\phi}_{I-}'(x_1) = B_{II} \phi_{II-}'(x_1) + \bar{B}_{II} \bar{\phi}_{II+}'(x_1) \tag{22}$$

Similarly, we can define a function, which automatically satisfies the condition (22) and is analytical on the whole plane except the cut along the interface crack, as

$$\omega(z) = \begin{cases} B_I \phi_I'(z) - \bar{B}_{II} \bar{\phi}_{II}'(z) & z \in L \\ B_{II} \phi_{II}'(z) - \bar{B}_I \bar{\phi}_I'(z), & z \in R \end{cases} \tag{23}$$

From Eqs. (21) and (23), we obtain

$$B_I \phi_I'(z) = N[\mathrm{i}\ \Phi'(z) + \bar{M}_{II} \omega(z)], \tag{24a}$$

$$\bar{B}_{II} \bar{\phi}_{II}'(z) = B_I \phi_I'(z) - \omega(z), \quad z \in L; \tag{24b}$$

and

$$B_{II} \phi_{II}'(z) = \bar{N}[\mathrm{i}\ \Phi'(z) + \bar{M}_I \omega(z)], \tag{25a}$$

$$\bar{B}_I \bar{\phi}_I'(z) = B_{II} \phi_{II}'(z) - \omega(z) \quad z \in R \tag{25b}$$

In the above equations, the following matrix was used:

$$N^{-1} = M_I + \bar{M}_{II} = M_I + M_{II} \tag{26}$$

Since M_I and M_{II} are real symmetric, so is N. Furthermore, define

$$H = M_I + M_{II} \tag{27}$$

Therefore, the boundary traction conditions along the interface crack surface give

$$B_I \phi_{I+}'(x_1) + B_{II} \phi_{II-}'(x_1) - \omega_-(x_1) = -p_0(x_1), \tag{28a}$$

$$B_{II} \phi_{II-}'(x_1) + B_I \phi_{I+}'(x_1) - \omega_+(x_1) = -p_0(x_1) \tag{28b}$$

Subtraction of Eq. (28b) from (28a)) yields

$$\omega_+(x_1) - \omega_-(x_1) = 0 \tag{29}$$

which implies that the $\omega(z)$ is continuous on the whole interface.

By the analytical continuation principle [15], the function $\omega(z)$ is analytical on the whole plane. But according to Liouville's theorem [15], this $\omega(z)$ must be a constant function in the whole domain. However, the condition that this function should vanish at infinity means this constant function must be identically zero in the whole plane, i.e.,

$$\omega(\mathbf{z}) = 0, \quad \text{for all } \mathbf{z} \tag{30}$$

Either Eq. (28a) or (28b) leads to a general Hilbert equation in matrix notation:

$$\Phi_+'(x_1) + \Phi_-'(x_1) = i\ Hp_0(x_1), \quad a < x_1 < b \tag{31}$$

The homogenous equation corresponding to the general Hilbert equation (31) can be written as

$$X_+(x_1) + X_-(x_1) = 0, \quad a < x_1 < b \tag{32}$$

where

$$X(z) = \frac{1}{\sqrt{(z-a)(z-b)}} \mathrm{diag}[1,1,1] \tag{33}$$

A solution which vanishes at infinite could be [16]

$$\Phi'(z) = \frac{1}{2\pi i} X(z) \int_{ab} \frac{[X(x_1)]_+^{-1} H[\mathrm{i}\ p_0(x_1)] dx_1}{x_1 - z} \tag{34}$$

Specifically, for constant applied loading, one has (see the Appendix)

$$\Phi'(z) = \mathrm{diag}\left(1 - \frac{z-(a+b)/2}{\sqrt{(z-a)(z-b)}}\right) \frac{H}{2}(ip_0) \tag{35}$$

Integrating Eq. (35) results in

$$\Phi(z) = \mathrm{diag}[z - \sqrt{(z-a)(z-b)}] \frac{H}{2}(ip_0) \tag{36}$$

where the constant contributing rigid body motion is omitted.

Next, let us consider some fracture characterizing parameters such as the crack tip field intensity factors, extended displacement discontinuities near the crack tips, and the energy release rate.

From the equations (24a) and (25b), the extended traction along the interface could be expressed as

$$\mathbf{t}(x_1) = N\ i\Phi_+(x_1) + \bar{N}\ i\Phi_-(x_1) = H^{-1}[i\ \Phi_+(x_1) + i\ \Phi_-(x_1)] \tag{37}$$

We shall show that the right-hand side of Eq. (37) is real, as required.

Substituting the stress function (34) to traction expression (37) leads to

$$\mathbf{t}(x_1) = -p + [X_+(x_1) + X_-(x_1)]\left(x_1 - \frac{a+b}{2}\right) p_0/2 \tag{38}$$

When Eq. (32) is employed, the traction along the interface reads:

$$\mathbf{t}(x_1) = \begin{cases} -p_0 + [(x_1-a)(x_1-b)]^{-1/2} \mathrm{diag}\left(x_1 - \dfrac{a+b}{2}\right) p_0, & x_1 < a \text{ and } b < x_1 \\ -p_0, & a < x_1 < b \end{cases} \tag{39}$$

which is a real vector as expected.

Then the extended tractions at a distance "r" ahead of the crack tip such as "b" (Fig. 1) can be expressed in the form of

$$\mathbf{t}(r) = (2\pi\ r)^{-1/2}\sqrt{\frac{\pi(b-a)}{2}}p_0 = (2\pi\ r)^{-1/2}[K_\sigma, K_D, K_B]^T \tag{40}$$

where K's are real numbers and defined as

$$K_\sigma = \sqrt{\frac{\pi(b-a)}{2}}\sigma_{32}^\infty, \quad K_D = \sqrt{\frac{\pi(b-a)}{2}}\Delta D_2^0,$$

$$K_B = \sqrt{\frac{\pi(b-a)}{2}}\Delta B_2^0 \tag{41}$$

These K's may be called the extended stress intensity factors (ESIFs). If we let

$$K = [K_\sigma, K_D, K_B]^T \tag{42}$$

then the expression (42) becomes

$$K = \sqrt{\frac{\pi(b-a)}{2}}p_0 \tag{43}$$

with p_0 defined in (16).

One may also extend the conventional crack open displacement (COD) to piezo-magneto-electric materials. From Eqs. (13), (14), and (19), this extended crack open displacement (ECOD) may readily be evaluated by

$$\Delta\mathbf{u}(x_1) = \mathbf{u}_+^I(x_1) - \mathbf{u}_-^{II}(x_1) = \Phi_+(x_1) - \Phi_-(x_1) = \begin{cases} [(x_1-a)(b-x_1)]^{1/2}H\ p_0, & a < x_1 < b \\ 0, \quad x_1 < a \quad \text{or} \quad b < x_1; \end{cases} \tag{44}$$

Then the ECOD at a small distance "r" behind the tip of the interface crack may read

$$\Delta\mathbf{u}(r) = \sqrt{\frac{r}{2\pi}}H(2K) \tag{45}$$

also an expected real vector.

Now, the energy release rate, G, can be computed and it reads

$$G = \frac{1}{2}\lim_{\delta\to 0^+}\frac{1}{\delta}\int_0^\delta \mathbf{t}(r)^T\Delta\mathbf{u}(\delta - r)\mathrm{d}r = \frac{1}{2}K^THK \tag{46}$$

One may realize that all the expressions derived so far include the unknown components D_2^0 and B_2^0 of the electro-magnetic field inside the crack. There are two approaches to determine these unknowns. The first method views the crack as a degenerated hole, using the continuous conditions on the hole surface to determine the electric-magnetic fields. This method may work well for monolithic material as shown in literature such as in [12], because of the convenient affine mapping function. But it is hard to extend this method derived for monolithic materials to the bimaterial media because of the differences in the material properties between the two constituents of a bimaterial system. To offset this difficulty, here, another approach, called the "energy method," is proposed. As one may know, when a remote load starts to apply, an electric-magnetic field begins to build up inside the interface crack. This newly built field causes reactions to fields induced by the applied loading inside the whole material system. One may see that the energy release rate, G, is a saddle surface with respect to variables, D_2^0 and B_2^0, the electric-magnetic field inside the interface crack. This means for each value of G, there exist many corresponding sets of D_2^0 and B_2^0 except at the stationary point, in which only a unique D_2^0 and B_2^0 corresponds to a unique value of G.

Therefore, the value of D_2^0 and B_2^0 at the stationary point could be the final competition result of the above-mentioned interaction. Then one would have following equations:

$$\frac{\partial G}{\partial D_2^0} = H_{12}\sigma_{32}^\infty + H_{22}\Delta D_2^0 + H_{32}\Delta B_2^0 = 0, \tag{47a}$$

$$\frac{\partial G}{\partial B_2^0} = H_{13}\sigma_{32}^\infty + H_{23}\Delta D_2^0 + H_{33}\Delta B_2^0 = 0 \tag{47b}$$

which leads to

$$D_2^0 = D_2^\infty - \Delta D_2^0 = D_2^\infty - (H_{23}H_{31} - H_{21}H_{33})/(H_{22}H_{33} - H_{23}^2)\sigma_{32}^\infty,$$

$$B_2^0 = B_2^\infty - \Delta B_2^0 = B_2^\infty - (H_{21}H_{32} - H_{22}H_{31})/(H_{22}H_{33} - H_{23}^2)\sigma_{32}^\infty \tag{48}$$

where H_{ij} $(i,j=1,2,3)$ are elements of the bimaterial matrix H defined in (27). The result of (48) can be shown the same as those in literature if the two media of this current bimaterial are identical. This agreement justifies the above energy method. From the result in (48), one may see that the electric-magnetic field inside the interface crack is a function of the bimaterial property under given remote applied loading.

One may also observe from (47a) that if one wants $D_2^0 \to 0$ without magnetic field, then H_{22} needs to approach a very big value. This is called electrically impermeable. The parameter λ_e, introduced by McMeeking [17], is used to characterize the electric permeability. A similar parameter, λ_m, could be defined from the observation made on (47b), in which if $B_2^0 \to 0$ without electric field, then H_{33} has to approach a very big value, a phenomenon called magnetically impermeable. These two parameters λ_e and λ_m have the relationship of $\lambda_m/\lambda_e = (\varepsilon_0/\mu_0)/(H_{22}/H_{33})$.

Therefore, for an impermeable interface crack, $D_2^0 = B_2^0 = 0$ and the ESIF can be expressed as

$$K = \sqrt{\frac{\pi(b-a)}{2}}[\sigma_{32}^\infty, D_2^\infty, B_2^\infty]^T \tag{49}$$

The energy release rate for this interface crack reads

$$G_{imp} = \frac{1}{4}K^THK = \frac{\pi(b-a)}{8}[H_{11}(\sigma_{32}^\infty)^2 + H_{22}(D_2^\infty)^2 + H_{33}(B_2^\infty)^2 + 2H_{21}\sigma_{32}^\infty D_2^\infty + 2H_{31}\sigma_{32}^\infty B_2^\infty + 2H_{32}D_2^\infty B_2^\infty] \tag{50}$$

For a permeable interface crack, $\lambda_e = \lambda_m = 0$, the D_2^0 and B_2^0 are given by Eq. (48), and the ESIF can be expressed as

$$K = \sqrt{\frac{\pi(b-a)}{2}}[\sigma_{32}^\infty, \Delta D_2^0, \Delta B_2^0]^T \tag{51}$$

The corresponding energy release rate reads

$$G_{perm} = \frac{\pi(b-a)}{8}\frac{\det(H)}{\det(\hat{H})}(\sigma_{32}^\infty)^2 \tag{52}$$

where the matrix $\hat{H}$, a principal submatrix of H, is

$$\hat{H} = \begin{pmatrix} H_{22} & H_{23} \\ H_{23} & H_{33} \end{pmatrix} \tag{53}$$

and det( ) is the determinant of a square matrix.

One interesting observation from Eq. (52) is that, though the energy release rate, G, is independent of the applied electric-magnetic load, it is affected by electric-magnetic properties of the two constituents of the bimaterial media.

4 Numerical Results

In this section, the influence of the material property mismatches between the two constituents of the bimedia and the effects from magneto-electric coupling on the interface crack

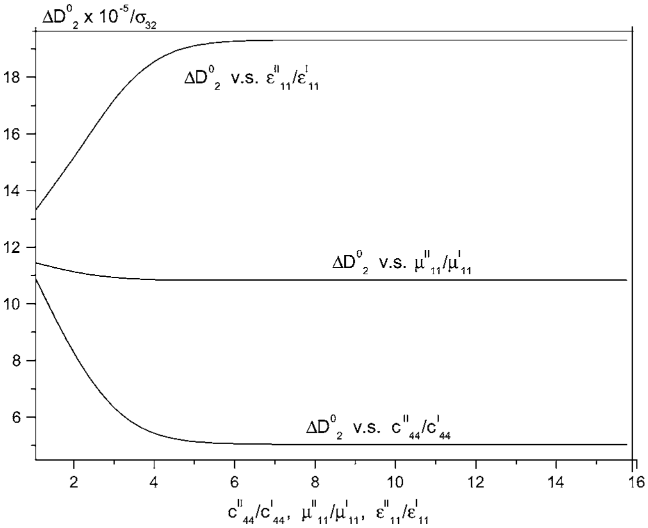

Fig. 2 ΔD_2^0 versus the bimaterial properties

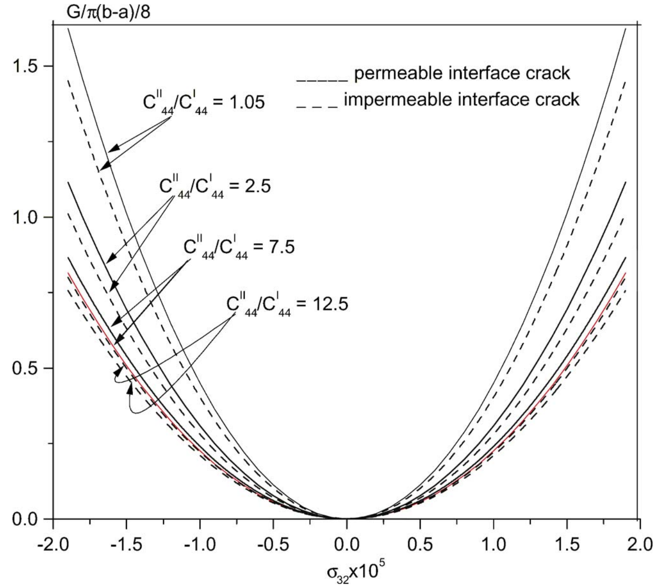

Fig. 4 Energy release rate, *G*, versus the stiffness ratio c^{II}_{44}/c^{I}_{44}, for an interface crack under pure mechanical loading

growth behavior will be demonstrated by some numerical results. The basic data for the material properties selected here are similar to those in [6]. These constants read as c^I_{44}=43.7 GPa; e^I_{15} =8.12 C/m^2; ε^I_{11}=7.86×10^{-9} C/Vm; α^I_{11}=0.0; ϱ^I_{15} =165.0 N/Am; μ^I_{11}=180.5×10^{-6} Ns^2/C^2, for the upper medium (medium "I"); and c^{II}_{44}=44.6 GPa; e^{II}_{15}=3.48 C/m^2; ε^{II}_{11}=3.42 ×10^{-9} C/Vm; α^{II}_{11}=0.0; ϱ^{II}_{15}=385.0 N/Am; μ^{II}_{11}=414.5 ×10^{-6} Ns^2/C^2, for the lower medium (medium "II").

Figures 2 and 3 present the influences of the bimaterial property mismatches $c^{II}_{44}/c^I_{44}, \varepsilon^{II}_{11}/\varepsilon^I_{11}$, and μ^{II}_{11}/μ^I_{11} on ΔD^0_2 and ΔB^0_2, which relate to the magneto-electric field, D^0_2 and B^0_2, inside the interface crack by Eq. (17). One may easily see from Fig. 2 that the electric displacement ΔD^0_2 decreases as the degree of anisotropy of these two constituents of the bimedia, defined by c^{II}_{44}/c^I_{44}, increases, while it increases as the electric permittivity ratio, $\varepsilon^{II}_{11}/\varepsilon^I_{11}$, increases. But it practically does not change as the magnetic permeability ratio, μ^{II}_{11}/μ^I_{11}, increases. The magnetic induction field ΔB^0_2 decreases as c^{II}/c^I and μ^{II}_{11}/μ^I_{11} increase, while it increase as $\varepsilon^{II}_{11}/\varepsilon^I_{11}$ increases, as shown in Fig. 3. One can also see the ΔD^0_2 and ΔB^0_2 do not vary with the increase of $c^{II}_{44}/c^I_{44}, \varepsilon^{II}_{11}/\varepsilon^I_{11}$, and μ^{II}_{11}/μ^I_{11} after they reach some value.

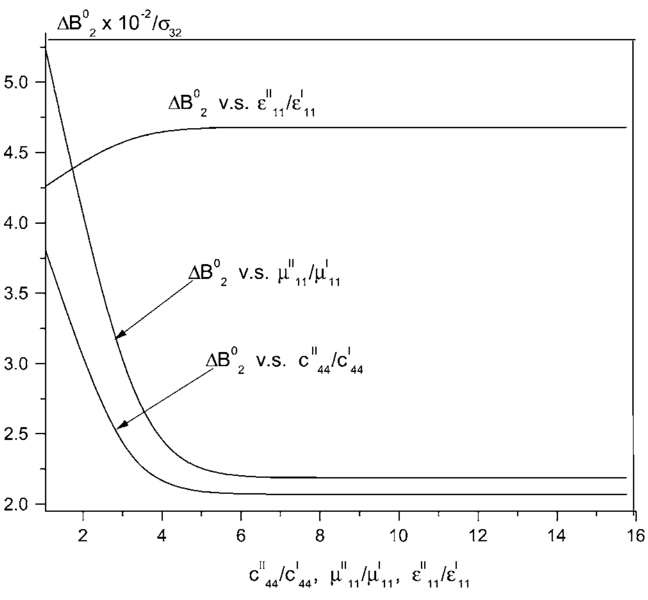

Fig. 3 ΔB_2^0 versus the bimaterial properties

Figure 4 shows the influence on the energy release rate, G, of the mismatch of the degree of anisotropy for the bimaterial media under pure mechanical tension. The G decreases as the c^{II}_{44}/c^I_{44} increase, both for permeable and impermeable interface cracks. It can also be seen that when c^{II}_{44}/c^I_{44} reaches some value (around 12.5 for this bimedia), the G almost does not vary with the increase in the mismatch on c_{44} between the two constituents of the bimaterial media. Another interesting result observed from this figure is that for a given σ_{23}, G_{perm} is larger than G_{imp}. This observation shows that the electric-magnetic field inside the interface crack may have an interaction with the stress field inside the bimaterial system, thus it has an influence on the propagation behavior of the interface crack. This observation may also suggest that the design of a piezo-electro-magneto-elastic bimaterial system based on a permeable assumption is more conservative than based on impermeable assumption.

Figures 5–7 show the influences on G from the directions of applied D^∞_2 and B^∞_2, respectively. Figure 5 shows the results for loading D^∞_2 and σ_{32}, Fig. 6 for B^∞_2 and σ_{32}, while Fig. 7 is for combined loading D^∞_2, B^∞_2, and σ_{32}. In these figures, a negative G can be observed under certain mechanically applied load, namely σ^{rtd}_{32}, for a given D^∞_2 and/or B^∞_2. These negative values on G may suggest that the applied electric-magnetic loading would retard the propagation of an interface crack in piezo-electro-magnetic bimaterials, a result which was also found in Ref. [12] for cracks in monolithic piezo-electro-magnetic materials. The σ^{rtd}_{32} varies as the direction of D^∞_2 or B^∞_2 revises. One can also observe that there exists a direction in which the combined loading applied would make σ^{rtd}_{32} reach its maximum and minimum value.

Figures 8 and 9 more clearly show the retarding effects, respectively, of $\varepsilon^{II}_{11}/\varepsilon^I_{11}$ and μ^{II}_{11}/μ^I_{11} on the energy release rate G under pure loading D^∞_2 or B^∞_2. In these two pictures, the value of G is always negative since the applied mechanical loading σ_{32} is zero. The G increases as $\varepsilon^{II}_{11}/\varepsilon^I_{11}$ or μ^{II}_{11}/μ^I_{11} increases, a result consist with the observation in Figs. 11 and 12.

Plotted in Figs. 10–12 are, correspondingly, the influences of c^{II}_{44}/c^I_{44}, $\varepsilon^{II}_{11}/\varepsilon^I_{11}$, and μ^{II}_{11}/μ^I_{11} on the energy release rate, G, under combined electric, magnetic, and mechanical loading for an impermeable interface crack. The plotting in solid line is for the

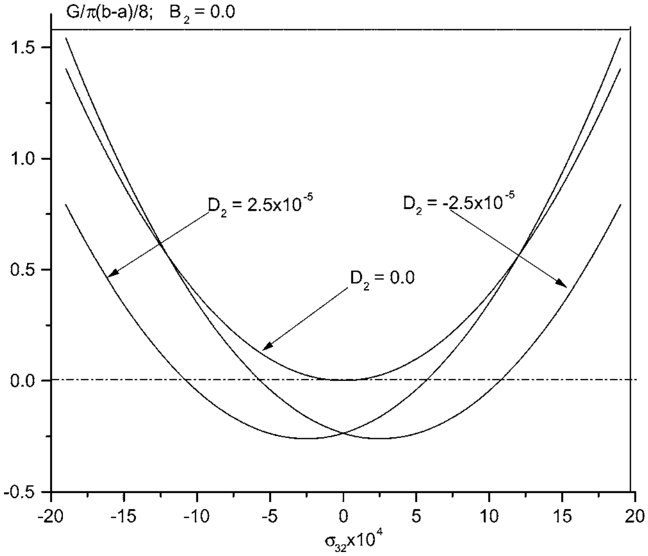

Fig. 5 The effect of the direction of the applied D_2 on the energy release rate, G, for an impermeable interface crack

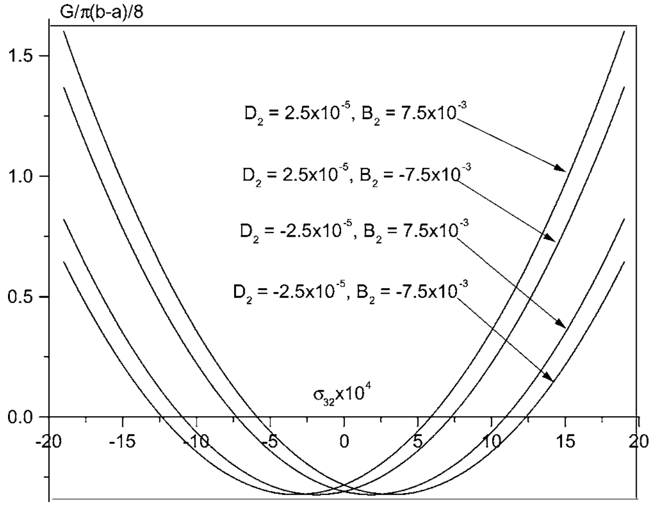

Fig. 7 The effect of the directions of the combined applied B_2 and D_2 on the energy release rate, G, for an impermeable interface crack

positive direction in D_2^∞ and B_2^∞, the dashed line is for revised direction in D_2^∞ and B_2^∞. The G decreases as the c_{44}^{II}/c_{44}^{I} increases and keeps almost unchanged when c_{44}^{II}/c_{44}^{I} reaches a certain value for both applied loading directions, as shown in Fig. 10. On the contrary, the G increases as the $\varepsilon_{11}^{II}/\varepsilon_{11}^{I}$ and $\mu_{11}^{II}/\mu_{11}^{I}$ increase, respectively. The observations in these figures may suggest that a reasonable selection in the mechanical and electric-magnetic properties for the two constituents of a bimaterial media may lower the energy release rate, making this bimedia much safer with regard to propagation of cracks.

Finally, it should be mentioned that the important contribution of our paper is the novel procedure, which has been developed to solve for the electric-magnetic fields inside an interface crack in a general bimaterial. The exact agreement of the results from this method with the results from the mapping method for the special case of homogeneous material (i.e., no bimaterial) in the literature, which, again, is the only case solved in the literature, provides validity for our "energy method" approach. It should be noted at this point that the contribution of the electric-magnetic fields inside a crack is very important for the devices being made of piezo-magneto-electro-elastic materials, since these fields may interfere with the desirable signals of electric-magnetic fields, like in broadband detecting devices. The results of our study could offer tentative guidelines for the damage-tolerant design of the devices.

5 Conclusions

In the present paper, the mode III interface crack in dissimilar piezo-magneto-electro-elastic bimaterial media is investigated in Stroh's formulism. In this study, the electric-magnetic field inside the interface crack is also considered and an "energy method" is proposed for obtaining the solution to this electric-magnetic field. Two types of interface cracks, namely permeable and impermeable cracks, are addressed. All the solutions are derived in closed form. The following conclusions can be reached from the results in this study:

1. The "energy method" is a very effective way to derive a solution to the electric-magnetic field inside a crack, thus

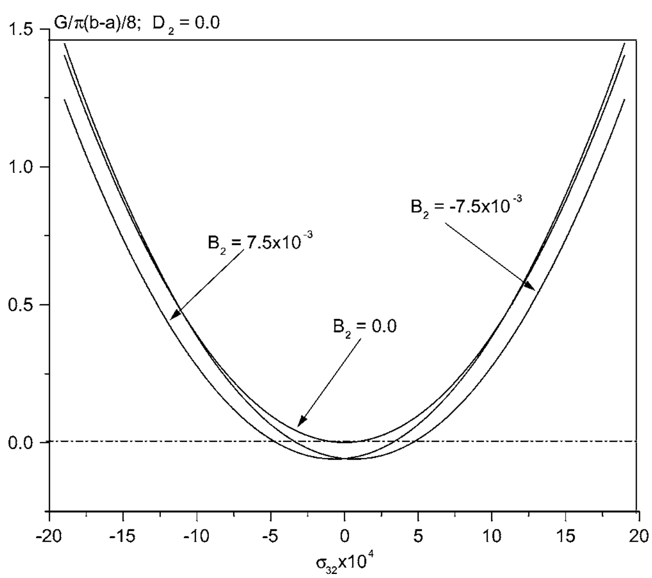

Fig. 6 The effect of the direction of the applied B_2 on the energy release rate, G, for an impermeable interface crack

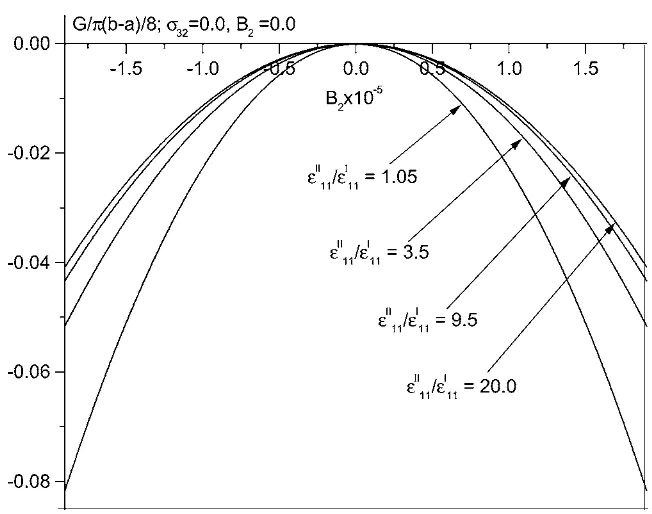

Fig. 8 Energy release rate, G, versus the electric permittivity ratio, $\varepsilon_{11}^{II}/\varepsilon_{11}^{I}$, for an impermeable interface crack under pure D_0^∞ loading

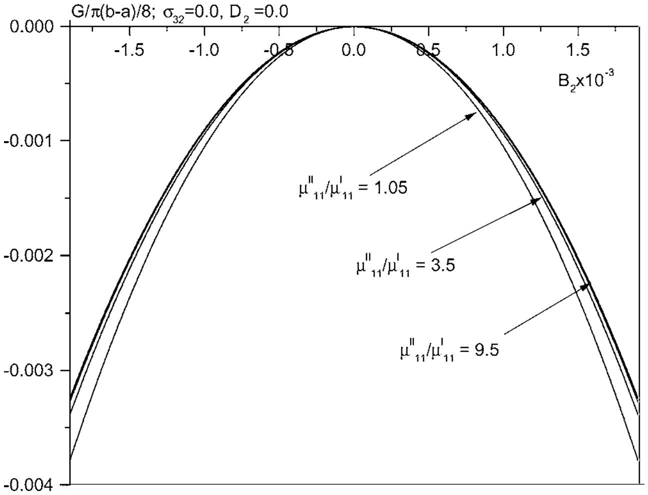

Fig. 9 Energy release rate, *G*, versus the magnetic permeability ratio, $\mu_{11}^{II}/\mu_{11}^{I}$, for an impermeable interface crack under pure B_0^∞ loading

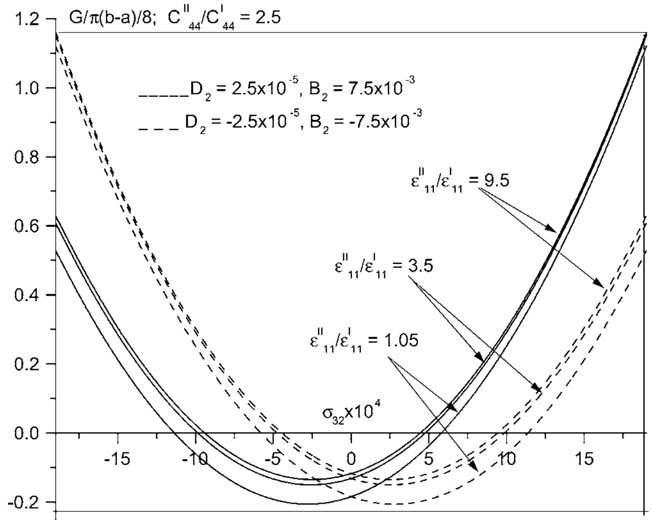

Fig. 11 Energy release rate, *G*, versus the electric permittivity ratio, $\varepsilon_{11}^{II}/\varepsilon_{11}^{I}$, for an impermeable interface crack under combined applied loading

solving the whole interface crack problem when the electric-magnetic field inside a crack is taken into account.

2. The mismatches of c_{44}, ε_{11}, and μ_{11} between the two constituents of a bimaterial media have strong effects on the potential propagation of a mode III interface crack. There exists an optimal selection on c_{44}, ε_{11}, and μ_{11} that would minimize the energy release rate for this mode III interface crack.
3. The directions of the applied loading D_2^∞ and B_2^∞ also have an effect on the possible growth of the interface crack in a piezo-electro-magneto-elastic bimaterial media.
4. The applied electric and/or magnetic loading D_2^∞ and B_2^∞ usually retard the propagation of the mode III interface crack.

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Appendix: Contour Integral for $\Phi(z)'$

The method used here can be viewed as the generalization of the technique in [16, 110, and 70] which is for a single equation. Let γ be a contour which includes the arc $\overline{ab}$, and let this contour shrink into the arc $\overline{ab}$. Then for $q(x_1)$ constant

$$\int_\gamma \frac{[X(\xi)]^{-1}N^{-1}}{\xi - z}d\xi = \int_{\overline{ab}} \frac{[X_+(x_1)]^{-1}N^{-1}}{x_1 - z}dt + \int_{\overline{ba}} \frac{[X_-(x_1)]^{-1}\bar{N}}{x_1 - z}dx_1 = \int_{\overline{ab}} \frac{[X_+(x_1)]^{-1}N^{-1}}{x_1 - z}dx_1 - \int_{\overline{ab}} \frac{[X_-(x_1)]^{-1}N^{-1}}{x_1 - z}dx_1 \tag{A1}$$

From Eq. (32), one could have

$$X_-(x_1) = -\bar{N}^{-1}NX_+(x_1), \quad a < x_1 < b \tag{A2}$$

Substituting Eq. (A2) into (A1) leads

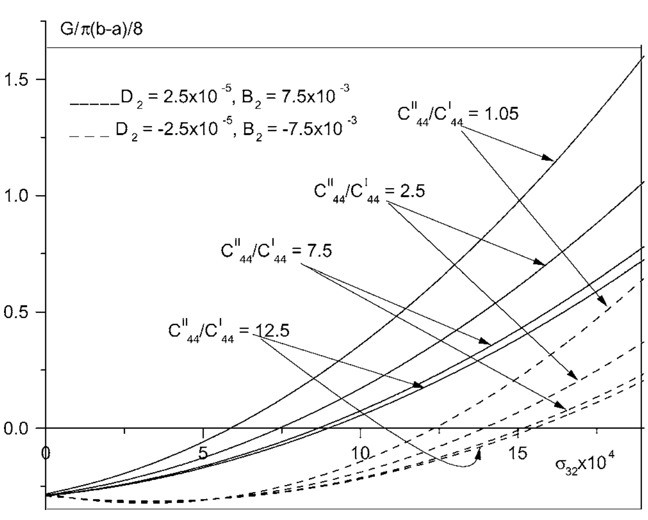

Fig. 10 Energy release rate, *G*, versus the stiffness ratio, c_{44}^{II}/c_{44}^{I}, for an impermeable interface crack under combined applied loading

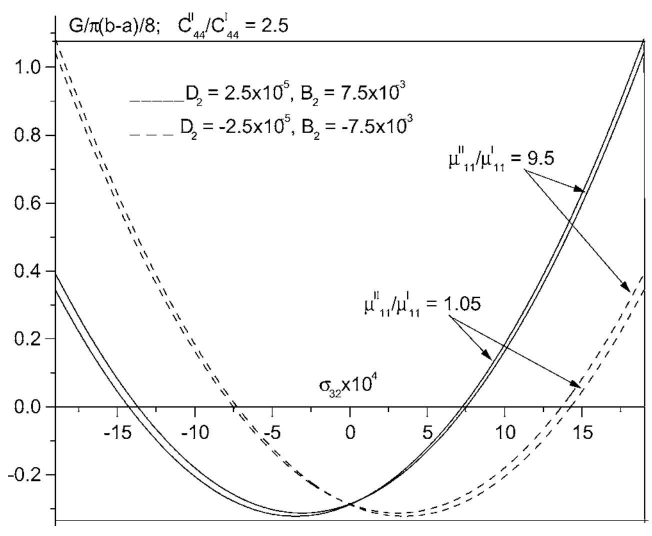

Fig. 12 Energy release rate, *G*, versus the magnetic permeability ratio, $\mu_{11}^{II}/\mu_{11}^{I}$, for an impermeable interface crack under combined applied loading

$$\int_{\gamma} \frac{[X(\xi)]^{-1}N^{-1}}{\xi - z} d\xi = \int_{\overline{ab}} \frac{[X_{+}(x_1)]^{-1}N^{-1}[I+\bar{N}N^{-1}]}{x_1 - z} dx_1 \quad \text{(A3)}$$

Then,

$$\int_{\overline{ab}} \frac{[X_{+}(x_1)]^{-1}N^{-1}}{x_1 - z} dx_1 = \int_{\gamma} \frac{[X(\xi)]^{-1}N^{-1}[I+\bar{N}N^{-1}]^{-1}}{\xi - z} d\xi$$

$$= \int_{\gamma} \frac{[X(\xi)]^{-1}[N+\bar{N}]^{-1}}{\xi - z} d\xi \quad \text{(A4)}$$

Since

$$N = \bar{N} = H^{-1} \quad \text{(A5)}$$

then,

$$\Phi'(z) = \frac{1}{2\pi i} X(z) \int_{\overline{ab}} \frac{[X_{+}(x_1)]^{-1}N^{-1}[ip]}{x_1 - z} dx_1$$

$$= \mathrm{diag}\left(1 - \frac{z-(a+b)/2}{\sqrt{(z-a)(z-b)}}\right)\frac{H}{2}(ip) \quad \text{(A6)}$$

References

[1] Alshits, V. I., Darinskii, A. N., and Lothe, J., 1992, "On the Existence of Surface Waves in Half-Infinite Anisotropic Elastic Media with Piezoelectric and Piezomagnetic Properties," Wave Motion, **16**, pp. 265–283.
[2] Benveniste, Y., 1995, "Magnetoelectric Effect in Fibrous Composites with Piezoelectric and Piezomagnetic Phases," Phys. Rev. B, **51**(22), pp. 16424–16427.
[3] Bichurinm, M. I., Kornev, V. M., Petrov, V. M., Tatarenko, A. S., Kiliba, Y. V., and Srinivasan, G., 2001, "Theory of Magnetoelectric/Magnetostrictive Multilayer Composites," Phys. Rev. B, **64**, pp. 094409-1–094409-6.
[4] Bichurin, M. I., Filippov, D. A., Petrov, V. M., Laletsin, V. M., and Paddubnaya, N., 2003, "Resonance Magnetoelectric Effect in Layered Magnetostrictive-Piezoelectric Composites," Phys. Rev. B, **68**, pp. 132408-1–132408-4.
[5] Pan, E. 2002, "Three-Dimensional Green's Functions in Anisotropic Magneto-Electro-Elastic Bimaterials," Math. Phys., **53**, pp. 815–838.
[6] Sih, G. C., and Song, Z. F., 2003, "Magnetic and Electric Poling Effects Associated with Crack Growth in $BaTio_3-CoFe_2O_4$ Composites," Theor. Appl. Fract. Mech., **39**, pp. 209–227.
[7] Chung, M. Y., and Ting, T. C. T., 1995, "The Green Function for a Piezoelectric Piezomagnetic Anisotropic Elastic Medium with an Elliptic Hole or Rigid Inclusion," Philos. Mag. Lett., **72**(6), pp. 405–410.
[8] Gao, C. F., Kessler, H., and Balke, H., 2003, "Crack Problems in Magnetoelectroelastic Solids. Part I: Exact Solution of a Crack," Int. J. Eng. Sci., **41**, pp. 969–981.
[9] Gao, C. F., Kessler, H., and Balke, H., 2003, "Crack Problems in Magnetoelectroelastic Solids. Part I: General Solution of Collinear Cracks," Int. J. Eng. Sci., **41**, pp. 969–981.
[10] Song, Z. F., and Sih, G. C., 2003, "Crack Initiation Behavior in Magnetoelectroelasstic Composite Under In-Plane Deformation," Theor. Appl. Fract. Mech. **39**, pp. 189–207.
[11] Gao, C. F., Tong, P., and Zhang, T. Y., 2003, "Interfacial Crack Problems in Magneto-Electroelastic Solids," Int. J. Solids Struct., **41**, pp. 2105–2121.
[12] Gao, C. F., Tong, P., and Zhang, T. Y., 2004, "Fracture Mechanics for a Mode III Crack in a Magnetoelectroelastic Solid," Int. J. Solids Struct., **41**, pp. 6613–6629.
[13] Nan, C. W., 1994, "Magnetoelectric Effect in Composites of Piezoelectric and Piezomagnetic Phases," Phys. Rev. B, **50**(9), pp. 6082–6088.
[14] Stroh, A. N., 1958, "Dislocations and Cracks in Anisotropic Elasticity," Philos. Mag., **8**(3), pp. 625–646.
[15] Rudin, W., 1987, *Real and Complex Analysis*, McGraw-Hill, New York.
[16] Muskhelishvili, N. I., 1953, *Some Basic Problems of the Mathematical Theory of Elasticity*, P. Noordoff and Company, New York.
[17] McMeeking, R. M., 1989, "Electrostrictive Forces Near Crack Like Flaws," J. Appl. Math. Phys., **40**, pp. 615–627.