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THE STATICS AND DYNAMICS OF THE MOORING LINES OF A GUYED TOWER FOR DESIGN APPLICATIONS

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SUMMARY

The statics and dynamics of the supporting cables of compliant structures are of primary importance in studying the behavior of the overall system. In this study efficient analytic solutions are presented for the guyed tower that reduce significantly the computational effort required, and allow a wide parametric search, which is essential in the early design phases.

First, the cable statics are presented and the importance of the concept of the effective tension is discussed. Analytic solutions including the elasticity effects are derived.

Next, the cable dynamics are studied and analytic solutions are developed using perturbation techniques. In particular, the effects of elasticity and catenary shape are presented since they can alter the nature of the dynamic response. The form of the solutions allows a simple treatment of the dynamics of multi-leg systems.

The dynamics of the guyed tower are studied and the effect of the clump weight is assessed in the form of a softening spring. The inertia and drag forces of the clump weight are identified as sources of dynamic amplification and phase lag. The use of perturbation techniques allows an efficient study of the dynamic behavior of the system.

The effect of nonlinearities is discussed and analytic techniques to account for such effects are presented.

The theory presented is illustrated by studying a specific guyed tower in 1,500 ft. of water.

INTRODUCTION

The dynamics of structures supported by mooring lines are complicated because of the blending of the dynamics of the structure with those of the cables. The need for efficient tools is particularly pressing at the early design phases, when no detailed system configuration exists so as to exercise it by using numerical techniques, such as finite elements or finite differences. At the same time, the most important parameters are to be selected at this phase, with very significant consequences for the subsequent design phases.

The complexity of the dynamics of a single mooring line is such that simplistic models lead to unreliable predictions. The fact that the governing equations are nonlinear partial differential equations with variable coefficients forces many investigators to employ numerical techniques at a very early stage, with significant consequences for the cost and flexibility of the design procedure.

This study is presented so as to indicate some simplifying facts concerning the behavior of mooring lines. In particular, the major factors contributing to the response of a cable are indicated and a simple but comprehensive dynamic analysis is outlined. The nonlinearities are also studied, but it is the belief of the authors that their effect can be assessed very efficiently by using analytic techniques with a better qualitative understanding than by employing numerical techniques.

It should be emphasized that the present paper focuses on the preliminary design phase and the results that are derived are simple but efficient solutions allowing a wide parametric search. These results are by no means general solutions to the cable dynamic problem, since such analytic expressions are impossible to obtain, but they are applicable within the frame of selecting the various parameters involved so as to minimize excessive dynamic phenomena.

The results have been derived for a guyed tower to demonstrate how to obtain simple solutions in an application-oriented manner.

GUYED TOWER

The guyed tower is one of the concepts that is being studied as an economical alternative to the jacket platform for water depths between 1,000 and 2,000 ft.

The concept has been described in detail in several publications [1] [2] [3] [4]. The system employs a slender truss-type tower supporting the deck on which all working equipment is installed. The lower end of the tower is connected to a special foundation which imposes small rotational restraint on the structure. Proposed foundations include a spud can and ungrouted piles extending along the full length of the tower. The lateral support is provided by a number of cables whose upper end is secured on the deck by cable grips, while they pass around fairleads located below the water level and then they extend radially to clump weights located on the seafloor. The clump weights are connected through horizontal lines to the anchors. Buoyancy tanks are provided so as to reduce the foundation load and to provide additional lateral restoring moment (figure 1).

The tensioning system could be used to change the pretension level so as to handle extreme weather conditions.

The primary objective is to obtain a compliant system whose first natural frequency, treating the tower as a rigid body, is below the wave frequency range (typically between 0.2 and 1.5 rad/sec). This objective places the guyed tower at the opposite end with respect to the wave frequency range than the jacket platform, whose first natural period for shallow water is typically 2-4 sec.

The problems that arise include the dynamic behavior of all subsystems involved, especially of the mooring lines; the structural integrity of the tower and the cables both in extreme loads and fatigue; and the damage stability of the system.

MOORING LINES

The mooring line is capable of supporting external forces by re-adjusting its configuration, given that it can support only positive tension which is tangent at each point to the line shape. This fact poses limitations to the holding capacity of a mooring line in the case of dynamic loads, because if the rate of change of the external force is fast, the cable configuration does not change as fast, resulting in poor holding capabilities. In addition, the configuration forms waves which may cause resonance phenomena and therefore large parasitic tensions.

The sag to length ratio is a very important parameter for cable dynamics. When the sag is very small, we obtain essentially a taut wire which can easily support dynamic forces along its axis, while it provides very little support against any fast varying transverse force. The longitudinal load can change fast except that its frequency range must remain below the first longitudinal (i.e. stretching) natural frequency. The longitudinal natural frequencies are given as:

$$(\omega_m)_\ell = \frac{m}{L}\sqrt{\frac{E}{\rho}} \tag{1}$$

where L is the cable length, E is Young's modulus and ρ the cable density, while the transverse natural frequencies are (assuming a constant tension T_o)

$$(\omega_n)_t = \frac{n}{L}\sqrt{\frac{T_o}{\frac{\pi}{4}\rho d^2}} \tag{2}$$

with d the cable diameter. For a steel cable the ratio $T_o/\frac{\pi}{4}d^2$ must remain below the elastic limit so by using some typical steel properties we obtain:

$$\frac{(\omega_m)_\ell}{(\omega_n)_t} = \frac{m}{n}\sqrt{\frac{E}{(T_o/\frac{\pi}{4}d^2)}} \approx \frac{m}{n}\,40 \tag{3}$$

We conclude that the first longitudinal frequencies are much larger than the first transverse frequencies. This is the reason why the cable can easily sustain dynamic loads in the longitudinal direction, while for normal wave and vortex shedding loads its longitudinal response is quasi-static (i.e. the longitudinal stretching can be represented by a distributed spring).

One particular aspect of taut cables is the possibility for parametric resonance [5]: The axial quasi-static stretching of the cable causes an oscillatory change in the tension which constitutes the restoring mechanism in the transverse dynamics. As a result, the transverse dynamics are characterized by an equation whose time dynamics are in the form of a Mathieu equation. Under certain combinations of wave frequency versus cable natural frequency, large transverse oscillations may occur.

When the sag is large the stretching becomes unimportant for the transverse dynamics, which are essentially the dynamics of a chain [6], [7]. As outlined in [8], we can define a tangential and a normal unit vector at any point along the static configuration and define a dynamic motion along the normal (transverse dynamics) and a motion along the tangential (axial dynamics). The solution consists of the sum of a solution that varies slowly along the cable length and is essentially a perturbation of the static configuration (catenary dynamics); and a solution which in the transverse direction resembles the motion of a taut string.

For moderate values of the sag (of the order of 1/8 of the length) both stretching and catenary dynamic effects are important. This was first recognized in [9] and a solution was obtained which unified the theories for small sag (taut wire) and large sag (chain dynamics).

The nonlinearities involved in the dynamics of a cable are composed of geometric nonlinearities (large amplitude motion); nonlinear stress-strain relations, especially for large dynamic tension; and nonlinear fluid forces which can be represented in the transverse direction as

$$F = \frac{1}{2}\rho_w C_D\, d\, U_n\, |U_n| \tag{4}$$

where F is the transverse force per unit length, ρ_w the density of the fluid, d the cable diameter and U_n the relative velocity between the fluid particles and the cable in the normal direction.

As we have already pointed out we are interested here in solutions useful for design. This means that we must identify the mechanisms leading to large dynamic motions but we need not study those extreme motions, which are known to cause failure. As a result we try to find the dynamic response with as simple a model as possible: From the three classes of nonlinearity indicated above only the nonlinear drag has a definite effect on the dynamics we are interested in.

When a system is lightly damped the (small) nonlinearities can be studied by using a double perturbation expansion in amplitude and time, as for example in the method of multiple scales [10]. The characteristic of the response is that it consists of a forced part and a homogenous part which may not be decaying.

On the contrary, a system with significant damping responds at the frequency of the forcing function while all other terms are very small or fast decaying. This is true for nonlinear damping as well and although the response is a nonlinear function of the amplitude of excitation there is no need to expand in multiple time scales.

The drag term (equation 4) is a nonlinear function of the velocity U_n so its effect depends on the amplitude of the forcing function. For a cable driven at its upper end the motion is usually equal to several times its diameter so the damping is important. This means that the method harmonic balance [11] or the similar method of equivalent linearization [11] can be used to ain the cable dynamics. This fact explains the success of applying the method of equivalent linearization on the problem of the forced response of slender cylinders in water driven at their upper end [8], [12].

For very small motions, the effect of the nonlinear damping is of the same order as the effect of the structural damping, i.e. the system is rather lightly damped. As a result, cable strumming [13] is characterized by phenomena of lightly damped systems such as frequency entrainment, multiple frequency response etc., and although we can not claim that we have a complete hydrodynamic model for vortex shedding, we can at least qualitatively describe cable strumming as the limit cycle of a lightly damped nonlinear system [15].

This concludes the review of the dynamics of mooring lines which is based primarily on physical argument rather than mathematics. In the sequel we develop quantitative techniques, we we will refer though to the arguments of this section quite frequently, in order to simplfy the equations used.

STATICS OF MOORING LINES

The static loads on the cable of a mooring line consist of the external forces, its own weight and the drag force from the current.

At each point of the cable configuration we define a tangential and normal unit vector, $\hat{t}$ and $\hat{n}$ respectively (Figure 2). Let all quantities referring to the unstretched cable have a subscript o. If W is the weight per unit length, d the cable diameter, ϕ the angle between the vector $\hat{t}$ and the horizontal, T the tension, U the local current velocity, s the coordinate along the cable, C_D the normal drag coefficient and C_f the (tangential) frictional coefficient, then the governing equations are [8]

$$T_e \frac{d\phi}{ds_o} = (W_o-B_o)\cdot\cos\phi + F_n(1+\tfrac{e}{2})\cdot\sin^2\phi\cdot U^2 \tag{5}$$

$$\frac{dT_e}{ds_o} = (W_o-B_o)\cdot\sin\phi - F_t(1+\tfrac{e}{2})\cdot\cos^2\phi\cdot U^2 \tag{6}$$

$$T_e = T+\tfrac{\pi}{4}d^2\rho g z \tag{7}$$

$$\frac{dx}{ds_o} = \cos\phi\cdot(1+e) \tag{8}$$

$$\frac{dy}{ds_o} = \sin\phi\cdot(1+e) \tag{9}$$

$$e = T_e/\tfrac{\pi}{4}d^2 E \tag{10}$$

where E is Young's modulus and z the vertical distance from the free surface, while

$$B_o = \tfrac{\pi}{4}d_o^2\rho g \tag{11}$$

$$F_n = \tfrac{1}{2}\rho\, C_d\, d_o \tag{12}$$

$$F_t = \tfrac{1}{2}\rho\, C_f \pi d_o \tag{13}$$

The derivation of equation (10) requires that Poisson's ratio is 0.5 which is an acceptable assumption at this stage. We must emphasize the fact that the hydrostatic force is normal to the cable configuration and not vertical. This leads to the formulation of the problem in terms of the <u>effective tension</u> T_e as defined in equation (7) rather than the actual tension. Essentially, we can say that all static solutions derived for cables in the air are valid for cables in water (in the absence of current) if the tension T is replaced by T_e.

This leads to surprising results because when the depth becomes large the actual tension may become negative, while the extension (which depends on T_e according to equation (10) is positive.

This is due to the "squeezing" action of the external hydrostatic pressure, which causes stretching in the longitudinal direction although the tension may be negative. This facet has been overlooked sometimes in the literature and may lead to significant errors.

If the mooring line employs a chain then the above are not applicable because each chain link is practically entirely submerged so the buoyancy is vertical and the effective tension is equal to the tension, i.e. for a chain equation (7) is replaced by

$$T_e = T \tag{7a}$$

The actual tension therefore in a chain is larger than the tension in a cable under the same loading conditions.

If the current action is ignored, since usually the current is a near surface phenomenon, then the <u>extensible catenary</u> solution can be obtained:

$$T_e = \frac{H}{\cos\phi} = \sqrt{H^2 + (V - W_1[L-s])^2} \tag{14}$$

$$x = \frac{H}{W_1}\left\{\sinh^{-1}\left[\frac{V-W_1(L-s)}{H}\right] - \sinh^{-1}\left[\frac{V-W_1L}{H}\right]\right\} + \frac{Hs}{AE} \tag{15}$$

$$y = \frac{H}{W_1}\left\{\sqrt{1+\left[\frac{V-W_1(L-s)}{H}\right]^2} - \sqrt{\left[1+\frac{V-W_1L}{H}\right]^2}\right\} + \frac{H}{AE}\left\{\frac{V}{H}s + \frac{W_1}{2H}(L-s)^2\right\} \tag{16}$$

$$\tan\phi = \frac{V}{H} - \frac{W_1}{H}(L-s) \tag{17}$$

where V is the vertical and H the horizontal force at the top end, L is the unstretched cable length, and:

$$W_1 = W_o - B_o \tag{18}$$

$$A = \frac{\pi}{4}d^2 \tag{19}$$

Note that s is the unstretched coordinate and the subscript o has been ommitted for convenience. The stretched length becomes:

$$L_e = L + \frac{1}{2AEW_1}\{V\sqrt{V^2+H^2} + H^2\sinh^{-1}(\frac{V}{H})\} \tag{20}$$

while it is reminded that

$$\sinh^{-1}(x) = \ln[x + \sqrt{1+x^2}] \qquad \text{if } x \geq o \tag{21}$$

If the force at the top is large compared to the net weight of the cable, i.e.

$$F = \sqrt{V^2+H^2} \gg W_1 L$$

then a shallow sag configuration is obtained, while elasticity effects are important. By using a perturbation expansion with W_1L/F and F/AE the small parameters we can obtain a simple solution to the static equations. By keeping the first two orders we find

$$T_e = F - W_1 \sin\phi_o (L-s) \tag{22}$$

$$\phi = \phi_o + \frac{W_1}{F}\cdot\cos\phi_o \cdot s + \frac{F_n}{F}\int_o^s U^2(s_1)ds_1 + \phi_b \tag{23}$$

$$x = \cos\phi_o \cdot s + \cos\phi_o \frac{F}{EA}s - \sin\phi_o\left[\frac{W_1}{F}\cos\phi_o \frac{s^3}{6} + \frac{F_n}{F}\int_o^s\int_o^{s_1} U^2(s_2)\, ds_2\, ds_1 + \phi_b s\right] \tag{24}$$

$$y = \sin\phi_o \cdot s + \sin\phi_o \cdot \frac{F}{EA}s + \cos\phi_o \cdot \left[\frac{W_1}{F}\cos\phi_o \frac{s^3}{6} + \frac{F_n}{F}\int_o^s\int_o^{s_1} U^2(s_2)\, ds_2\, ds_1 + \phi_b s\right] \tag{25}$$

where if D is the water depth

$$\sin\phi_o = \frac{D}{L} \tag{25}$$

while

$$\phi_b = \tan^{-1}\left(\frac{H}{V}\right) - \frac{W_1}{F} \cdot \cos\phi_o L - \frac{F_n}{F} \int_o^L U^2(s_1)\, ds_1 \tag{27}$$

It should be mentioned that the original equations (5) through (13) are easy to implement in a finite difference scheme in the computer and then solve them iteratively fully accounting for current and elasticity effects. The analytic solutions are of value so as to check the numerical solutions, but primarily to derive efficiently the overall statics of the multi-leg system.

DYNAMICS OF MOORING LINES

By using the normal and tangential vectors defined in figure 2 we can derive the equations of motion in the normal and tangential directions: (The effect of the current on the added mass is omitted).

$$m_o \frac{\partial u}{\partial t} - m_o v \frac{\partial \phi}{\partial t} = -W_1 \cdot \sin\phi + \frac{\partial T_e}{\partial s_o} - R_t (1+\frac{e}{2}) \tag{28}$$

$$(m_o+a_o)\frac{\partial v}{\partial t} + m_o u \frac{\partial \phi}{\partial t} = -W \cdot \cos\phi + T_e \frac{\partial \phi}{\partial s_o} + R_n (1+\frac{e}{2}) \tag{29}$$

where m_o is the cable mass per unit unstretched length, a_o the added mass per unit length, u is the tangential and v the normal cable velocity and R_t, R_n are the fluid forces in the $\hat{t}$, $\hat{n}$ directions respectively:

$$R_n = (\frac{\pi}{4} d_o^2 \rho + a_o) \frac{\partial V_n}{\partial t} + F_n (V_n - v) |V_n - v| \tag{30}$$

$$R_t = F_t (V_t - u) |V_t - u| \tag{31}$$

where V_n is the normal and V_t the tangential velocity of the fluid. Also, the following compatability equations must be used.

$$\frac{\partial u}{\partial s_o} - v \frac{\partial \phi}{\partial s_o} = \frac{\partial e}{\partial t} \tag{32a}$$

$$\frac{\partial v}{\partial s_o} + u \frac{\partial \phi}{\partial s_o} = \frac{\partial \phi}{\partial t}(1+e) \tag{32b}$$

T_e and e are defined again by equations (7) and (10) respectively.

It is important to compare the dynamics of the overall system with those of the cables, in order to decide for the type of governing equations to be used. Let us consider small amplitude motions of the cable around an average static configuration defined by a tension $T_o(s)$ and $\phi_o(s)$. Then if we subtract the static equations from equation (28) through (32) we obtain

$$(m_o+a_o) \frac{\partial v}{\partial t} = T_o \frac{\partial \phi_1}{\partial s} + \frac{\partial T_o}{\partial s} \phi_1 - F_n v |v| + T_1 \frac{\partial \phi}{\partial s} \tag{33}$$

$$m_o \frac{\partial u}{\partial t} = -W_1 \cos\phi_o \phi_1 + \frac{\partial T_1}{\partial s} \tag{34}$$

$$\frac{\partial v}{\partial s} = \frac{\partial \phi_1}{\partial t} - u \frac{\partial \phi_o}{\partial s} \tag{35}$$

$$\frac{\partial u}{\partial s} = v \frac{\partial \phi_o}{\partial s} + \frac{1}{EA} \frac{\partial T_1}{\partial t} \tag{36}$$

where ϕ_1, T_1 are the time varying parts of ϕ, T_e respectively, while the subscript o has been ommitted from s again for convenience. Note that the nonlinear drag force has been included in equation (33) because, according to the discussion in an earlier section, this will govern the type of technique to be used. No fluid particle motion has been assumed in the equations (33) - (36).

Since the mooring lines are under large tension the effect of elasticity is as important as the effect of the sag. This is the reason for including the elasticity term in equation (36). Note that still no longitudinal elastic waves are present as explained in an earlier section. The solution of equations (33) through (36) has been derived for a horizontal, flat catenary in [9], while the solution for an inelastic cable has been derived in [8].

Using the data provided in Appendix 1 and the solution derived in Appendix 2, the natural frequencies are derived as shown in Table 1 (i.e. by taking $F_n=0$).

It is important to determine the effect of the damping term. If F_n is small, a multiple ale technique must be applied, while for larger F_n the method of harmonic balance is appropriate. .ce the cable will be forced to follow the motion of the tower the excursions will be large compared to the cable diameter, and the contribution of the damping is important. As a result the method of harmonic balance can be used.

Assuming a single frequency excitation at the top, of amplitude a and frequency ω_o, following the standard procedure of the harmonic balance [11], we replace the nonlinear term $F_n V|V|$ with the equivalent linear term bV, where [8]:

$$b = \frac{4\ F_n}{3\pi\omega_o}\ \alpha \tag{37}$$

while the response will be predominantly at the exciting frequency ω_o.

GUYED TOWER DYNAMIC

The natural frequency of the rigid body dynamics of the guyed tower is placed below the range of wave frequencies, typically around 0.25 rad/sec. This fact combined with the results of table 1 indicates that the dynamics of the cables and the rigid body dynamics of the system are separated by a factor of 4.

When we consider the dynamic response of the tower at low frequencies, therefore (when it is expected to move significantly), we can consider the cables to respond quasi-statically. There is one important consideration though: The clump weight is subject to inertia and drag forces, so that its response will be dynamic, i.e. there will be dynamic amplification and phase difference in its response, although the cables behave essentially as (nonlinear) springs.

(a) Quasi-static Modeling

Let us first adopt a quasi-static modeling of the mooring lines. Then the analytic solutions for the statics can be used to derive the force-displacement relation. The first part of the cable connects the upper end of the clump weight to the tower and its configuration is that of a shallow sag cable so that equations (22) through (27) can be used, or, if the current can be neglected, the elastic catenary equations (14) through (21) provide better accuracy. The distri- ed clump weight behaves clearly as a heavy inelastic catenary, so that equations (14) through) can be used by neglecting the elasticity terms.

The third part of the line is the cable connecting the lower end of the clump weight to the anchor. Its transverse motions are small so it can be modelled essentially as an equivalent spring.

This modeling neglects the inertia and drag forces on the clump weight, as well as the frictional forces between the weight and the bottom. It is used to provide a first rough estimate of the dynamic behavior of the tower.

By matching the solutions for each of the three parts we obtain a relation between the vertical and horizontal forces at the upper end and the horizontal displacement at the top. Figure 4 shows a plot of the horizontal force versus horizontal displacement for a single cable (data from Appendix 1). When several cables participate we can project all forces along the direction of motion and obtain an overall force-displacement relation as shown in figure 5 for 20 symmetrically placed cables (the graph depicts an odd function so only half of the curve is shown).

The relation thus obtained is that of a nonlinear spring: For displacements below a critical value the relation is that of a weakly hardening spring; above the critical value the relation changes fast to a softening spring. In physical terms the clump weight allows relatively large restoring forces for moderate displacements, while for large displacemnts it allows large motion so that the tension level is kept below the breaking limit.

It is interesting to study the effect of such a nonlinear spring on the rigid body dynamics of the tower. As we have already mentioned, damping is a very serious consideration when treating the dynamics of nonlinear systems, so we start by considering the effect of the nonlinear damping: We write the equation of motion of the tower by equating the moments acting on the structure with the bottom as reference point. In order to make the analysis possible we write separate drag terms for the tower velocity and the fluid particle velocity:

$$(I+I_a)\ddot{\phi} + C\dot{\phi}|\dot{\phi}| = M_W + F_h\cdot D + F_v\cdot D\phi \tag{38}$$

where I is the moment of inertia of the tower, I_a the added moment of inertia, C the nonlinear damping coefficient, M_w the excitation moment, F_n^a the horizontal cable force and F_V the vertical cable force. D is the distance from the bottom point to the cable attachment and ϕ the angular motion of the tower. It is convenient to define an equivalent drag diameter of the tower d_D, and an equivalent inertia diameter d_I

$$d_D = \sum_{i=1}^{N} d_i \tag{39}$$

$$d_I = \sqrt{\sum_{i=1}^{N} d_i^2} \tag{40}$$

where d_i (i=1, 2,...N) are the diameters of the N members of which the tower section consists. Then

$$I_a = \rho\frac{\pi}{4} d_I^2 \int_0^D z^2 dz = \rho\frac{\pi}{12} d_I^2 D_o^3 \tag{41}$$

$$C = \frac{1}{2}\rho\, C_D\, d_D \int_0^D z^3 dx = \frac{1}{8}\rho\, C_D\, d_D\, D_o^4 \tag{42}$$

where ρ is the water density, D_o the water depth and C_D the drag coefficient. Using figure 5 we can derive a polynomial approximation to the horizontal force-displacement curve in the form

$$F_h(x) = -\alpha_1 x - \alpha_3 x^3 + \begin{cases} 0 & x \le x_o \\ \ell_1(x-x_o) + \ell_3(x^3-x_o^3) & x > x_o \end{cases} \tag{43}$$

where $\alpha_1, \alpha_3, \ell_1, \ell_3$ curve-fitting constants and x_o the clump-lifting displacement. The approximation is very good as shown in figure 5 for the specific example considered. For the vertical force it is sufficient to consider

$$F_v(x) = -F_{vo} \tag{44}$$

Next we change from the angle ϕ to the horizontal displacement x, i.e.

$$x(t) = \phi(t)\, D \tag{45}$$

and then nondimensionalize by setting

$$u(t) = \frac{x(t)}{x_o} \tag{46}$$

$$t = t\sqrt{\frac{\alpha_1+F_{vo}D}{I+I_a}} = t\omega_o \tag{47}$$

so as to obtain

$$\ddot{u} + C^*\, \dot{u}|\dot{u}| + u = f^*(u) + F\sin(\omega_1\tau) \tag{48}$$

where

$$C^* = \frac{C\ x_o}{I+I_a}\ . \tag{49}$$

$$f^*(u) = \begin{cases} 0 & u \le 1 \\ \{\ell_1(u-1) + x_o^2\,\ell_3(u^3-1)\}\ \frac{1}{(I+I_a)\omega_o^2} & u > 1 \end{cases} \tag{50}$$

while F is the nondimensional excitation moment amplitude and if ω is the frequency of excitation

$$\omega_1 = \frac{\Omega}{\omega_o} \tag{51}$$

By definition ω_o is the natural frequency of the tower for small displacements around the vertical equilibrium. It is interesting to use the data provided in appendix 1 and derive a numerical value for the coefficients of (48):

$$\ddot{u} + 0.3566\, \dot{u}\,|\dot{u}| + u = F^*(u) + F \sin(\omega_1 \tau) \tag{52}$$

$$f^*(u) = \begin{cases} 0 & u < 1 \\ 0.2289(u-1) + (u^3-1) & u > 1 \end{cases} \tag{53}$$

$$\omega_o = 0.2275 \text{ rad/sec} \tag{54}$$

The perturbation techniques are expected to provide reliable data in this case since all nonlinearities have relatively small coefficients. This is indeed the case as simulations have shown and it is important to note that the damping term has a relatively large coefficient: As simulations have verified the guyed tower response can be considered to be heavily damped.

The nondimensional form of the equation of motion is convenient in assessing the effect of nonlinearities, it should be noted though that, as equation (46) indicates, the form of the equation is correct for amplitudes of the order of x_o (i.e. for u of the order of 1). For small amplitudes we must nondimensionalize with respect to a smaller amplitude, in which case the value of C^* will be significantly smaller, i.e. the small motions of a guyed tower are lightly damped.

Figure 7 shows the relation between nondimensional amplitude (with respect to x_o)and frequency ω_1 for small damping, as derived by using perturbation techniques (see appendix 3). The softening-hardening spring form is clearly shown while some areas are unstable (thick lines) so that jump phenomena can occur. For the typical drag coefficients of a guyed tower, the resulting damping is large so that the method of harmonic balance can be used instead, i.e. the tower responds only at the frequency of excitation.

(b) Dynamic Modeling

In the previous section the analysis was based on a quasi-static modeling of the cables. As shown in the section on cable dynamics, this is adequate for modeling the inclined upper part the cable for frequencies close to the natural frequency of the tower since the first cable natural frequency is 4 times higher. The only part of the cable that is not adequately modelled by a quasi-static model is the clump weight, which is bulky and heavy so that its inertia and drag forces are significant.

This means that the motions of the clump weight will introduce dynamic amplification and phase lag in the overall response, although the cable response can be modelled as quasi-static. This observation can save us significant computational effort since we do not need to model the distributed behavior of the cable which is known to cause numerical problems unless special care is taken for the fast longitudinal dynamics.

Let us model the inclined cable connecting the clump weight to the tower as a nonlinear spring and the cable between the clump weight and the anchor as a linear spring. We can model the clump weight as a distributed mass subject to added mass and fluid drag forces. No waves develop along the clump weight so its space configuration can be approximated by a parabola for the part which has lifted from the bottom and a straight line lying on the bottom for the remaining part. Other approximate techniques such as suggested in [17] could also be used.

The simulation scheme is significantly simpler than a general model modeling the cables by finite difference or element techniques. Figure 8 demonstrates the dynamic amplification and phase lag introduced by the dynamics of the clump weight.

By ommitting damping forces such as the friction between the clump weight and the bottom, oscillations may appear in a simulation scheme which in reality are quickly damped by those neglected forces. This is a source of trouble especially if numerical models are used for the cables, which are very sensitive to stretching oscillations.

CONCLUSIONS

The purpose of this paper is to indicate some analytical techniques that can provide significant insight in the static and dynamic behavior of moored structures and therefore assist the designer in selecting appropriate values for the parameters involved especially at the early design phases.

The guyed tower, like other compliant structures, has a low rigid-body natural frequency by design, while the mooring lines are under significant tension so that the natural frequencies of the cables are far from the frequency range where significant guyed tower motions are obtained. As a result, we can model the cables as nonlinear springs and concentrate on the proper simulation of such components as the clump weight, whose effect on the dynamics of the tower is significant. A simple overall model is obtained providing flexibility for configuration changes and efficiency of computation for a wide parametric search.

The dynamics of the cables are studied as sources of parasitic forces, while their structural integrity must be guaranteed against excessive dynamic tension or fatique. Some efficient solutions are derived for the linear dynamics of stretched cables, which agree with recent developments in cable dynamics. These solutions are particularly useful to analyze the behavior of multi-leg systems.

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APPENDIX 1: Data

To demonstrate the techniques described here, the following example has been used:

Water depth	$= D_o = 1,500$ ft
Tower height	$= D_t = 1,600$ ft
Distance from fairlead to seafloor =	
	$= D = 1,400$ ft
Clump weight	$= W_c = 200$ kips
Clump weight length	$= \ell_c = 150$ ft
Length of unstretched cable from fairlead to clump weight =	
	$= L = 3,300$ ft
Total cable length (unstretched) =	
	$= L_t = 4,600$ ft
Initial tension at the cable upper end =	
	$= T = 300$ kips
Cable Young's modulus	$= E = 4.3*10^9$ lb/ft^2
Cable diameter	$= d = 3.5$ inch
Tower weight	$= 4*10^7$ lbs
Tower buoyancy	$= 3*10^7$ lbs
Drag diameter	$= d_D = 55$ ft
Inertia diameter	$= d_I = 19$ ft
Drag coefficient	$= C_D = 0.7$
Number of cables	$= 20$

APPENDIX 2: Cable Dynamic Solution

We will use equations (33) through (36) of the text to derive the linear dynamics of the cable. As shown in [8], the overall dynamic response consists of a part which is slowly varying with space (called in the sequel the slow solution) and a part which is wave-like and varies fast with respect to space (fast solution). In [8], the slow part was related to the catenary effects, because the systems considered involved considerable sag.

In the present case, large tensions are involved so the elasticity effects are significant and must be included in the slow solution.

First, we consider all dynamic quantities as varying sinusoidally in time, i.e.

$$u = \tilde{u}\, e^{i\omega t} \tag{A.1}$$

$$v = \tilde{v}\, e^{i\omega t} \tag{A.2}$$

$$\tilde{T}_1 = \tilde{T}_1\, e^{i\omega t} \tag{A.3}$$

$$\phi_1 = \tilde{\phi}_1\, e^{i\omega t} \tag{A.4}$$

$$e = \tilde{e}\, e^{i\omega t} \tag{A.5}$$

We ommit all nonlinear terms and obtain a homogeneous set of equations expressed in terms of ξ, η instead of u, v

$$u = \frac{d\xi}{dt} = i\omega\tilde{\xi}\, e^{i\omega t} \tag{A.6}$$

$$v = \frac{d\eta}{dt} = i\omega\tilde{\eta}\, e^{i\omega t} \tag{A.7}$$

i.e. ξ,η are the tangential and normal displacements respectively. Then:

$$-\omega^2(m_o+a_o)\tilde{\eta} = \frac{d}{ds}\left[T_o\tilde{\phi}_1\right] + \tilde{T}_1\frac{d\phi_o}{ds} \tag{A.8}$$

$$-\omega^2 m_o\,\tilde{\xi} = -W_1\cos\phi_o\,\tilde{\phi}_1 + \frac{d\tilde{T}_1}{ds} \tag{A.9}$$

$$\frac{d\tilde{\eta}}{ds} = \tilde{\phi}_1 - \tilde{\xi}\frac{d\phi_o}{ds} \tag{A.10}$$

$$\frac{d\tilde{\xi}}{ds} = \tilde{\eta}\frac{d\phi_o}{ds} + \frac{\tilde{T}_1}{EA} \tag{A.11}$$

Let $\alpha = d\phi_o/ds$ where for shallow sag cables, α will be a small quantitiy (order ε) and similarly all changes with respect of s of the static quantities will be small.

Let us first derive a solution which is fast varying with space, i.e. $\tilde{\eta}(s)$, $\tilde{\xi}(s)$. Then $\tilde{\xi}(s)$ must be of order ε compared to $\tilde{\eta}(s)$ so that to leading order from (A.8)

$$\tilde{T}_1 = \frac{1}{\alpha}\left\{M\omega^2\tilde{\eta} + \frac{d}{ds}\left[T_o\,\tilde{\phi}_1\right]\right\} \tag{A.12}$$

By using (A.10) and (A.11) to first order, we obtain from (A.12)

$$\frac{d\tilde{T}_1}{ds} = -\frac{d}{ds}\left\{\frac{M\omega^2}{\alpha^2}\frac{d\xi}{ds} + \frac{T_o}{\alpha}\frac{d^2}{ds^2}\left[\frac{1}{\alpha}\frac{d\tilde{\xi}}{ds}\right]\right\} \tag{A.13}$$

where $M = m_o + a_o$. From equation (A.9) to first order

$$\frac{d\tilde{T}_1}{ds} = 0 + 0(\varepsilon) \tag{A.14}$$

so that

$$M\omega^2\tilde{\eta} + \frac{d}{ds}\left[T_o\frac{d\tilde{\eta}}{ds}\right] = 0 \tag{A.15}$$

By using the WKB method as in [8] we find

$$\tilde{\eta} = \left[C_1\cos[W(s)] + C_2\sin[W(s)]\right]\frac{1}{\sqrt[4]{T_o/M}} \tag{A.16}$$

$$W(s) = \omega\int_o^s\frac{ds}{\sqrt{T_o/M}} \tag{A.17}$$

By using (A.11) to first order, we find

$$\tilde{\xi} = \left[C_1\sin[W(s)] - C_2\cos[W(s)]\right]\frac{\sqrt[4]{T_o/M}}{\omega}\,\alpha \tag{A.18}$$

Next, we derive a solution which is slowly varying with respect to s, with $\tilde{\eta}$ and $\tilde{\xi}$ of the same order. We note that

$$\frac{d\tilde{\xi}}{ds} = 0(\varepsilon\tilde{\xi}) \tag{A.19}$$

so that from (A.8)

$$\tilde{T}_1 = -\frac{1}{\alpha}M\omega^2\tilde{\eta} + 0(\varepsilon^2) \tag{A.20}$$

By combining (A.20) and (A.11) we find to first order

$$\tilde{T}_1 = \frac{-M\omega^2\,AE}{\alpha^2 AE - M\omega^2}\frac{d\tilde{\xi}}{ds} \tag{A.21}$$

By combining (A.21) and (A.9) we find to first order

$$\tilde{\xi} = \frac{M}{m}AE\frac{d}{ds}\left[\frac{1}{\alpha^2 AE - M\omega^2}\frac{d\tilde{\xi}}{ds}\right] \tag{A.22}$$

For a shallow catenary such as the one considered here, subject to a surface current, the value of α is constant to first order as easily seen from equations (5) and (6) of the text: If $T \gg W.L$ (in order to have a shallow catenary)

$$\frac{d\phi}{ds} \simeq \frac{W}{T_o}\cos\phi_o + O(\varepsilon^2) \tag{A.23}$$

where T_o is the tension at the top and ϕ_o the angle between the line connecting the cable ends and the horizontal. Let

$$P^2 = \alpha^2 \left\{1 - \frac{M\,\omega^2}{EA\alpha^2}\right\} \frac{m}{M} \tag{A.24}$$

so that (A.22) becomes

$$\tilde{\xi} = \frac{1}{P^2} \frac{d^2\tilde{\xi}}{ds^2} \tag{A.25}$$

or

$$\tilde{\xi} = C_3 e^{Ps} + C_4\, e^{-Ps} \tag{A.26}$$

and by using (A.11) and (A.21)

$$\tilde{\eta} = (C_3\, e^{Ps} - C_4\, e^{-Ps}) \frac{\alpha}{P} \frac{m}{M} \tag{A.27}$$

In order to find the natural frequencies we add the slow and fast solutions, i.e.

$$\tilde{\xi}(s) = \frac{\sqrt[4]{T_o(s)/M}}{\omega}\, \alpha \left[C_1 \sin[w(s)] - C_2 \cos[W(s)]\right] + C_3\, e^{Ps} + C_4\, e^{-Ps} \tag{A.28}$$

$$\tilde{\eta}(s) = \frac{1}{\sqrt[4]{T_o(s)/M}}\, C_1 \cos[W(s)] + C_2 \sin[W(s)] + \frac{\alpha}{P}\left[C_3\, e^{Ps} - C_4 e^{-Ps}\right]\frac{m}{M} \tag{A.29}$$

and apply the boundary conditions

$$\tilde{\xi}(0) = \tilde{\xi}(L) = 0 \tag{A.30}$$

$$\tilde{\eta}(0) = \tilde{\eta}(L) = 0 \tag{A.31}$$

The resulting equation is:

$$\sinh(PL)\, \sin(\overline{W}_o) \left[1 - \frac{\alpha_1^4}{P^2 k_1 k_2}\right] + \left[\cosh(PL)\, \cos(\overline{W}_o) - 1\right] \frac{\alpha_1^2}{P}\left\{\frac{1}{k_1} + \frac{1}{k_2}\right\} = 0$$

where

$$\overline{W}_o = \int_o^L \frac{\omega\ ds}{\sqrt{T(s)/M}} \tag{A.33}$$

$$k_1 = \frac{\omega}{\sqrt{T(o)/M}} \qquad k_2 = \frac{\omega}{\sqrt{T(L)/M}} \qquad \alpha_1 = \alpha\frac{m}{M} \tag{A.34}$$

Note that if we consider a horizontal catenary, then $k_1 = k_2 = k$. If the elasticity $EA \to \infty$ then $P \to \alpha$ and equation (A.32) becomes to first order in α_1:

$$\tan\frac{kL}{2} \cdot \left\{\tan\left(\frac{kL}{2}\right) - \frac{kL}{2}\right\} = 0 \tag{A.35}$$

which provides the symmetric and antisymmetric natural frequencies of a shallow sag chain [9].

If we let $\alpha \to 0$ then :

$$P \to i \frac{\omega}{\sqrt{E/\rho}} = i\mu$$

and equation (A.32) becomes:

$$\sin(kL) \cdot \sin(\mu L) = 0 \tag{A.36}$$

which provides the natural frequencies (transverse and longitudinal) of a taut wire.

APPENDIX 3: Weakly Nonlinear Dynamics

Equations (48) and (50) of the text describe the dynamics of the guyed tower when the mooring system is modelled quasi-statically. Since the coefficients of the nonlinear terms are small relative to 1 (order ε) we can use the method of multiple scales to derive the response up to first order [10]. Essentially the technique is based on the remark that the nonlinearity changes the amplitude of the response, but also the period (or equivalently the phase). As a result, both the amplitude and the phase are modified by expanding the amplitude in a power series in ε and by using two time scales t and εt.

In the case of $\omega_1 = 1$ (i.e. $\Omega = \omega_o$) we expect resonance, i.e. a single frequency response which is excited by relatively small forcing amplitude (order ε). To first order the method of multiple scale predicts

$$u^{(o)}(t) = a(\varepsilon t)\cos(\omega_1 t + \gamma(\varepsilon t)) \tag{B.1}$$

where a and γ are slowly varying functions of time. By using the next order equation derived from (48) we find a set of equations that a and γ must satisfy, otherwise they produce secular terms [10].

$$\frac{da}{dt} = -\delta(a)\,a - \frac{F}{(1+\omega_1)}\cos\gamma + \frac{4C^*}{3\pi}a^2 \tag{B.2}$$

$$\frac{d\gamma}{dt} = \omega_e(a) - \omega_1 + \frac{F}{a(1+\omega_1)}\sin\gamma \tag{B.3}$$

where [16]:

$$\delta(a) = \frac{1}{2\pi a}\int_o^{2\pi} f^*(a,\psi)\sin\psi\, d\psi \tag{B.4}$$

$$\omega_e^2(a) = 1 - \frac{1}{\pi a}\int_o^{2\pi} f^*(a,\psi)\cos\psi\, d\psi \tag{B.5}$$

In the case of a steady state response $\dot{a}=0, \dot{\gamma} = 0$ so equations (B.2) and (B.3) provide

$$\omega_1 = \sqrt{\omega_e^2(a) \pm \sqrt{\left(\frac{F}{a}\right)^2 - 4\left(\delta(a) + \frac{4\,C^*a}{3\pi}\right)^2}}$$

By using equation (B.6) we can draw the curve relating the amplitude and the frequency of excitation ω_1. Note that in order for (B.6) to provide real ω_1 the radical must be positive so the maximum amplitude, is the solution of the equation

$$\frac{F}{2} = a\left\{\delta(a) + \frac{4\,C^*}{3\pi}\right\}$$

i.e. strongly dependent on the damping coefficient C* as expected.

When the frequency of excitation ω_1 is far from 1 the system is responding with a significant amplitude only if the excitation is large (order 1). The response consists of a term of frequency ω_1 and a term of frequency $\omega_o=1$ (natural frequency of the system) which may not decay depending on the relation between ω_1 and ω_o (possibility of subharmonics and superharmonics).

TABLE 1

First Natural Frequencies of the Cable in (rad/sec)

Symmetric Modes	Antisymmetric Modes
$\omega_1 = 1.18$	$\omega_1 = 0.89$
$\omega_2 = 1.59$	$\omega_2 = 1.80$
$\omega_3 = 2.27$	$\omega_3 = 2.70$

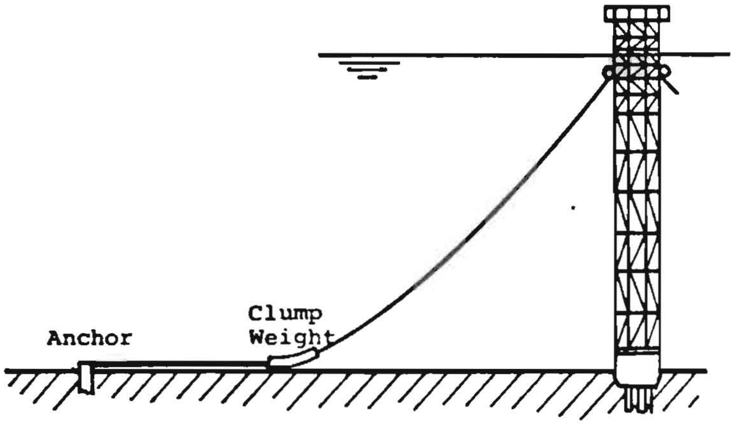


Figure 1: Guyed Tower

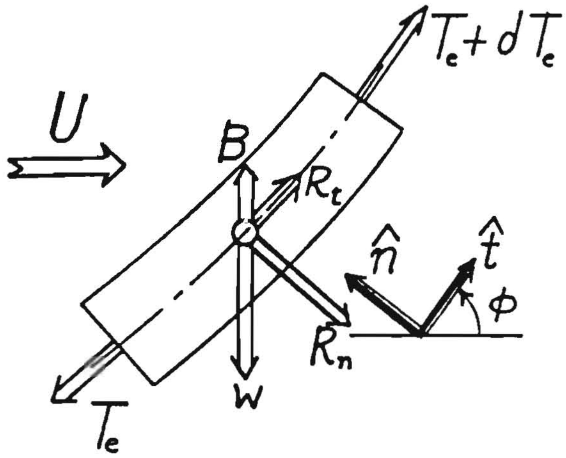


Figure 2: Forces on a cable element

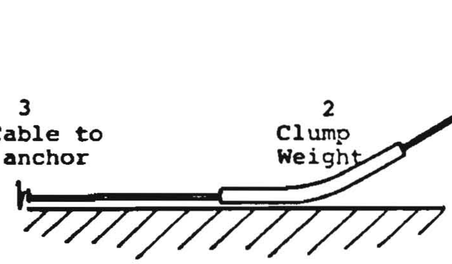


Figure 3

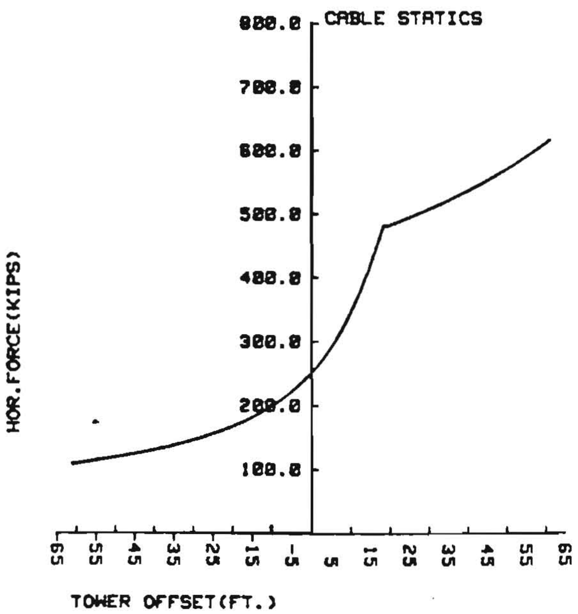


Figure 4

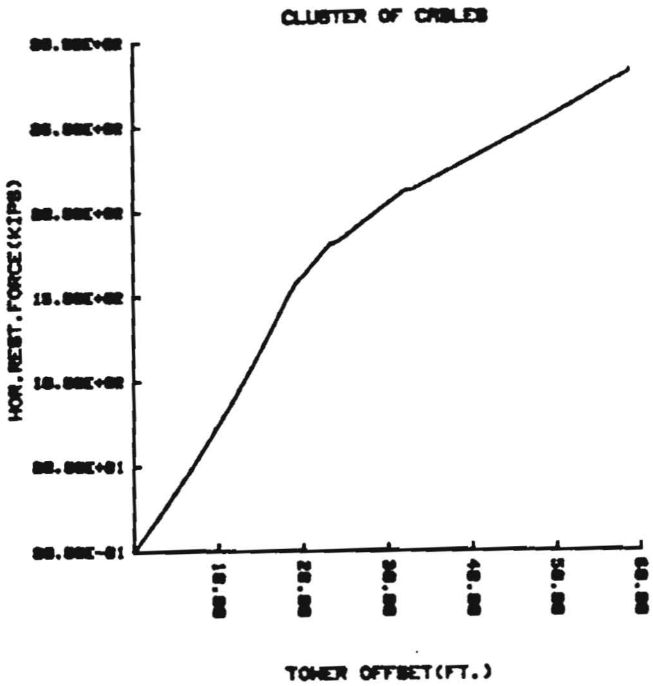
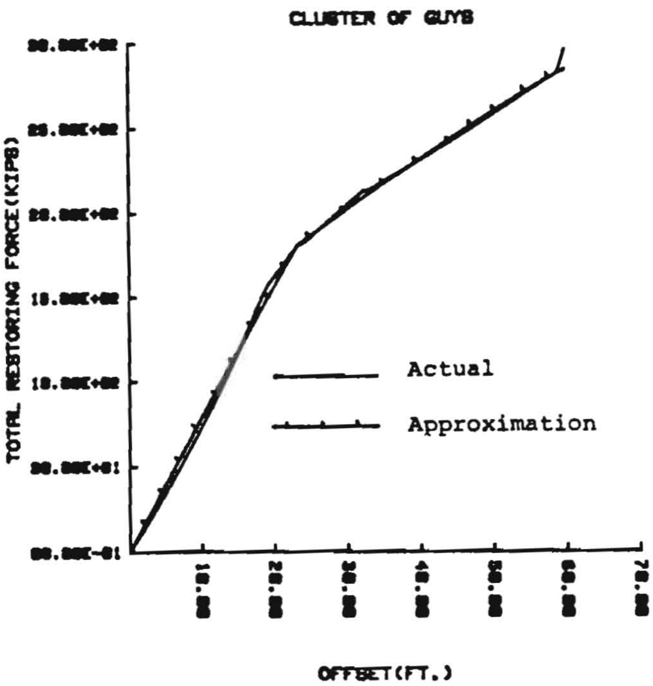


Figure 5



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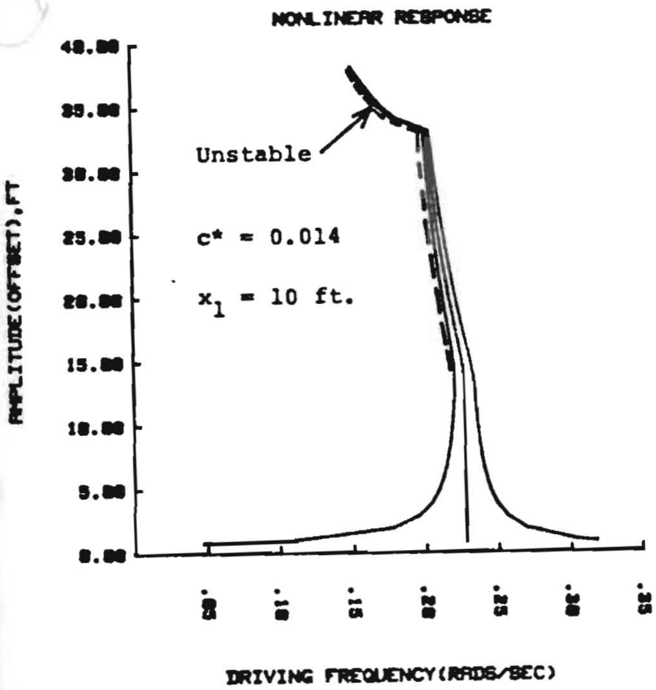

Figure 7

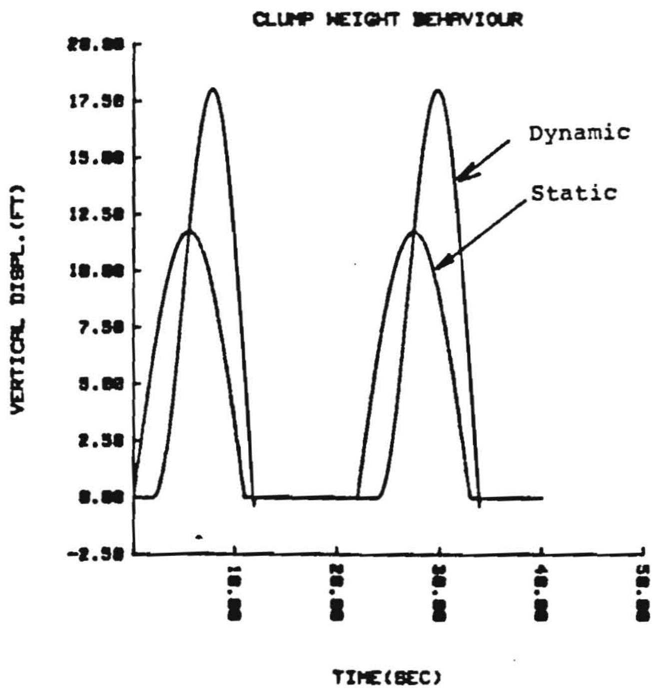

Figure 8