ANISOTROPY AND LOCALIZATION OF PLASTIC DEFORMATION

Proceedings of PLASTICITY '91:
The Third International Symposium on
Plasticity and Its Current Applications

Edited by

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ELSEVIER APPLIED SCIENCE
LONDON and NEW YORK

FRACTURE ANALYSIS OF MICROLAMINATE COMPOSITES WITH PLASTICALLY DEFORMING MATRICES

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ABSTRACT

Composite systems consisting of sheet reinforcement and polymeric or metallic matrix which can undergo plastic deformation present unique and unexplored issues with respect to the mechanics of their deformation, especially in the presence of cracks. For example, microlaminates consisting of layers of metal sheet reinforced with sheets of nonmetallic composite material are a family of new structural composite material systems.

In this class of composites, which consist of very strong and stiff sheets in a plastic matrix, the elastic and plastic distributions of stress and strain around a crack can be determined by assuming that the transverse displacements around the crack are negligible, by analogy from the corresponding case for longitudinal shear.

An infinite yield strength in the reinforcing sheets then gives a corresponding plastic zone and on a microscopic scale, the plastic flow within the matrix between the last cracked and the first uncracked reinforcing sheet is discussed and used to determine the stress concentration in the uncracked sheet. Other issues associated with the quantitative understanding of crack growth are discussed.

INTRODUCTION

Microlaminates consisting of layers of metal sheet reinforced with sheets of nonmetallic composite material are a family of new structural composite material systems. An example is the aramid aluminum laminates which consist of layers of thin aluminum alloy sheet bonded by adhesive impregnated with high strength unidirectional Aramid fibers.

The issues addressed in the present paper pertain to any material system with sheet reinforcement and plastically deforming matrix (metal or polymetric) and are therefore not restricted to the particular aramid/aluminum system. In general, the toughness of high strength materials can be enhanced in at least three ways. First, introducing free surfaces normal to the crack direction allows geometrical readjustments and transfer of the load to neighboring elements. The free surfaces may be manufactured, as in a fibrous structure, or may be developed during fracture, as with secondary cracks that blunt the main crack. Second, the toughness is enhanced by a certain amount of viscosity, so that cracks become blunt under low loads and can then withstand greater occasional overloads than if they had remained sharp. A third method of toughening is by using a matrix that deforms plastically, so that its yield strength will limit the stress applied to the high strength elements. Such plastic yielding not only occurs in metallic crystals but is also an approximation to the non-linear viscoelastic flow of polymers subjected to elastic strains of more than a few percent.

Consider a composite with elastic reinforcing elements in sheet form and a plastic matrix. Since the reinforcing elements are in sheet form, plane strain may be assumed.

In this work, we seek the microscopic distribution of stress and strain at the tip of a crack normal to the reinforcing sheet.

ANALYSIS

The Classical Anisotropic Elasticity of a Sheet-Reinforced Composite. The configuration under study, shown in Fig. 1, consists of relatively stiff and strong reinforcing sheets of thickness t_r, bonded in a relatively compliant, plastically deforming matrix of thickness t_m. The question of concern is the stress distribution in the unbroken platelet next to the tip of a crack. Intuitively one expects the reinforcing sheets to slide over each other with the plastically deforming material acting as a kind of solid lubricant. If the sheets are stiff enough, almost all the deformation will be in the vertical direction. If so, the analysis is much simplified by the analogy with longitudinal shear (Mode III) [1]. We therefore first consider the anisotropic elasticity of the material to see the extent to which the displacement can be assumed unidirectional.

In regions of moderate stress and strain gradients, each layer is subjected to the same strain ϵ_{22} in the x_2 direction and the same stress σ_{11} in the x_1 direction, as well as being under plane strain so that $\epsilon_{33} = 0$. When these conditions are imposed on the three-dimensional stress-strain relations for each layer, regarded as isotropic with a common Poissson's ratio ν and moduli of elasticity E_m and E_r for the matrix and reinforcing layers, respectively, an averaging process gives

$$\epsilon_{22} = -\sigma_{11}\frac{\nu(1+\nu)(t_m+t_r)}{E_m t_m + E_r t_r} + \sigma_{22}\frac{(1-\nu^2)(t_m+t_r)}{E_m t_m + E_r t_r} , \tag{1}$$

$$\epsilon_{11} = \sigma_{11}\left[\left(\frac{t_m}{E_m}+\frac{t_r}{E_r}\right)\frac{(1-2\nu)(1+\nu)}{(t_m+t_r)(1-\nu)} + \frac{\nu(1+\nu)(t_m+t_r)}{(1-\nu)(t_m E_m + t_r E_r)}\right] -$$

$$-\sigma_{22}\frac{\nu(1+\nu)(t_m+t_r)}{E_m t_m + E_r t_r} , \quad \gamma_{12} = \sigma_{12}\left(\frac{t_m}{G_m}+\frac{t_r}{G_r}\right)/(t_m+t_r) . \tag{2}$$

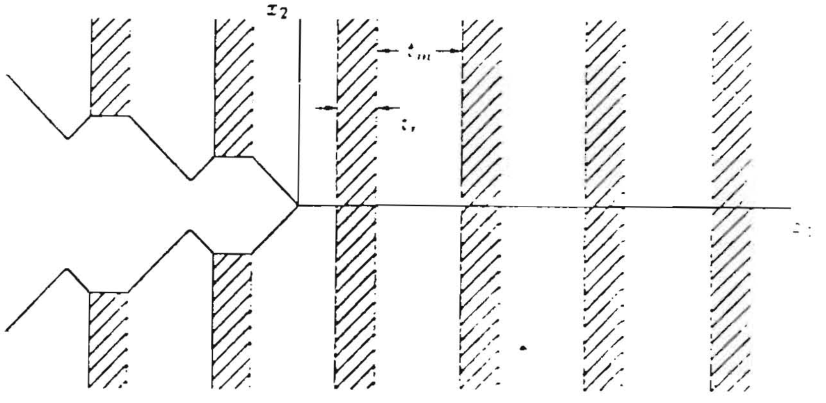


Figure 1. Crack in a laminated composite with ductile matrix.

As an example, consider the following properties: $E_r/E_m = 10$, $\nu_r = \nu_m = 0.3$, $t_m/(t_m + t_r) = 0.7$. In terms of the modulus of elasticity of the reinforcing sheet, Eqs. (1,2) give: $E_r\epsilon_{11} = (6.93)\sigma_{11} - (1.054)\sigma_{22}$, $E_r\epsilon_{22} = -(1.054)\sigma_{11} + (2.46)\sigma_{22}$, $E_r\gamma_{12} = (18.98)\sigma_{12}$. Note that the transverse strain ϵ_{11} due to a longitudinal stress σ_{22} is indeed small. This means that the effect of transverse strains on the longitudinal stress is also small and suggests that the assumption of an analysis based on displacements in the x_2 direction could be appropriate.

__Anisotropic Elastic Stress and Strain Distributions around a Crack Tip.__ The stress and strain fields around the tip of a crack in an anisotropic medium under plane strain conditions for cracks at arbitrary angles to the axes of anisotropy have been presented in Refs 2 and 3. In our case the stress-strain relation reduce to

$$\epsilon_{11} = a_{11}\sigma_{11} + a_{12}\sigma_{22}\ , \quad \epsilon_{22} = a_{12}\sigma_{11} + a_{22}\sigma_{22}\ , \quad \gamma_{12} = a_{66}\sigma_{12}\ . \tag{3}$$

The crack tip stress distributions are given in terms of the roots μ_i of an equation formed from the compliance coefficients of Eq. (3):

$$a_{11}\mu^4 + (2a_{12} + a_{66})\mu^2 + a_{22} = 0\ . \tag{4}$$

Solving for μ_i,

$$\mu_1^2, \mu_2^2 \simeq -\frac{2a_{12} + a_{66}}{a_{11}}\ , \quad -\frac{a_{22}}{2a_{12} + a_{66}}\ . \tag{5}$$

Note that the coefficients of compliance a_{ij} in comparison to their magnitude, indicate that the values of μ will be imaginary, and that μ_1 is large compared with μ_2. Neglecting μ_2 with respect to μ_1, the equations given in Ref 2 for the stress distribution in terms of a stress intensity factor k_1 (defined below) are:

$$\sigma_{11} \simeq \frac{k_1}{\sqrt{2r}}\mathrm{Re}\left[\frac{\sqrt{a_{22}/a_{11}}}{\sqrt{\cos\theta + i\sqrt{a_{66}/a_{11}}\sin\theta}}\right] = \frac{k_1}{\sqrt{2}}\mathrm{Re}\left[\frac{\sqrt{a_{22}/a_{11}}}{\sqrt{x_1 + ix_2\sqrt{a_{66}/a_{11}}}}\right]\ , \tag{6a}$$

$$\sigma_{22} \simeq \frac{k_1}{\sqrt{2}}\mathrm{Re}\left[\frac{1}{\sqrt{x_1 + ix_2\sqrt{a_{22}/a_{66}}}}\right]\ , \tag{6b}$$

$$\sigma_{12} \simeq \frac{k_1}{\sqrt{2}}\mathrm{Re}\left[\frac{i\sqrt{a_{22}/a_{66}}}{\sqrt{x_1 + ix_2\sqrt{a_{66}/a_{11}}}} - \frac{i\sqrt{a_{22}/a_{66}}}{\sqrt{x_1 + ix_2\sqrt{a_{22}/a_{66}}}}\right]\ . \tag{6c}$$

The stress intensity factor k_1 differs from one frequently used in that the factor $\pi^{1/2}$ disappears from a number of equations. In particular, for a crack of half length, c, in a body subjected to a tensile stress σ_∞,

$$k_1 = \sigma_\infty\sqrt{c} = K_I/\sqrt{\pi}\ . \tag{7}$$

The corresponding equations for longitudinal shear (Mode III) in an isotropic material are

$$\sigma_{1\prime 3\prime} = -\frac{k_{3\prime}}{\sqrt{2}}\mathrm{Re}\left[\frac{i}{(x_{1\prime} + ix_{2\prime})^{1/2}}\right] , \quad \sigma_{2\prime 3\prime} = \frac{k_{3\prime}}{\sqrt{2}}\mathrm{Re}\left[\frac{1}{(x_{1\prime} + ix_{2\prime})^{1/2}}\right] . \tag{8}$$

To complete the analogy, turn to the equations for displacement. For plane strain with large values of μ_1, the equations given in Ref 2 become

$$u_1 = k_1\sqrt{2}\ \mathrm{Re}\left[a_{12}(x_1 + \mu_2 x_2)^{1/2} - \sqrt{a_{11}a_{22}}(x_1 + \mu_1 x_2)^{1/2}\right] , \tag{9a}$$

$$u_2 = k_1\sqrt{2}\ \mathrm{Re}\left[-i\sqrt{a_{22}a_{66}}(x_1 + \mu_2 x_2)^{1/2}\right] . \tag{9b}$$

For Mode III deformation in anisotropic material

$$u_{3\prime} = k_3\sqrt{2}\ \mathrm{Re}\left[-i(1/G)(x_{1\prime} + ix_{2\prime})^{1/2}\right] . \tag{10}$$

Comparison of Eqs. (9b) and (10) along with (6b,c) and (8) gives the following analogy between variables in isotropic longitudinal shear and anisotropic plane strain tension:

$$u_{3\prime} \to u_2 \ ; \ G \to 1/\sqrt{a_{22}a_{66}} \ ; \ x_{2\prime} \to x_2\sqrt{a_{22}/a_{66}} \ , \tag{11}$$

$$x_{1\prime} \to x_1 \ ; \ \sigma_{1\prime 3\prime} \to \sigma_{12}\sqrt{a_{66}/a_{22}} \ ; \ \sigma_{2\prime 3\prime} \to \sigma_{22} \ ; \ k_{3\prime} = \sigma_{2\prime 3\prime}\sqrt{c} \to k_1 = \sigma_{22\infty}\sqrt{c} \ . \tag{12}$$

<u>Yielding at the Crack Tip.</u> For predicting fracture we need the stress in front of the crack. Applying the procedure by Rice [4, 5] to the present problem, we find that well away from the plastic zone, the solution approaches the elastic stress distribution:

$$\sigma_{12} = \frac{-k\sin\theta\prime/2}{\sqrt{2r\prime}} , \qquad \sigma_{22} = k\sqrt{\frac{a_{66}}{a_{22}}}\frac{\cos\theta\prime/2}{\sqrt{2r\prime}} , \tag{13}$$

where

$$\theta\prime = \tan^{-1}\left(x_2\sqrt{a_{22}/a_{66}}/x_1\right) , \qquad r\prime = \frac{\sqrt{x_1^2 + x_2^2 a_{22}/a_{66}}}{(k_1/k)^2(a_{22}/a_{66})} .$$

Close to, but ahead of the crack tip ($|\bar{z}\prime| \ll$ and $x_1 > 0$), the solution becomes

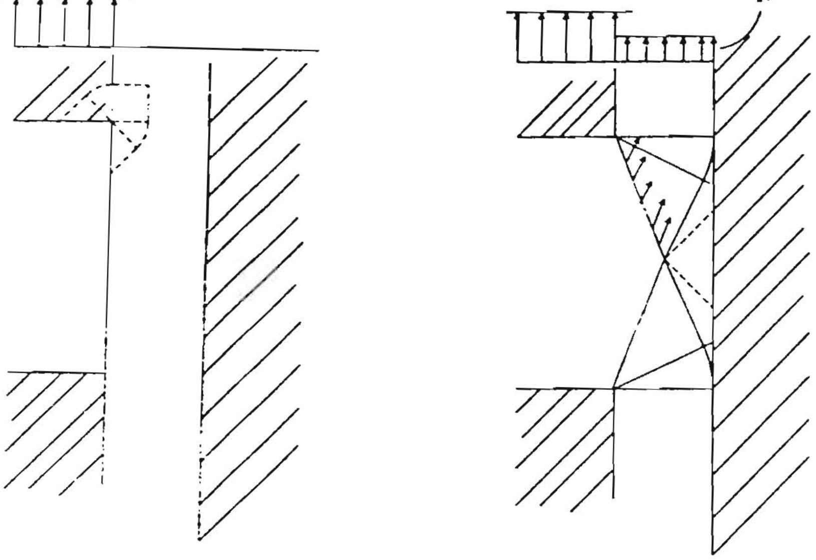
$$\frac{\sigma_{12}}{k} = -\frac{\theta\prime}{\pi/2} , \qquad \frac{\sigma_{22}}{k} = \frac{2}{\pi}\sqrt{\frac{a_{66}}{a_{22}}}\ln\frac{\pi^2}{4r\prime} . \tag{14}$$

<u>Fully Plastic Flow within a Layer of Matrix.</u> The stress and strain distribution within the matrix layer at the tip of the crack are needed to predict whether or not the layer will delaminate and thus blunt the crack. The displacements of the reinforcing sheets at the crack tip set boundary conditions on the stress and strain distribution in the matrix layer. Further boundary conditions are set by the transverse tension σ_{11}, which tends to suck matrix material away from the crack tip, increasing the thickness of the laminar layer. This increased thickness will in turn decrease σ_{11}. So far, no

such analysis is available. Therefore we neglect this thickening action and inquire what transverse stress develops from two nonunique, fully plastic flow fields.

The simplest flow field consists of pure sliding along one interface, as shown in Fig. 2a. The slip line field shown by the dashed lines demonstrates that plastic flow need not spread out from the corner if the rest of the matrix remains rigid. The stress distribution is indeterminate, except for a yield strength in shear k along the active interface. Any strain-hardening would tend to thicken the zone of plastic flow; so we seek a field with flow more widely distributed through the matrix layer.

A field with plastic flow in the matrix is shown in Fig. 2b. It was derived assuming geometrical similarity as the crack opens. A gradually thickening layer of the matrix is drawn upwards at half the displacement rate of the cracked reinforcing sheet.



a) Stationary matrix b) Sliding matrix

Figure 2. Plastic flow in matrix.

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