Nonlinear Response of a Shallow Sandwich Shell With Compressible Core to Blast Loading

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This paper investigates the nonlinear dynamic response of a shallow sandwich shell subject to blast loading with consideration of core compressibility. The shallow shell consists of two laminated composite or metallic face sheets and an orthotropic compressible core. Experimental results and finite element simulations in literature have shown that the core exhibits considerable compressibility when a sandwich panel is subjected to impulse loading. To address this issue properly in the analysis, a new nonlinear compressible core model is proposed in the current work. The system of governing equations is derived by means of Hamilton's principle in combination with the Reissner–Hellinger's variational principle. The analytical solution for the simply supported shallow shell is formulated using an extended Galerkin procedure combined with the Laplace transform. Numerical results are presented. These results demonstrate that this advanced sandwich model can capture the transient responses such as the stress shock wave effect and the differences in the transient behaviors of the face sheets and the core when a sandwich shadow shell is subjected to a blast loading. However, in the steady state dynamic stage, all the displacements of the face sheets and the core tend to be identical. This model can be further used to study the energy absorption ability of the core and the effects of different material and geometrical parameters on the behaviors of sandwich structures subject to blast loading. [DOI: 10.1115/1.2937154]

Keywords: shallow sandwich shell, blast, impact, laminated face sheets, compressible core, dynamic response, sudden loading

1 Introduction

The response of suddenly loaded structural configurations is essential in ensuring their integrity. Sudden loading can occur, for example, due to blast from an explosive device and this entails both distributed particle impact from the explosion fragments and the overpressure from the shock wave. This study deals with the dynamic response of sandwich shallow shells to a blast pressure pulse. There have been indeed recently many efforts to investigate the blast response of structures and suggest ways of mitigating their detrimental effects through an optimal sandwich construction design. Several papers have addressed various aspects of the problem [1–7]. A typical sandwich structure consists of two stiff metallic/composite face sheets and a soft honeycomb/foam core. This layout gives the sandwich material system the integrity of high stiffness and strength with little resultant weight penalty and high-energy absorption capability and has led to many successful applications of sandwich structures in the construction of marine vessels, aerospace vehicles, and civil infrastructure.

In the study of the response of a sandwich structure to a static loading or a dynamic loading of long duration, it has been customary to neglect the deformation of the core in the transverse direction [8,9]. The core would then be considered infinitely rigid in the thickness direction and assumed to only carry the shear stresses. Though there are two transversely compressive core models proposed in literature [10–12], the transversely rigid core model has been found to be working well in most of the studies involving static or dynamic-long-duration loading. However, experimental and numerical results [3–7] have shown that the core undergoes significant deformation when the sandwich structure experiences a sudden, impulsive loading and the core plays an important role in the absorption of the impact energy. Therefore, a model including the core transverse flexibility would offer a better prediction over the classic transversely rigid core model in the study of the transient response of sandwich structures. A detailed look into the two currently available transversely compressive core models would reveal that the transverse strains in the models in Refs. [12,10,11] are constant and linear functions with regard to the variable in the transverse direction, respectively. However, the observations in Refs. [6,7] clearly demonstrate that the core transverse deformation/strain is highly nonlinear with regard to the variable in the thickness direction. Therefore, a more refined core model is needed in order to obtain a better understanding of the dynamic behavior of a sandwich construction under sudden, blast loading. Furthermore, up to date, most of the studies for the response of sandwich construction to blast loading have focused on flat panels or plates. Very few works on this topic are available for the sandwich shallow shell configuration, which is very often used in engineering construction, for example, in ship hulls. Therefore, the investigation of the behavior of the sandwich shallow shell to blast loading has both practical and theoretical importance.

In this paper, we shall properly address these issues by first proposing an advanced sandwich shallow shell model that accounts for the highly nonlinear compressibility of the core. The transient behavior of the face sheets and the core will be analyzed in some detail. We organize this paper as follows: A nonlinear transversely compressible core theory is proposed in Sec. 2. In the model, the strain of the core in the transverse direction is no longer constant or linear but a third order function with regard to the transverse variable. The derivation of the governing equations

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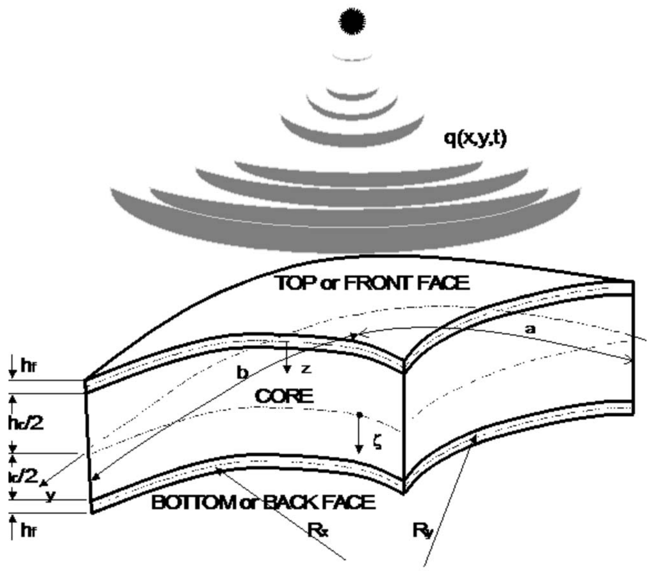

Fig. 1 A sandwich shallow shell subject to a sudden, blast impact

and boundary and initial conditions is presented in Sec. 3. These unknowns in the equations are highly coupled in terms of both spatial and time variables. The solution procedure for solving the nonlinear partial governing equations is presented in Sec. 4. Applications and discussions of numerical results are given in Sec. 5. In sec. 6, we present some conclusions and related work in the future.

2 Formulation

2.1 Basic Assumptions. The sandwich shell treated here is composed of two thin faces of high stiffness and a thick soft core (Fig. 1), whose compressibility will also be taken into account. Because of the core compressibility, the widely used [4] assumption that the transverse displacements of the two face sheets and the core displacements are equal will not be adopted in this study. We shall employ the following assumptions:

1. The face sheets satisfy the Kirchhoff–Love assumptions and the thicknesses are small compared with the overall thickness of the sandwich section. In the current study, the two face sheets are further assumed to have identical thickness.
2. The curvatures of each shell in the two directions may not be equal and the total thickness of the shell section is small compared to its radii of curvatures.
3. The core is compressible in the transverse direction, that is, its thickness may change.
4. The bonding between the face sheets and the core is assumed to be perfect.
5. Uniformly distributed shock wave pressure on the front face of the shell, will be considered. The intensities of the loading would range from causing indentation to core crushing or initiating face damage.

2.2 Kinematics for the Thin Face Sheets and the Compressible Core of a Sandwich Shallow Shell. Thin Face Sheets. Let a Cartesian coordinates system (x,y,z) be on the middle plane of the core, as shown in Fig. 1. The middle surfaces of the two face sheets and the core can then be defined in terms of a set of curvilinear coordinates (α,β,ζ) as $x=x(\alpha,\beta)$, $y=y(\alpha,\beta)$, and $z=(\alpha,\beta)$. Considering the shallow shell assumptions that the terms $z_{,\alpha}^2$ and $z_{,\beta}^2$ can be neglected in comparison to unity [13], the curvilinear coordinate system can be approximated by the Cartesian coordinates in the middle surface and the transverse displacements through the thickness can be approximated by the middle surface displacement for a thin shallow shell. We also define by ζ a global transverse coordinate from the midsurface of the core, as opposed to the local transverse coordinate from the midsurface of each phase (face sheet or core), which is denoted by z.

For thin face sheets, the transverse displacements can be viewed as undergoing no change through the thickness. Therefore, the displacements in the face sheets can be expressed as

$$u^t(x,y,\zeta,t) = u_o^t(x,y,t) - \left(\zeta + \frac{h_c+h_f}{2}\right)w_{,x}^t(x,y,t) \qquad (1a)$$

$$v^t(x,y,\zeta,t) = v_o^t(x,y,t) - \left(\zeta + \frac{h_c+h_f}{2}\right)w_{,y}^t(x,y,t) \qquad (1b)$$

$$w^t(x,y,\zeta,t) = w^t(x,y,t), \quad -h_f - \frac{h_c}{2} \leqslant \zeta \leqslant -\frac{h_c}{2} \qquad (1c)$$

for the top face sheet, and

$$u^b(x,y,\zeta,t) = u_o^b(x,y,t) - \left(\zeta - \frac{h_c+h_f}{2}\right)w_{,x}^b(x,y,t) \qquad (2a)$$

$$v^b(x,y,\zeta,t) = v_o^b(x,y,t) - \left(\zeta - \frac{h_c+h_f}{2}\right)w_{,y}^b(x,y,t) \qquad (2b)$$

$$w^b(x,y,\zeta,t) = w^b(x,y,t), \quad \frac{h_c}{2} \leqslant \zeta \leqslant \frac{h_c}{2} + h_f \qquad (2c)$$

for the bottom face sheet. Omitting the superscripts t and b, the nonlinear strain-displacement relations for the face sheets can take the following form:

$$[\epsilon] = \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = [\epsilon_o] + z[k] = \begin{bmatrix} \epsilon_{ox} + zk_x \\ \epsilon_{oy} + zk_y \\ \gamma_{oxy} + zk_{xy} \end{bmatrix}, \quad z = \zeta \pm \frac{h_c+h_f}{2} \qquad (3)$$

in which the "$\pm$;" sign in the variable z corresponds to the top and bottom face sheets, respectively, and $[\epsilon_o]$ is the middle surface strain given by

$$[\epsilon_o] = \begin{bmatrix} \epsilon_{ox} \\ \epsilon_{oy} \\ \gamma_{oxy} \end{bmatrix} = \begin{bmatrix} u_{o,x} + \frac{1}{2}w_{,x}^2 + w/R_x \\ v_{o,y} + \frac{1}{2}w_{,y}^2 + w/R_y \\ u_{o,y} + v_{o,x} + w_{,x}w_{,y} + 2w/R_{xy} \end{bmatrix} \qquad (4)$$

Moreover, $[k]$ is the curvature

$$[k] = \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} -w_{,xx} \\ -w_{,yy} \\ -2w_{,xy} \end{bmatrix} \qquad (5)$$

where R_x, R_y, and R_{xy} are the radii of the curvature for the middle surfaces.

Higher Order Theory for Compressible Cores. The compressibility in the thickness direction of the core can be important for the absorption ability of a sandwich shallow shell subject to a suddenly applied loading. This compressibility implies that the displacement in the thickness direction should be a function of the variable in the transverse direction and satisfy equilibrium equations and continuity conditions along the face sheets/core interface. In literature, this function is often approximated in a linear or quadratic form [4,10]. In this paper, a fourth order nonlinear core theory is formulated. Details of the formulation are given in Appendix A. Based on this model, the transverse displacement can be expressed as follows:

$$w^c(x,y,\zeta,t)=\left(1-\frac{2\zeta^2}{h_c^2}-\frac{8\zeta^4}{h_c^4}\right)w_o^c(x,y,t)+\left(\frac{2\zeta^2}{h_c^2}+\frac{8\zeta^4}{h_c^4}\right)\overline{w}(x,y,t)-\left(\frac{\zeta}{h_c}+\frac{4\zeta^3}{h_c^3}\right)\hat{w}(x,y,t),\quad -\frac{h_c}{2}\leqslant\zeta\leqslant\frac{h_c}{2} \tag{6}$$

and the in-plane displacements in the core are

$$u^c(x,y,\zeta,t)=\overline{u}(x,y,t)-\frac{\zeta}{h_c/2}\hat{u}(x,y,t)+\zeta\frac{h_f}{h_c}w^c_{,x}(x,y,\zeta,t) \tag{7a}$$

$$v^c(x,y,\zeta,t)=\overline{v}(x,y,t)-\frac{\zeta}{h_c/2}\hat{v}(x,y,t)+\zeta\frac{h_f}{h_c}w^c_{,y}(x,y,\zeta,t) \tag{7b}$$

where $w_0^c(x,y,t)$ is the transverse displacement of the middle surface of the core, $\overline{w}(x,y,t)$ and $\hat{w}(x,y,t)$, $\overline{u}(x,y,t)$ and $\hat{u}(x,y,t)$, and $\overline{v}(x,y,t)$, and $\hat{v}(x,y,t)$ are defined in Appendix A.

This leads to the following strain-displacement relations for the core:

$$\epsilon_z^c=\left(-\frac{1}{2h_c}+\frac{2\zeta}{h_c^2}-\frac{6\zeta^2}{h_c^3}+\frac{16\zeta^3}{h_c^4}\right)w^t(x,y,t)-\left(\frac{4\zeta}{h_c^2}+\frac{32\zeta^3}{h_c^4}\right)w_o^c(x,y,t)+\left(\frac{1}{2h_c}+\frac{2\zeta}{h_c^2}+\frac{6\zeta^2}{h_c^3}+\frac{16\zeta^3}{h_c^4}\right)w^b(x,y,t) \tag{8a}$$

$$\gamma_{xz}^c=-\frac{2}{h_c}\hat{u}(x,y,t)+\eta_1(\zeta)w^t_{,x}(x,y,t)+\eta_2(\zeta)w^c_{o,x}(x,y,t)+\eta_3(\zeta)w^b_{,x}(x,y,t) \tag{8b}$$

$$\gamma_{yz}^c=-\frac{2}{h_c}\hat{v}(x,y,t)+\eta_1(\zeta)w^t_{,y}(x,y,t)+\eta_2(\zeta)w^c_{o,y}(x,y,t)+\eta_3(\zeta)w^b_{,y}(x,y,t) \tag{8c}$$

in which

$$\eta_1(\zeta)=-\frac{1}{2}\left(1+2\frac{h_f}{h_c}\right)\frac{\zeta}{h_c}+\left(1+3\frac{h_f}{h_c}\right)\frac{\zeta^2}{h_c^2}-2\left(1+4\frac{h_f}{h_c}\right)\frac{\zeta^3}{h_c^3}+4\left(1+5\frac{h_f}{h_c}\right)\frac{\zeta^4}{h_c^4} \tag{9a}$$

$$\eta_2(\zeta)=\left(1+\frac{h_f}{h_c}\right)-2\left(1+\frac{3h_f}{h_c}\right)\frac{\zeta^2}{h_c^2}-8\left(1+\frac{5h_f}{h_c}\right)\frac{\zeta^4}{h_c^4} \tag{9b}$$

$$\eta_3(\zeta)=\frac{1}{2}\left(1+2\frac{h_f}{h_c}\right)\frac{\zeta}{h_c}+\left(1+3\frac{h_f}{h_c}\right)\frac{\zeta^2}{h_c^2}+2\left(1+4\frac{h_f}{h_c}\right)\frac{\zeta^3}{h_c^3}+4\left(1+5\frac{h_f}{h_c}\right)\frac{\zeta^4}{h_c^4} \tag{9c}$$

The core is considered undergoing large rotation with a small displacement; therefore, the in-plane strains can be neglected.

2.3 Constitutive Relations. The equations developed so far can be applied to general materials. In the following sections, we shall assume the face sheets to be orthotropic laminated composites and the core to be orthotropic as well. The stress-strain relationship for any layer of the face sheets is

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{bmatrix}=\begin{bmatrix}Q_{11}&Q_{12}&Q_{16}\\ Q_{12}&Q_{22}&Q_{26}\\ Q_{16}&Q_{26}&Q_{66}\end{bmatrix}\begin{bmatrix}\epsilon_x\\ \epsilon_y\\ \gamma_{xy}\end{bmatrix}\quad\text{or } [\sigma]=[Q][\epsilon] \tag{10}$$

where Q_{ij} for $i,j=1,2,6$ are the plane-stress reduced stiffness coefficients. With Eqs. (3)–(5), (8*a*)–(8*c*), and (10), one can compute the resultants for the top/front face sheet of the sandwich shallow shell:

$$[N^t]=\begin{bmatrix}N_x^t\\ N_y^t\\ N_{xy}^t\end{bmatrix}=\int_{-h_c/2-h_f}^{-h_c/2}[\sigma^t]d\zeta=\int_{-h_c/2)-h_f}^{-h_c/2}[Q^t][\epsilon^t]d\zeta=[A^t][\epsilon_0^t]+[B^t][k^t] \tag{11a}$$

$$[M^t]=\begin{bmatrix}M_x^t\\ M_y^t\\ M_{xy}^t\end{bmatrix}=\int_{-h_c/2-h_f}^{-h_c/2}[\sigma^t]\left(\zeta+\frac{h_c+h_f}{2}\right)d\zeta=[B^t][\epsilon_o^t]+[D^t][k^t] \tag{11b}$$

in which the stiffness coefficients are

$$[A_{ij}^t,B_{ij}^t,D_{ij}^t]=\int_{-h_c/2-h_f}^{-h_c/2}Q_{ij}\times\left[1,\zeta+\frac{h_c+h_f}{2},\left(\zeta+\frac{h_c+h_f}{2}\right)^2\right]d\zeta,\quad i,j=1,2,6 \tag{12}$$

Applying a similar procedure, one can obtain the following resultant expressions for the bottom/back face sheet:

$$[N^b]=[A^b][\epsilon_0^b]+[B^b][k^b] \tag{13a}$$

$$[M^b]=[B^b][\epsilon_o^b]+[D^b][k^b] \tag{13b}$$

with the stiffness coefficients reading as

$$[A_{ij}^b,B_{ij}^b,D_{ij}^b]=\int_{h_c/2}^{h_c/2+h_f}Q_{ij}\times\left[1,\zeta-\frac{h_c+h_f}{2},\left(\zeta-\frac{h_c+h_f}{2}\right)^2\right]d\zeta,\quad i,j=1,2,6 \tag{14}$$

The stress-strain relations for an orthotropic core can be written as

$$\sigma_z^c=E^c\epsilon_z^c,\quad \tau_{xz}^c=G_{xz}^c\gamma_{xz}^c,\quad \tau_{yz}^c=G_{yz}^c\gamma_{yz}^c \tag{15}$$

3 Governing Equations

The equations of motion and appropriate boundary conditions can be derived using Hamilton's principle. The sandwich shell is subjected to a sudden loading $q(x,y,t)$ on the front face sheet. Let the strain energy be denoted by U, the external potential by W, and the kinetic energy by T, then the variational principle is stated as

$$\delta\int_{t_o}^{t_1}[T-(U-W)]dt=0 \tag{16}$$

in which

$$\delta T=\int_{t_o}^{t_1}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\left[\int_{-h_c/2-h_f}^{-h_c/2}\rho^t(\dot{u}^t\delta\dot{u}^t+\dot{v}^t\delta\dot{v}^t+\dot{w}^t\delta\dot{w}^t)d\zeta+\int_{-h_c/2}^{h_c/2}\rho^c(\dot{u}^c\delta\dot{u}^c+\dot{v}^c\delta\dot{v}^c+\dot{w}^c\delta\dot{w}^c)d\zeta+\int_{h_c/2}^{h_c/2+h_f}\rho^b(\dot{u}^b\delta\dot{u}^b+\dot{v}^b\delta\dot{v}^b+\dot{w}^b\delta\dot{w}^b)d\zeta\right]dxdydt \tag{17}$$

$$\delta U=\int_{t_o}^{t_1}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2}\left[\int_{-h_c/2-h_f}^{-h_c/2}(\sigma_x^t\delta\epsilon_x^t+\sigma_y^t\delta\epsilon_y^t+\tau_{xy}^t\delta\gamma_{xy}^t)d\zeta+\int_{-h_c/2}^{h_c/2}(\sigma_z^c\delta\epsilon_z^c+\sigma_{xz}^c\delta\gamma_{xz}^c+\tau_{yz}^c\delta\gamma_{yz}^c)d\zeta+\int_{h_c/2}^{h_c/2+h_f}(\sigma_x^b\delta\epsilon_x^b+\sigma_y^b\delta\epsilon_y^b+\tau_{xy}^b\delta\gamma_{xy}^b)d\zeta\right]dxdydt \tag{18}$$

$$\delta W = \int_{t_o}^{t_1}\int_{-b/2}^{b/2}\int_{-a/2}^{a/2} q(x,y,t)\delta w^t dxdydt \tag{19}$$

where ρ is the mass density. The superscript t in Eqs. (18), (17), and (19) denotes the corresponding values for the top face sheet whereas t when appearing in the variable list of the functions refers to time. The equation of motion and the boundary conditions can be obtained by substituting the stress-strain relations (10) and (15) and displacements (6) and (7*a*)–7(c) into Eqs. (18), (17), and (19), then into Eq. (16) and employing integration by parts. This results in seven equations, three for each face sheet and one for the core. There are seven unknowns: u_o^t, v_o^t, w^t, w_o^c, u_o^b, v_o^b, and w^b.

The resulting equations for the top face sheet are

$$N_{x,x}^t + N_{xy,y}^t - \left(\rho^t h_f + \rho^c \frac{h_c}{3}\right)\ddot{u}_o^t - \rho^c \frac{h_c}{6}\ddot{u}_o^b + \rho^c \frac{h_c h_f}{420}(23\ddot{w}_{,x}^t + 17\ddot{w}_{o,x}^c - 5\ddot{w}_{,x}^b) - G_{xz}^c\left[\frac{1}{h_c}(u_o^t - u_o^b) - \frac{11}{15}w_{o,x}^c - \alpha_4(w_{,x}^t + w_{,x}^b)\right] = 0 \tag{20}$$

$$N_{xy,x}^t + N_{y,y}^t - \left(\rho^t h_f + \rho^c \frac{h_c}{3}\right)\ddot{v}_o^t - \rho^c \frac{h_c}{6}\ddot{v}_o^b + \rho^c \frac{h_c h_f}{420}(23\ddot{w}_{,y}^t + 17\ddot{w}_{o,y}^c - 5\ddot{w}_{,y}^b) - G_{yz}^c\left[\frac{1}{h_c}(v_o^t - v_o^b) - \frac{11}{15}w_{o,y}^c - \alpha_4(w_{,y}^t + w_{,y}^b)\right] = 0 \tag{21}$$

and

$$M_{x,xx}^t + 2M_{xy,xy}^t + M_{y,yy}^t + (N_x^t w_{,x}^t)_{,x} + (N_{xy}^t w_{,x}^t)_{,y} + (N_{yx}^t w_{,y}^t)_{,x} + (N_y^t w_{,y}^t)_{,y} - \left(\rho^t h_f + \frac{29}{315}\rho^c h_c\right)\ddot{w}^t - \rho^c \frac{37h_c}{630}\left(\ddot{w}_o^c - \frac{11}{37}\ddot{w}^b\right) + \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \times\left[\left(\rho^t \frac{h_f^3}{12} + \rho^c \frac{19h_c h_f^2}{1155}\right)\ddot{w}^t + \frac{\rho^c h_c h_f^2}{27720}(199\ddot{w}_o^c - 61\ddot{w}^b)\right] - \rho^c \frac{h_c h_f}{420}[23(\ddot{u}_{o,x}^t + \ddot{v}_{o,y}^t) + 5(\ddot{u}_{o,x}^b + \ddot{v}_{o,y}^b)] + \alpha_1 h_c(G_{xz}^c w_{,xx}^t + G_{yz}^c w_{,yy}^t) + \alpha_2 h_c(G_{xz}^c w_{o,xx}^c + G_{yz}^c w_{o,yy}^c) - \alpha_3 h_c(G_{xz}^c w_{,xx}^b + G_{yz}^c w_{,yy}^b) - \alpha_4[G_{xz}^c(u_{o,x}^t - u_{o,x}^b) + G_{yz}^c(v_{o,y}^t - v_{o,y}^b)] - (N_x^t/R_x^t + 2N_{xy}^t/R_{xy}^t + N_y^t/R_y^t) - \frac{61}{21}\frac{E^c}{h_c}\left(w^t - \frac{358}{305}w_o^c + \frac{53}{305}w^b\right) + q(x,y,t) = 0 \tag{22}$$

in which $\alpha_i (i=1,\ldots,4)$ are constants in terms of the ratio of face thickness and core thickness as follows:

$$\alpha_1 = \frac{29}{315} + \frac{373}{630}\frac{h_f}{h_c} + \frac{247}{252}\left(\frac{h_f}{h_c}\right)^2, \quad \alpha_2 = \frac{37}{630} + \frac{37}{630}\frac{h_f}{h_c} - \frac{383}{630}\left(\frac{h_f}{h_c}\right)^2 \tag{23a}$$

$$\alpha_3 = \frac{11}{630} + \frac{11}{630}\frac{h_f}{h_c} - \frac{23}{180}\left(\frac{h_f}{h_c}\right)^2, \quad \alpha_4 = \frac{2}{15} + \frac{h_f}{2h_c} \tag{23b}$$

A similar set of equations for the motion of the bottom face sheet can be derived, and this is listed in Appendix B.

The equations of motion for the compressible core are

$$\alpha_5 h_c(G_{xz}^c w_{o,xx}^c + G_{yz}^c w_{o,yy}^c) + \alpha_2 h_c[G_{xz}^c(w_{,xx}^t + w_{,xx}^b) + G_{yz}^c(w_{,yy}^t + w_{,yy}^b)] - \frac{194}{315}\hat{\rho}w_o^c - \frac{37h_c}{630}\rho^c(\ddot{w}^t + \ddot{w}^b) - \frac{17h_f h_c}{210}\rho^c(\hat{u}_{,x} + \hat{v}_{,y}) + \frac{181h_f^2 h_c}{6930}\rho^c\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[\ddot{w}_o^c + \frac{199}{724}(\ddot{w}^t + \ddot{w}^b)\right] - \frac{358}{105}\frac{E^c}{h_c}(2w_o^c - w^t - w^b) - \frac{11}{15}G_{xz}^c(u_{o,x}^t - u_{o,x}^b) - \frac{11}{15}G_{yz}^c(v_{o,y}^t - v_{o,y}^b) = -0 \tag{24}$$

where

$$\alpha_5 = \frac{194}{315} + \frac{194}{315}\frac{h_f}{h_c} + \frac{383}{315}\left(\frac{h_f}{h_c}\right)^2 \tag{25}$$

Assuming that the sandwich shells are made of orthotropic materials and substituting Eq. (4) into Eqs. (13*a*) and (13*b*) and then Eqs. (20)–(22), one can rewrite the nonlinear governing equations for the top face sheet as

$$A_{11}^t u_{o,xx}^t + A_{66}^t u_{o,yy}^t + (A_{12}^t + A_{66}^t)v_{o,xy}^t - \frac{G_{xz}^c}{h_c}(u_o^t - u_o^b) - \left(\rho^t h_f + \rho^c \frac{h_c}{3}\right)\ddot{u}_o^t - \rho^c \frac{h_c}{6}\ddot{u}_o^b = \hat{f}_1^t \tag{26a}$$

$$(A_{21}^t + A_{66}^t)u_{o,xy}^t + A_{66}^t v_{o,xx}^t + A_{22}^t v_{o,yy}^t - \frac{G_{yz}^c}{h_c}(v_o^t - v_o^b) - \left(\rho^t h_f + \rho^c \frac{h_c}{3}\right)\ddot{v}_o^t - \rho^c \frac{h_c}{6}\ddot{v}_o^b = \hat{f}_2^t \tag{26b}$$

$$D_{11}^t w_{,xxxx}^t + 2(D_{12}^t + 2D_{66}^t)w_{,xxyy}^t + D_{22}^t w_{,yyyy}^t + \frac{61}{21}\frac{E^c}{h_c}\left(w^t - \frac{358}{305}w_o^c + \frac{53}{305}w^b\right) + \left(\rho^t h_f + \rho^c \frac{29h_c}{315}\right)\ddot{w}^t + \rho^c \frac{37h_c}{630}\left(\ddot{w}_o^c - \frac{11}{37}\ddot{w}^b\right) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[\left(\rho^t \frac{h_f^3}{12} + \rho^c \frac{19h_f^2 h_c}{1155}\right)\ddot{w}^t + \frac{\rho^c h_f^2 h_c}{27720} \times(199\ddot{w}_o^c - 61\ddot{w}^b)\right] - \alpha_1 h_c(G_{xz}^c w_{,xx}^t + G_{yz}^c w_{,yy}^t) - \alpha_2 h_c(G_{xz}^c w_{o,xx}^c + G_{yz}^c w_{o,yy}^c) + \alpha_3 h_c(G_{xz}^c w_{,xx}^b + G_{yz}^c w_{,yy}^b) = q(x,y,t) + \hat{f}_3^t \tag{26c}$$

in which

$$\hat{f}_1^t = -G_{xz}^c\left[\frac{11}{15}w_{o,x}^c + \alpha_4(w_{,x}^t + w_{,x}^b)\right] - A_{11}w_{,x}^t w_{,xx}^t - (A_{12} + A_{66})w_{,y}^t w_{,xy}^t - A_{66}w_{,x}^t w_{,yy}^t - A_{11}\frac{w_{,x}^t}{R_x} - A_{12}\frac{w_{,x}^t}{R_y} - \rho^c \frac{h_f h_c}{420}(23\ddot{w}_{,x}^t + 17\ddot{w}_{o,x}^c - 5\ddot{w}_{,x}^b) \tag{27a}$$

$$\hat{f}_2^t = -G_{yz}^c\left[\frac{11}{15}w_{o,y}^c + \alpha_4(w_{,y}^t + w_{,y}^b)\right] - (A_{21}+A_{66})w_{,x}^t w_{,xy}^t - A_{66}w_{,xx}^t w_{,y}^t - A_{22}w_{,y}^t w_{,yy}^t - A_{21}\frac{w_{,y}^t}{R_x} - A_{22}\frac{w_{,y}^t}{R_y} - \rho^c\frac{h_f h_c}{420}(23\ddot{w}_{,y}^t + 17\ddot{w}_{o,y}^c - 5\ddot{w}_{,y}^b) \quad (27b)$$

$$\hat{f}_3^t = -\alpha_4[G_{xz}^c(u_{o,x}^t - u_{o,x}^b) + G_{yz}^c(v_{o,y}^t - v_{o,y}^b)] + (N_x^t w_{,x}^t)_{,x} + (N_{xy}^t w_{,x}^t)_{,y} + (N_{yx}^t w_{,y}^t)_{,x} + (N_y^t w_{,y}^t)_{,y} - \left(\frac{N_x^t}{R_x^t} + \frac{N_y^t}{R_y^t}\right) - \rho^c\frac{h_f h_c}{420}[23(\ddot{u}_{o,x}^t + \ddot{v}_{o,y}^t) + 5(\ddot{u}_{o,x}^b + \ddot{v}_{o,y}^b)] \quad (27c)$$

The first terms in the expressions for $\hat{f}_1^t$ and $\hat{f}_2^t$ reflect the effects of the higher order core theory, the second to fourth terms represent the effects from the von Karman nonlinear theory, the fifth to seventh terms represent the effects from the initial curvatures of the shallow shell, and the last terms can be viewed as the excitation produced by the transverse motion for the in-plane motion. Moreover, $\hat{f}_3^t$ includes the membrane-bending coupling effect. One can also see that the $\hat{f}_3^t$ includes the effects from the curvatures of the shell and the in-plane motion on the transverse motion. In $\hat{f}_1^t$, $\hat{f}_2^t$, and $\hat{f}_3^t$, we can further group the nonlinear terms and define

$$\hat{F}_1^t = A_{11}w_{,x}^t w_{,xx}^t + (A_{12}+A_{66})w_{,y}^t w_{,xy}^t + A_{66}w_{,x}^t w_{,yy}^t \quad (28a)$$

$$\hat{F}_2^t = (A_{21}+A_{66})w_{,x}^t w_{,xy}^t + A_{66}w_{,xx}^t w_{,y}^t \quad (28b)$$

$$\hat{F}_3^t = \frac{N_x^t}{R_x^t} + \frac{N_y^t}{R_y^t} - [(N_x^t w_{,x}^t)_{,x} + (N_{xy}^t w_{,x}^t)_{,y} + (N_{yx}^t w_{,y}^t)_{,x} + (N_y^t w_{,y}^t)_{,y}] \quad (28c)$$

Similarly, one can also recast the equations for core as follows:

$$\alpha_5 h_c(G_{xz}^c w_{o,xx}^c + G_{yz}^c w_{o,yy}^c) + \alpha_2 h_c[G_{xz}^c(w_{,xx}^t + w_{,xx}^b) + G_{yz}^c(w_{,yy}^t + w_{,yy}^b)] - \frac{194}{315}\rho^c h_c w_o^c - \frac{358}{105}\frac{E^c}{h_c}(2w_o^c - w^t - w^b) - \frac{37h_c}{630}\rho^c(\ddot{w}^t + \ddot{w}^b) + \frac{181h_f^2 h_c}{6930}\rho^c\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \times \left[\ddot{w}_o^c + \frac{199}{724}(\ddot{w}^t + \ddot{w}^b)\right] = \hat{f}^c \quad (29)$$

where

$$\hat{f}^c = \frac{17h_f h_c}{210}\rho^c(\hat{u}_{,x} + \hat{v}_{,y}) + \frac{11}{15}G_{xz}^c(u_{o,x}^t - u_{o,x}^b) + \frac{11}{15}G_{yz}^c(v_{o,y}^t - v_{o,y}^b) \quad (30)$$

Finally, for the bottom face sheet, the equations of motion become

$$A_{11}^b u_{o,xx}^b + A_{66}^b u_{o,yy}^b + (A_{12}^b + A_{66}^b)v_{o,xy}^b + \frac{G_{xz}^c}{h_c}(u_o^t - u_o^b) - \left(\rho^b h_f + \frac{h_c}{3}\rho^c\right)\ddot{u}_o^b - \rho^c\frac{h_c}{6}\ddot{u}_o^t = \hat{f}_1^b \quad (31a)$$

$$(A_{21}^b + A_{66}^b)u_{o,xy}^b + A_{66}^b v_{o,xx}^b + A_{22}^b v_{o,yy}^b + \frac{G_{yz}^c}{h_c}(v_o^t - v_o^b) - \left(\rho^b h_f + \frac{h_c}{3}\rho^c\right)\ddot{v}_o^b - \rho^c\frac{h_c}{6}\ddot{v}_o^t = \hat{f}_2^b \quad (31b)$$

$$D_{11}^b w_{,xxxx}^b + 2(D_{12}^b + 2D_{66}^b)w_{,xxyy}^b + D_{22}^b w_{,yyyy}^b + \frac{61}{21}\frac{E^c}{h_c}\left(\frac{53}{305}w^t - \frac{358}{305}w_o^c + w^b\right) + \left(\rho^b h_f + \rho^c\frac{29h_c}{315}\right)\ddot{w}^b + \rho^c\frac{37h_c}{630}\left(\ddot{w}_o^c - \frac{11}{37}\ddot{w}^t\right) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left[\left(\rho^b\frac{h_f^3}{12} + \rho^c\frac{19h_f^2 h_c}{1155}\right)\ddot{w}^b + \rho^c\frac{h_f^2 h_c}{27720}(199\ddot{w}_o^c - 61\ddot{w}^t)\right] + \alpha_3 h_c(G_{xz}^c w_{,xx}^t + G_{yz}^c w_{,yy}^t) - \alpha_2 h_c(G_{xz}^c w_{o,xx}^b + G_{yz}^c w_{o,yy}^b) - \alpha_1 h_c(G_{xz}^c w_{,xx}^c + G_{yz}^c w_{,yy}^c) = \hat{f}_3^b \quad (31c)$$

in which

$$\hat{f}_1^b = G_{xz}^c\left[\frac{11}{15}w_{o,x}^c + \alpha_4(w_{,x}^t + w_{,x}^b)\right] - A_{11}^b w_{,x}^b w_{,xx}^b - (A_{12}^b + A_{66}^b)w_{,y}^b w_{,xy}^b - A_{66}^b w_{,x}^b w_{,yy}^b - A_{11}^b\frac{w_{,x}^b}{R_x} - A_{12}^b\frac{w_{,x}^b}{R_y} + \rho^c\frac{h_f h_c}{420}(23\ddot{w}_{,x}^b + 17\ddot{w}_{o,x}^c - 5\ddot{w}_{,x}^t) \quad (32a)$$

$$\hat{f}_2^b = G_{yz}^c\left[\frac{11}{15}w_{o,y}^c + \alpha_4(w_{,y}^t + w_{,y}^b)\right] - (A_{21}^b + A_{66}^b)w_{,x}^b w_{,xy}^b - A_{66}^b w_{,xx}^b w_{,y}^b - A_{22}^b w_{,y}^b w_{,yy}^b - A_{21}^b\frac{w_{,y}^b}{R_x} - A_{22}^b\frac{w_{,y}^b}{R_y} - \rho^c\frac{h_f h_c}{420}(5\ddot{w}_{,y}^t - 17\ddot{w}_{o,y}^c - 23\ddot{w}_{,y}^b) \quad (32b)$$

$$\hat{f}_3^b = -\alpha_4[G_{xz}^c(u_{,x}^t - u_{,x}^b) + G_{yz}^c(v_{,y}^t - v_{,y}^b)] + (N_x^b w_{,x}^b)_{,x} + (N_{xy}^b w_{,x}^b)_{,y} + (N_{yx}^b w_{,y}^b)_{,x} + (N_y^b w_{,y}^b)_{,y} - \left(\frac{N_x^b}{R_x^b} + \frac{N_y^b}{R_y^b}\right) + \rho^c\frac{h_f h_c}{420}[5(\ddot{u}_{o,x}^t + \ddot{v}_{o,y}^t) + 23(\ddot{u}_{o,x}^b + \ddot{v}_{o,y}^b)] \quad (32c)$$

As before, we can group the nonlinear terms and define

$$\hat{F}_1^b = A_{11}^b w_{,x}^b w_{,xx}^b + (A_{12}^b + A_{66}^b)w_{,y}^b w_{,xy}^b + A_{66}^b w_{,x}^b w_{,yy}^b \quad (33a)$$

$$\hat{F}_2^b = (A_{21}^b + A_{66}^b)w_{,x}^b w_{,xy}^b + A_{66}^b w_{,xx}^b w_{,y}^b + A_{22}^b w_{,y}^b w_{,yy}^b \quad (33b)$$

$$\hat{F}_3^b = \frac{N_x^b}{R_x^b} + \frac{N_y^b}{R_y^b} - [(N_x^b w_{,x}^b)_{,x} + (N_{xy}^b w_{,x}^b)_{,y} + (N_{yx}^b w_{,y}^b)_{,x} + (N_y^b w_{,y}^b)_{,y}] \quad (33c)$$

4 Solution Procedure

In this section, the solution procedure for the dynamic response of sandwich shallow shells will be demonstrated through the study of the simply supported case. The boundary conditions along the $x=0$, a and $y=0$, b sides (Fig. 1) read as

$$u_0^t = 0, \quad u_0^b = 0, \quad v_0^t = 0, \quad v_0^b = 0, \quad w^t = 0, \quad w^c = 0, \quad w^b = 0 \quad (34)$$

and

$$M_{xx}^t = 0, \quad M_{xx}^b = 0 \quad \text{for } x = 0, a \quad (35a)$$

$$M_{yy}^t = 0, \quad M_{yy}^b = 0 \quad \text{for } y = 0, b \quad (35b)$$

The displacements can be assumed as

$$u_o^t=\sum_{m,n} U_{mn}^t(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$v_0^t=\sum_{m,n} V_{mn}^t(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{36a}$$

$$u_o^b=\sum_{m,n} U_{mn}^b(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$v_0^b=\sum_{m,n} V_{mn}^b(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{36b}$$

$$w^t=\sum_{m,n} W_{mn}^t(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$w^b=\sum_{m,n} W_{mn}^b(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{36c}$$

$$w_o^c=\sum_{m,n} W_{mn}^c(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

where $U_{mn}^t(t)$, $V_{mn}^t(t)$, $U_{mn}^b(t)$, $V_{mn}^b(t)$, $W_{mn}^t(t)$, $W_{mn}^b(t)$, and $W_{mn}^c(t)$ are unknown functions of time t. These displacements satisfy the boundary conditions. Substituting Eqs. (36a)–(36d) into Eqs. (26a)–(26c), (29), and (31a)–(31c) with $\hat{F}_i^t$, $\hat{F}_i^b(i=1,2,3)$, and $q(x,y,t)$ being expressed into the following form:

$$\hat{F}_1^t=\sum_{mn}\hat{F}_{1mn}^t(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$\hat{F}_2^t=\sum_{mn}\hat{F}_{2mn}^t(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \tag{37a}$$

$$\hat{F}_3^t=\sum_{mn}\hat{F}_{3mn}^t(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$
$$\hat{F}_1^b=\sum_{mn}\hat{F}_{1mn}^b(t)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{37b}$$

$$\hat{F}_2^b=\sum_{mn}\hat{F}_{2mn}^b(t)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}$$
$$\hat{F}_3^b=\sum_{mn}\hat{F}_{3mn}^b\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{37c}$$

$$q(x,y,t)=\sum_{mn}\hat{Q}_{mn}(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{37d}$$

we can obtain sets of second order ordinary differential equations with regard to the variable time in matrix form:

$$[\mathbf{M}_{mn}]\ddot{U}_{mn}(t)+[\boldsymbol{\varsigma}_{mn}]\dot{U}_{mn}(t)+[\boldsymbol{\kappa}_{mn}]\mathbf{U}_{mn}(t)=\mathbf{F}_{mn}(t) \tag{37e}$$

where $[\mathbf{M}_{mn}]$ is the equivalent mass matrix, $[\boldsymbol{\varsigma}_{mn}]$ is the damping coefficient matrix, and $[\boldsymbol{\kappa}_{mn}]$ is the equivalent spring constant matrix. These are 7×7 matrices for a given pair (m,n).

The displacement vector $\mathbf{U}_{mn}$ is defined as $\mathbf{U}_{mn}=[U_{mn}^t(t),V_{mn}^t(t),W_{mn}^t(t),W_{mn}^c(t),U_{mn}^b(t),V_{mn}^b(t),W_{mn}^b(t)]^T$ and the loading vector $\mathbf{F}_{mn}=[\hat{F}_{1mn}^t(t)+\hat{Q}_{mn}(t),\hat{F}_{2mn}^t(t),\hat{F}_{3mn}^t(t),0,\hat{F}_{1mn}^b(t),\hat{F}_{2mn}^b(t),\hat{F}_{3mn}^b(t)]^T$. The $\hat{F}_{jmn}^t(t)$, $\hat{F}_{jmn}^b(t)$, $\hat{Q}_{mn}(t)$ are obtained from Eqs. (37a)–(37d) as

$$\hat{Q}_{mn}(t)=\frac{4}{ab}\int_0^a\int_0^b q(x,y,t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{37f}$$

$$\hat{F}_{1mn}^t(t)=\frac{4}{ab}\int_0^a\int_0^b \hat{F}_1^t\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{37g}$$

$$\hat{F}_{1mn}^b(t)=\frac{4}{ab}\int_0^a\int_0^b \hat{F}_1^b\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{38}$$

with similar expressions for the rest of the $\hat{F}_{jmn}^t(t)$ and $\hat{F}_{jmn}^b(t)$.

Next, applying the Laplace transform

$$\tilde{U}(s)=L[U(t)](s)=\int_0^\infty U(t)e^{-st}dt \tag{39}$$

to Eq. (38), one can further obtain

$$(s^2[\mathbf{M}_{mn}]+s[\boldsymbol{\varsigma}_{mn}]+[\boldsymbol{\kappa}_{mn}])\tilde{U}_{mn}(s)=\tilde{\mathbf{F}}_{mn}(s) \tag{40}$$

In the Laplace space, the solution in terms of the displacements to Eq. (40) can be obtained without much difficulty if the loading vector $\tilde{\mathbf{F}}_{mn}=[\tilde{F}_{1mn}^t+\tilde{Q}_{mn}(s),\tilde{F}_{2mn}^t,\tilde{F}_{3mn}^t,0,\tilde{F}_{1mn}^b,\tilde{F}_{2mn}^b,\tilde{F}_{3mn}^b]^T$ is constant, then Eq. (40) is a set of linear algebraic equations, which can be solved directly for $\tilde{U}_{mn}=[\tilde{U}_{mn}^t,\tilde{V}_{mn}^t,\tilde{W}_{mn}^t,\tilde{W}_{mn}^c,\tilde{U}_{mn}^b,\tilde{V}_{mn}^b,\tilde{W}_{mn}^b]^T$ and then the displacements in time domain $U_{mn}=[U_{mn}^t(t),V_{mn}^t(t),W_{mn}^t(t),W_{mn}^c(t),U_{mn}^b(t),V_{mn}^b(t),W_{mn}^b(t)]^T$ can be recovered using the inverse Laplace transform without much difficulty. Subsequently, the solution for the displacements can be found by using Eqs. (36a)–(36d). However, the loading coefficients $\tilde{F}_{jmn}^t$ and $\tilde{F}_{jmn}^b$ were derived from expressions (28a)–(28c) and (33a)–(33c), which are nonlinear functions of the displacements. However, the right-hand side of Eq. (40), $\tilde{\mathbf{F}}_{mn}$, are nonlinear functions of $\tilde{U}_{mn}$. Therefore, an iterative procedure is developed as follows: (1) First, $\tilde{Q}_{mn}$ is a known function once the applied load is given. If the right-hand side of Eq. (40) is approximated by $\tilde{\mathbf{F}}_{mn}=[\tilde{Q}_{mn},0,0,0,0,0,0]^T$, then a first approximation to the solution is easily obtained as $\{\tilde{U}_{mn}(s)=s^2[\mathbf{M}_{mn}]+s[\boldsymbol{\varsigma}_{mn}]+[\boldsymbol{\kappa}_{mn}]\}^{-1}\tilde{\mathbf{F}}_{mn}$ (the superscript -1 denotes matrix inversion). (2) Application of the inverse Laplace transform to $\tilde{U}_{mn}(s)$ can lead to the corresponding solution $U_{mn}(t)$. Then, making use of Eqs. (36a)–(36d), (33a)–(33c), and (28a)–(28c), one can determine the functions $\hat{F}_1^t$, $\hat{F}_2^t$, $\hat{F}_3^t$ and $\hat{F}_1^b$, $\hat{F}_2^b$, $\hat{F}_3^b$ and then the corresponding to these Laplace transforms $\tilde{F}_1^t$, $\tilde{F}_2^t$, $\tilde{F}_3^t$ and $\tilde{F}_1^b$, $\tilde{F}_2^b$, $\tilde{F}_3^b$. (3) The next approximation for the displacements is found by solving Eq. (40) with the updated vector $\tilde{\mathbf{F}}_{mn}=[\tilde{F}_{1mn}^t+\tilde{Q}_{mn}(s),\tilde{F}_{2mn}^t,\tilde{F}_{3mn}^t,0,\tilde{F}_{1mn}^b,\tilde{F}_{2mn}^b,\tilde{F}_{3mn}^b]^T$. This procedure continues until the in-plane and transverse displacements are determined by the nth iteration with a convergence tolerance ϵ applied on the displacements normalized by the total height of the sandwich section, such that $\epsilon\leq 10^{-5}$ between two consecutive steps.

5 Applications and Discussions

The formulas and solution procedure in the foregoing sections can be applied to investigate the nonlinear transient response of a shallow shell subjected to a sudden blast loading [14]. Detailed analysis of a few example problems are presented in this section.

Uniformly Distributed and Exponentially Decaying Blast Loading on an Orthotropic Sandwich Shallow Shell. In this example, both the top and bottom face sheets of the sandwich shallow shell are made of E-glass/polyester composite material with stiffnesses (in GPa): $E_1=50.8$, $E_2=35.7$, $G_{12}=7.1$; Poisson's ratios: $\nu_{12}=0.35$, $\nu_{21}=0.246$; and mass density $\rho^{t,b}=1632\ \mathrm{kg/m^3}$. The ortho-

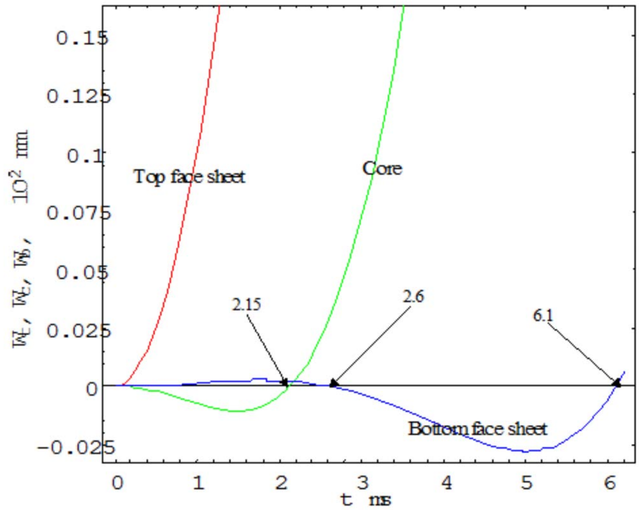

Fig. 2 Transient response of a sandwich shallow shell at the very early stage of blast loading

tropic core material has the following properties: $E^c = 1.005$ GPa, $G^c_{xz} = 120.6$ MPa, $G^c_{yz} = 75.8$ MPa, $\nu_c = 0.3$, and $\rho^c = 64$ kg/m^3. The top and bottom face sheets have an identical thickness $h_f = 1.0$ mm and the thickness of the core is $h_c = 20.0$ mm. The geometric dimensions of the shell (Fig. 1) read as (in mm): $a = 800$, $b = 500$, $R_x = 1000$, and $R_y = 600$. The top face sheet is subjected to blast loading, which is uniformly distributed over the entire sheet surface, but its intensity varies with time exponentially:

$$p_i(t) = q_m e^{-t/\alpha}, \quad t \geq 0 \tag{41}$$

We use in this example the values from Ref. [4]: $q_m = 60.86$ MPa and $\alpha = 3.33435$.

The results in Fig. 2 are the transverse displacements for the center points in the top face sheet, core middle plane, and bottom face sheet, respectively, within a few micrometers after the blast loading impact on the front surface of the top face sheet. An interesting phenomenon can be observed in the early blast loading stage: the top face sheet, core, and bottom face sheet behaviors are very different: (1) The displacement for the top face sheet increases with time positively (relative to the loading direction) due to the continuously applied loading; (2) the displacement for the middle plane in the core is negative when time $t \leqslant 2.15$ μs, then it becomes positive; and (3) the displacement of the bottom face sheet changes from positive to negative around time $t = 2.6$ μs, then it becomes positive again after time $t = 6.1$ μs. This phenomenon reflects the impact shock stress wave propagation in the sandwich shell. The blast loading impacts on the surface of the top face sheet at the instant of time $t = 0^+$ μs and induces a shock wave propagating through the thickness of the shell. During this time period, a negative pressure zone, which is similar to the cavitation zone in water, behind the shock wave front, is created in the core. This is why we see the displacement of the middle plane of the core negative up to $t_a = 2.15$ μs. This negative pressure zone reaches the bottom face sheet around $t_b = 2.6$ μs. The displacement of the bottom face sheet then becomes negative until $t_c = 6.1\,\mu$s, when this zone is dissipated. One may also interestedly note that the propagation velocity of the cavity zone is nonlinear since $t_c \geqslant 2t_b$. This observation clearly demonstrates that the nonlinear higher core theory in this paper can give us deep insight on what happens at the different phases of the sandwich construction when it is subjected to a blast loading.

Two points need to be further explained in Fig. 2. First, it can be observed that the midpoint back face displacement is double that of the midpoint core and this is because these two displacements are at different time instants and the back face and the core are different materials. There is actually no direct relation between them due to the core compressibility, which can mask the otherwise intuitively derived behavior, i.e., the core can expand in the process and therefore show larger back face displacement than at an earlier instant. Furthermore, as the wave propagates in the sandwich, energy is still added in the material system in this early time period, which can make up for the energy dissipation as the wave propagates in the sandwich. Therefore, the larger back face deflection in Fig. 2 is quite plausible. Second, it can be seen that during the early part of loading, the face sheets deform in one direction and the core in the opposite and this is due to the core compressibility and the shock wave propagation and reflection. It should be noted that the curve of the core deflection is the deflection of the initial midpoint of the core, not the current core midpoint location. In other words, in this early phase, a negative pressure zone is induced within the core and, since the core is compressible, it expands in the process of shock wave propagation. This negative pressure results in part of the sandwich structure deforming in the opposite direction. Therefore, the observed deflections are compatible.

The results plotted in Fig. 3 show that the popular assumption that the displacements of the middle planes in the top face sheet, the core, and the bottom face sheet are identical may be true only at some time instants. Most of the time, the transient responses of these three displacements are different, as will be further shown in the following discussion.

Presented in Fig. 4 is the transient response in terms of the displacements $[W_t(t), W_c(t), W_b(t)]$ for the center points in the middle plane of the sandwich shallow shell in a short time period ($0 \leqslant t \leqslant 6$ ms) after the blast loading is applied on the surface of the top face sheet. The maximum values of these displacements happen around time $t = 0.2$ ms and then decrease to near-half of the maximum values quickly. One can see that the solution converges as the time increases. The detailed drawings in Sections A and B show that the curves representing the displacements are tangled in the sense that the core midplane displacement exceeds that of the midplane bottom face sheet due to the compressibility of the core. This observation would further indicate that the nonlinear core theory may be a good model to study the behavior of sandwich structures subjected impact loading.

The behavior of the sandwich shell in the stage from the transient response to the steady dynamic response is demonstrated in Fig. 5. It can be seen that after time $t \geqslant 12$ ms, the sandwich structure enters into a steady state dynamic response region. One interesting result in the figure observed from the steady state dynamics response is that the curve for the displacement of the core is not in the middle between the curves of the top and bottom face sheets. This is due to the nonlinearity in the core transverse displacement.

Finally, Fig. 6 shows the stress profile σ_{zz} through the thickness and as a function of time. It can be seen that at the top face sheet, the stresses are always compressive and the highest in magnitude. The bottom face sheet shows lower stresses and they can even be at brief times tensile. This would indicate that damage would most likely initiate at the front (top) face sheet or even more likely at the front (top) face sheet/core interface. Such has been preliminary experimental evidence [15].

6 Conclusions

In this work, a higher order nonlinear core theory is proposed and is incorporated into the constitutive equations. A set of nonlinear governing equations is formulated and the solution procedure is obtained using the extended Galerkin method and the Laplace transform. Numerical results are presented to demonstrate the application of this higher order core model for the transient response of a composite sandwich shallow shell subject to blast loading. The observations obtained in the forgoing study suggest the following conclusions: (1) This nonlinear higher order core model can be used to capture the complex behavior such as cavi-

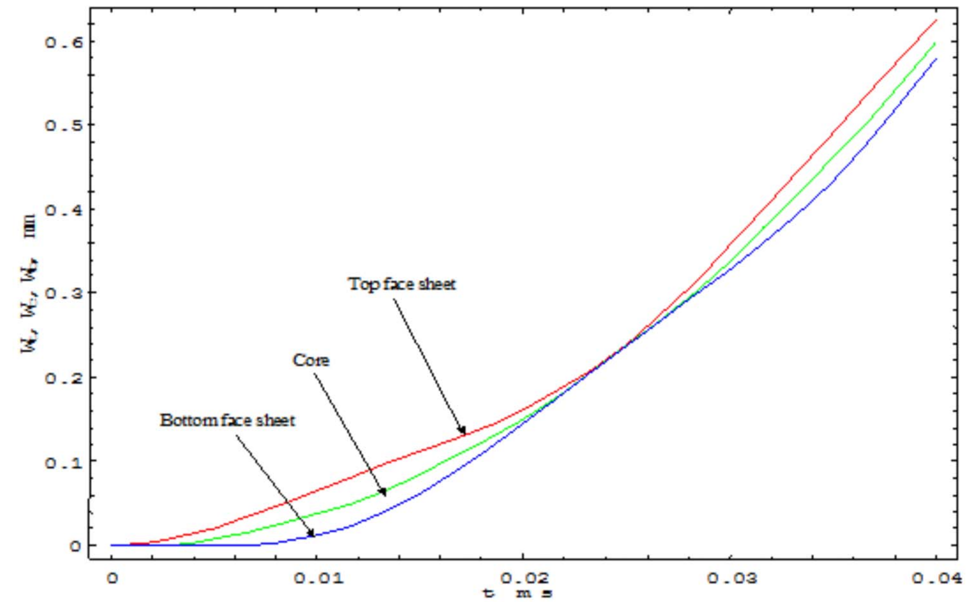

Fig. 3 Transient response of a sandwich shallow shell at a larger time scale of blast loading

tation in the core caused by the shock wave in the sandwich shell during the very early stage following the blast loading and the high levels of core thickness reduction; (2) the conventional assumption that the middle plane of the top face sheet, the core, and the bottom face sheet is identical may be not adequate in studying blast loading problems; (3) the highest in magnitude stresses are observed at the front (top) face sheet, which indicates that damage would most likely initiate at the front (top) face sheet/core interface.

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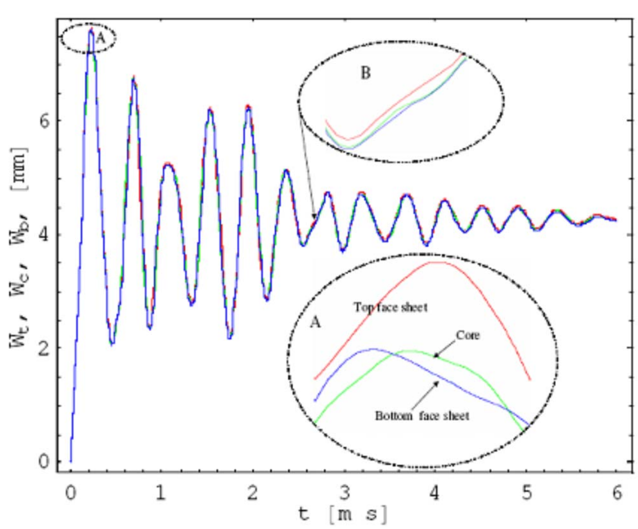

Fig. 4 Transient response of a sandwich shallow shell at an even larger time scale following blast loading

Appendix A: Derivation of the Fourth Order Nonlinear Compressible Core Theory

Let the displacements in the core be approximated by a fourth order polynomial in terms of the displacements in the top face sheet, middle plane of the core, and bottom face sheet. Then, the transverse displacement in the core can be expressed as

$$
\begin{aligned}
w^c(x,y,\zeta,t) &= \left(\beta_0 - \beta_2\frac{\zeta^2}{h_c^2} - \beta_4\frac{\zeta^4}{h_c^4}\right)w_o^c(x,y,t) \\
&\quad + \left(\beta_2\frac{\zeta^2}{h_c^2} + \beta_4\frac{\zeta^4}{h_c^4}\right)\bar{w}(x,y,t) \\
&\quad - \left(\beta_1\frac{\zeta}{h_c} + \beta_3\frac{\zeta^3}{h_c^3}\right)\hat{w}(x,y,t), \quad -\frac{h_c}{2} \le \zeta \le \frac{h_c}{2}
\end{aligned}
\tag{A1}
$$

in which $w_0^c(x,y)$ is the transverse displacement of the middle surface of the core, and $\bar{w}(x,y)$ and $\hat{w}(x,y)$ are, respectively, the average and difference of the middle surface transverse displacements for the two face sheets,

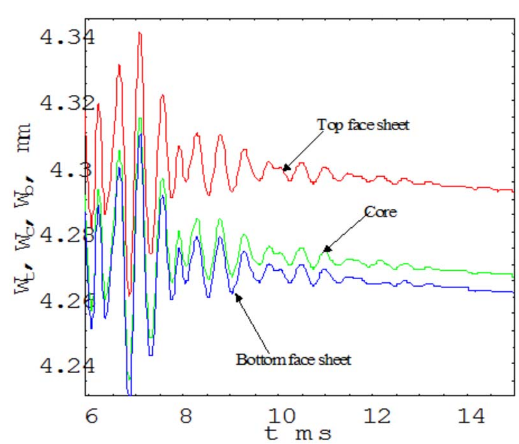

Fig. 5 The stage from transient to steady dynamic response for a sandwich shallow shell subject to blast loading

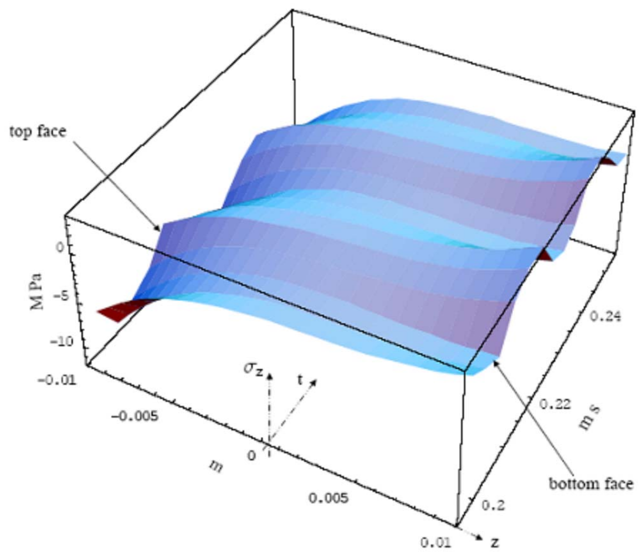

Fig. 6 Stress profiles following blast loading

$$\bar{w}(x,y,t)=\frac{1}{2}[w^t(x,y,t)+w^b(x,y,t)]$$
$$\hat{w}(x,y,t)=\frac{1}{2}[w^t(x,y,t)-w^b(x,y,t)] \tag{A2}$$

The in-plane displacements in the core can also be approximated as follows (fifth power of ζ):

$$u^c(x,y,\zeta,t)=\bar{u}(x,y,t)+\beta_5\frac{\zeta}{h_c}\hat{u}(x,y,t)+\zeta\frac{h_f}{h_c}w^c_{,x}(x,y,\zeta,t) \tag{A3a}$$

$$v^c(x,y,\zeta,t)=\bar{v}(x,y,t)+\beta_6\frac{\zeta}{h_c}\hat{v}(x,y,t)+\zeta\frac{h_f}{h_c}w^c_{,y}(x,y,\zeta,t) \tag{A3b}$$

where $\bar{u}(x,y,t)$, $\hat{u}(x,y,t)$ and $\bar{v}(x,y,t)$, $\hat{v}(x,y,t)$ are, again, respectively, the average and difference of the middle surface in-plane displacements for the two face sheets:

$$\bar{u}(x,y,t)=\frac{1}{2}[u^t_o(x,y,t)+u^b_o(x,y,t)]$$
$$\hat{u}(x,y,t)=\frac{1}{2}[u^t_o(x,y,t)-u^b_o(x,y,t)] \tag{A4a}$$

$$\bar{v}(x,y,t)=\frac{1}{2}[v^t_o(x,y,t)+v^b_o(x,y,t)]$$
$$\hat{v}(x,y,t)=\frac{1}{2}[v^t_o(x,y,t)-v^b_o(x,y,t)] \tag{A4b}$$

Therefore, there are seven constants β_i, $i=0,6$, to be determined from displacement continuity as follows.

Top face sheet/core interface, $\zeta=-h_c/2$:

$$u^c(x,y,\zeta,t)|_{\zeta=-h_c/2}=u^t_o(x,y,t)-\frac{h_f}{2}w^t_{,x}(x,y,t) \tag{A5a}$$

$$v^c(x,y,\zeta,t)|_{\zeta=-h_c/2}=v^t_o(x,y,t)-\frac{h_f}{2}w^t_{,y}(x,y,t) \tag{A5b}$$

$$w^c(x,y,\zeta,t)|_{\zeta=-h_c/2}=w^t(x,y,t) \tag{A5c}$$

Bottom face sheet/core interface, $\zeta=h_c/2$:

$$u^c(x,y,\zeta,t)|_{\zeta=h_c/2}=u^b_o(x,y,t)+\frac{h_f}{2}w^b_{,x}(x,y,t) \tag{A5d}$$

$$v^c(x,y,\zeta,t)|_{\zeta=h_c/2}=v^b_o(x,y,t)+\frac{h_f}{2}w^b_{,y}(x,y,t) \tag{A5e}$$

$$w^c(x,y,\zeta,t)|_{\zeta=h_c/2}=w^b(x,y,t) \tag{A5f}$$

Also, at the midsurface of the core, $\zeta=0$:

$$w^c(x,y,\zeta,t)|_{\zeta=0}=w^c_0(x,y,t) \tag{A5g}$$

Substitution of Eqs. (A1), (A3*a*), and (A3*b*) into the seven continuity conditions (A5*a*)–(A5*g*) leads to

$$\beta_0=\beta_1=1,\quad \beta_2=-2,\quad \beta_3=-4,\quad \beta_4=-8,\quad \beta_5=\beta_6=-1/2 \tag{A6}$$

Appendix B: The Governing Equations for the Bottom Face Sheet

One can see that the governing equations are nonlinear. Substituting equations Eq. (4) into Eqs. (13*a*) and (13*b*) and then Eqs. (20)–(22), the governing equations for the bottom face sheet can be written as

$$N^b_{x,x}+N^b_{xy,y}-\left(\rho^b h_f+\rho^c\frac{h_c}{3}\right)\ddot{u}^b_o-\rho^c\frac{h_c}{6}\ddot{u}^t_o+\rho^c\frac{h_f h_c}{420}(5\ddot{w}^t_{,x}-17\ddot{w}^c_{o,x}-23\ddot{w}^b_{,x})+G^c_{xz}\left[\frac{(u^t_o-u^b_o)}{h_c}-\frac{11}{15}w^c_{o,x}-\alpha_4(w^t_{,x}+w^b_{,x})\right]=0 \tag{B1a}$$

$$N^b_{xy,x}+N^b_{y,y}-\left(\rho^b h_f+\rho^c\frac{h_c}{3}\right)\ddot{v}^b_o-\rho^c\frac{h_c}{6}\ddot{v}^t_o+\rho^c\frac{h_f h_c}{420}(5\ddot{w}^t_{,y}-17\ddot{w}^c_{o,y}-23\ddot{w}^b_{,y})+G^c_{yz}\left[\frac{(v^t_o-v^b_o)}{h_c}-\frac{11}{15}w^c_{o,y}-\alpha_4(w^t_{,y}+w^b_{,y})\right]=0 \tag{B1b}$$

$$\begin{aligned}
&M^b_{x,xx}+2M^b_{xy,xy}+M^b_{y,yy}+(N^b_x w^b_{,x})_{,x}+(N^b_{xy}w^b_{,x})_{,y}+(N^b_{yx}w^b_{,y})_{,x}\\
&+(N^b_y w^b_{,y})_{,y}-\left(\rho^b h_f+\rho^c\frac{29h_c}{315}\right)\ddot{w}^b-\rho^c\frac{37h_c}{630}\left(\ddot{w}^c_0-\frac{11}{37}\ddot{w}^t\right)\\
&+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left[\left(\rho^t\frac{h_f^3}{12}+\rho^c\frac{19h_f^2 h_c}{1155}\right)\ddot{w}^b+\rho^c\frac{h_f^2 h_c}{27720}(199\ddot{w}^c_o\right.\\
&\left.-61\ddot{w}^t)\right]+\rho^c\frac{h_f h_c}{420}[5(\ddot{u}^t_{o,x}+\ddot{v}^t_{o,y})+23(\ddot{u}^b_{o,x}+\ddot{v}^b_{o,y})]\\
&-\alpha_3 h_c(G^c_{xz}w^t_{,xx}+G^c_{yz}w^t_{,yy})+\alpha_2 h_c(G^c_{xz}w^b_{,xx}+G^c_{yz}w^b_{,yy})\\
&+\alpha_1 h_c(G^c_{xz}w^c_{o,xx}+G^c_{yz}w^c_{o,yy})-\alpha_4[G^c_{xz}(u^t_{o,x}-u^b_{o,x})+G^c_{yz}(v^t_{o,y}\\
&-v^b_{o,y})]-\left(\frac{N^b_x}{R^b_x}+\frac{N^b_y}{R^b_y}\right)-\frac{61}{21}\frac{E^c}{h_c}\left(\frac{53}{305}w^t-\frac{358}{305}w^c_o+w^b\right)=0
\end{aligned} \tag{B1c}$$

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