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Nonlinear High-Order Response of Imperfect Sandwich Beams with Delaminated Faces

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Delaminations within face sheets are often observed when a sandwich structure is exposed to impact loads. The buckling and postbuckling behavior of sandwich beams with delaminated faces is investigated. The governing nonlinear equations, boundary conditions, and continuity conditions are formulated through variational principles. The beam construction consists of two stiff composite laminated face sheets and a soft core of a foam or a low-strength honeycomb type. A higher-order theory is used for the core that accounts for the nonlinear distortion of the plane of the section of the core and the compressibility in the thickness direction. The delamination considered is an interface crack, in which the substrate includes the transversely flexible core. As an illustration, the results of numerical computation for a simply supported beam are presented to show the effects of a delamination inside the face sheets on the behavior of sandwich structures under compressive loading.

Nomenclature

A^i	=	axial rigidity matrices of face sheets
B^i	=	stretching and bending coupling matrices of face sheets
b	=	width of face sheets
c	=	thickness of core
D^i	=	flexural rigidity matrices of face sheets
d_b, d_t	=	thickness of bottom face sheet and top face sheet, respectively
E_c, E_f	=	elastic modulus of core and face sheets, respectively
G_c	=	shear modulus of core
L	=	length of the sandwich panel
M_{xx}, N_{xx}	=	resultant bending moment and in-plane force, respectively
m_{xx}, n_{xx}	=	external moment and axial force, respectively
P, P_{cr}	=	applied load and buckling load, respectively
q_z	=	external transversal load
U, V	=	strain energy and external potential, respectively
u	=	axial displacement
w	=	transversal displacement
$\bar{w}$	=	imperfection of face sheets
γ^c	=	shear strain of the core
ϵ_{xx}	=	axial strain of face sheet
ϵ_{zz}	=	transversal strain of the core
ϵ_{0xx}	=	axial strain of face sheet measured along reference plane
κ_{xx}	=	curvature of face sheet
σ_{xx}	=	axial stress of face sheet
σ_{zz}	=	transverse stress of the core
τ^c	=	shear stress of the core

Subscripts and Superscripts

b	=	bottom face sheet
c	=	core
t	=	top face sheet

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I. Introduction

A TYPICAL sandwich structure is composed of two thin metallic or composite laminated faces and a thick soft core made from foam or a low-strength honeycomb. Because of their exceptional properties such as high stiffness and strength with little resultant weight penalty, thermal and acoustic insulation, smooth aerodynamic surface in a high-speed range, high-energy absorption capability, etc., sandwich structures have been used as parts of aircraft, marine vessels, ground vehicles, and offshore structures. Especially for structures operating in hostile environments with high-temperature, moisture, or pressure fields, a sandwich structure can offer an attractive alternative to a conventional construction.

Research into sandwich structural behavior and failure modes can be traced to after World War II, with several papers published between 1945 and 1955, on the strength and stability. Extensive and very complete listings of early works on sandwich structures up to the end of the 1960s have been summarized by Plantema[1] and Allen.[2] The basic assumption adopted for the core by most of the early researchers was that the core only has a shear rigidity, that is, it is incompressible in the thickness direction and the plane of the section remains plane after deformation, that is, classic core theory or antiplane core. The two basic buckling modes, the global buckling of the beam/panel and the wrinkling/local buckling of the face sheets/skins, were considered uncoupled. The global buckling was studied by considering the whole structure as an equivalent beam incorporating the shear stiffness of the core, and the local buckling was investigated by considering the isolated skins as beams resting on an elastic foundation (which is provided by the core) in the thickness direction.[3] In the 1990s, high-order theories were proposed for the core.[4,5] The high-order theory formulation takes into account the effects of the flexibility of the core in the transverse direction and the shear rigidity of the core simultaneously. However, the effects of delaminations within the skins on the behavior of a sandwich structure have not been adequately studied so far, although some papers on delaminations in a sandwich structure have provided models for a debond between the skin and the core.

Delamination is a very common failure phenomenon in laminated composites, especially those exposed to impact loads. These delaminations may deteriorate the performance of the structure under compressive loading.[6,7] A large number of studies on the behavior of delamination buckling and postbuckling in composites has been carried out by many researchers, including Chai et al.,[8] who used a one-dimensional model to simulate the characteristics of delamination growth, and Sheinman and Kardomateas,[7] who studied the energy release rate for an arbitrary stacking sequence. Although the general principles are not very different, delamination failure in a sandwich structure is just beginning to be explored in detail.

In this regard, differences in the behavior of delamination buckling and postbuckling within a sandwich structure from that of a laminated composite structure may arise because the substrate in a delaminated sandwich structure includes a much different kind of material, namely, a transversely flexible core made of foam or low strength honeycomb.

The purpose of present research is to investigate in detail this often-observed damage mode in sandwich structures. Here, the term delaminated faces refers to two possible situations: a delamination within the face sheet or a debonding at the interface between the face sheet and the core. The higher-order core theory is incorporated into the constitutive relationships for the structure to take into account the effects of the interaction between two face sheets and core on the buckling and postbuckling behavior of the delaminated sandwich panels. Using this higher-order core theory, we can calculate the peeling stress, which otherwise may be difficult to be obtained by classic core theory. Here peeling stress is the normal stress that separates the face sheets and the core in the normal direction. It is one of the two stresses (the other one is the shear stress between the faces sheets and the core material) that contributes to the debonding of sandwich structure. By the use of higher-order core theory, we formulate a group of the nonlinear governing equations, boundary conditions, and continuity conditions that can be used to deal with the nonlinear behavior of delaminated sandwich panels with/without imperfections under various loading and boundary conditions. The results of simply supported panels with a delamination under compressive loading are compared with those of the nondelamination case, and it is shown that delaminations have significant effects on the failure of a sandwich structure.

II. Formulation

Kinematics

The nonlinear theory developed in this paper is based on the following assumptions:

1) The thin face sheets are considered as Kirchhoff–Love beams with bending and axial rigidities.

2) The imperfections are small and are only imposed on the face sheets.

3) The core has shear resistance and is free of longitudinal normal stresses, but its transverse compressibility is considered, that is, its thickness may change.

The delamination considered is through the entire width. The sandwich beam with a delamination in the upper face sheet can be subdivided into four zones for the upper skin (delaminated skin zone, substrate skin zone in the delamination region, and two base skin zones outside the delaminated region), and three zones for the core and lower skin (one zone in the delamination region and two base zones outside the delaminated region), as shown in Fig. 1.

The displacement field is defined by the longitudinal component $u(x,z)$ in the x direction and the transverse component $w(x,z)$ in the z direction. Here x is measured from the left boundary of the structure to the right, and z measured upward from the reference plane of the skin. The displacements of the skins are of the form

$$u(x,z) = u_0(x) + z w_{,x}(x), \qquad w(x,z) = w_0(x) = w(x) \quad (1)$$

where the subscript 0 denotes values measured at reference plane of the skin. Notice that a uniform transverse deflection is assumed

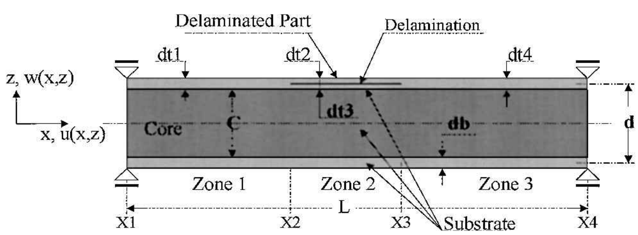

Fig. 1 Geometry of a simply supported sandwich beam with a delamination inside the top face sheet.

along the z direction for the skins and, therefore, the subscript is omitted for w.

The strain field in the skins is given as follows:

$$\epsilon_{xx}(x,z) = \epsilon_{0xx}(x) + z\kappa_{xx}(x)$$

$$\epsilon_{0xx} = u_{0,x} + \tfrac{1}{2} w_{,x}(w_{,x} + 2\bar{w}_{,x}), \qquad \kappa_{xx} = -w_{,xx} \quad (2)$$

where κ_{xx} is the change in curvature, $\bar{w}(x)$ is the initial imperfection, and the subscript ,x denotes a derivative with respect to x.

The strains and the curvature can be written in terms of the axial force N_{xx} and the bending moment M_{xx} as follows:

$$\epsilon_{0xx} = \alpha_1^i N_{xx} + \alpha_2^i M_{xx}, \qquad \kappa_{xx} = \alpha_3^i N_{xx} + \alpha_4^i M_{xx} \quad (3)$$

where $i = t, b$ and α_1^i are defined in terms of the stiffness matrices A^i, B^i, and D^i (e.g., see Sheinman and Kardomateas[7]).

The core is considered undergoing a small displacement with large rotation, and the corresponding strain field is of the following form:

$$\epsilon^c = w^c_{,z_c}, \qquad \gamma^c = u^c_{,z_c} + w^c_{,x} \quad (4)$$

where the superscript c denotes the core, and ,z_c denotes derivative with respect to z_c; this z_c is measured downward from the upper core/skin interface.

Under the postulation of perfect bonding between face sheets and the core, the corresponding compatibility conditions at the core/skin interfaces can be written as follows.

At the upper interface ($z_c = 0$):

$$u^c(x,0) = u_0^t(x) - (d_t/2) w^t_{,x}(x), \qquad w^c(x,0) = w^t(x) \quad (5)$$

At the lower interface ($z_c = c$):

$$u^c(x,c) = u_0^b(x) + (d_b/2) w^b_{,x}(x), \qquad w^c(x,c) = w^b(x) \quad (6)$$

where t and b denote upper/top skin and lower/bottom skins, respectively, and d_i, $i = t$ or b, is the thickness of the corresponding upper or lower skin.

Constitutive Relationships for the Skins and the Core

The general stress–strain relationship is $[\sigma] = \bar{Q}\ [\epsilon]$, where Q is the stiffness matrix. For the present problem, relations in the form of resultant forces and bending moments in the skins can be written as follows:

$$N_{xx}^{t_i} = A_{11}^{t_i}\epsilon_{0xx}^{t_i} + B_{11}^{t_i}\kappa_{xx}^{t_i}, \qquad M_{xx}^{t_i} = B_{11}^{t_i}\epsilon_{0xx}^{t_i} + D_{11}^{t_i}\kappa_{xx}^{t_i} \quad (7)$$

where $i = 1, 2, 3, 4$ for the four zones of the upper/top skin. Similar expressions hold for the lower/bottom skin.

The higher-order theory adopted for the relationship between stress and strain fields in the core is of the following form[5]:

$$\sigma_{zz}^{t_i}(x,z) = \frac{w^{b_i} - w^{t_i}}{c} E_c + \frac{c}{2}\tau^{c_i}_{,x}$$

$$\sigma_{zz}^{b_i}(x,z) = \frac{w^{b_i} - w^{t_i}}{c} E_c - \frac{c}{2}\tau^{c_i}_{,x} \quad (8a)$$

$$\sigma_{zz}^{c_i}(x,z_c) = \frac{w^{b_i} - w^{t_i}}{c} E_c + \tau^{c_i}_{,x}\left(\frac{c}{2} - z_c\right)$$

$$w^{c_i}(x,z_c) = \frac{\left(c z_c - z_c^2\right)}{2E_c}\tau^{c_i}_{,x} + \left(1 - \frac{z_c}{c}\right) w^{t_i} + \frac{z_c}{c} w^{b_i} \quad (8b)$$

$$u^{c_i}(x,z_c) = -\frac{\left(c z_c^2/2 - z_c^3/3\right)}{2E_c}\tau^{c_i}_{,xx} + \frac{z_c\tau^{c_i}}{G_c} + \left(\frac{z_c^2}{2c} - z_c - \frac{d_{t_i}}{2}\right) w^{t_i}_{,x} - \frac{z_c^2}{2c} w^{b_i}_{,x} + u_0^{t_i} \quad (8c)$$

where $t_i = t_1, t_3, t_4$ and $b_i = b_1, b_2, b_3$; $c_i = c_1, c_2, c_3$ for the core in the three zones; and G_c and E_c are shear and elastic moduli of the core, respectively.

Nonlinear Governing Equations

When the variational principle is applied, the nonlinear governing equations with the appropriate boundary conditions can be derived. The sandwich beam/panel is subjected to the compound actions of an external axial force n^i_{xx}, transverse force q^i_z, and moment m^i, $i = t$ or b. At an equilibrium state, the first variation of the total potential energy, $\Pi = U - V$, consisting of strain energy U and external potential V of applied loading, vanishes. These can be written as

$$\delta U = \iiint_{v_t} \sigma^t_{xx}\delta\epsilon^t_{xx}\,\mathrm{d}v + \iiint_{v_b} \sigma^b_{xx}\delta\epsilon^b_{xx}\,\mathrm{d}v + \iiint_{v_c} \tau^c\delta\gamma^c\,\mathrm{d}v + \iiint_{v_c} \sigma^c_{zz}\delta\epsilon^c_{zz}\,\mathrm{d}v \tag{9}$$

$$\delta V = \int_0^L \left(n^i_{xx}\delta u^i_0 + q^i_z\delta w^i + m^i\delta w^i_{,x}\right)\mathrm{d}x + F_{\text{ext}}\delta X \tag{10}$$

where $F_{\text{ext}} = N^i_{\text{ext}}$, M^i_{ext}, P^i_{ext} and $X = u^i_0$, w^i, $w^i_{,x}$, respectively.

Substitution of Eq. (8) and integration by parts yields sets of nonlinear governing equations.

Upper/top skin:

$$-N^{t_i}_{xx,x} - b\tau^{c_j}\Big|_{z_c=0} - n^{t_i}_{xx} = 0 \tag{11}$$

$$-M^{t_i}_{xx,xx} - \left[N^{t_i}_{xx}\left(w^{t_i}_{,x} + \bar{w}^{t_i}_{,x}\right)\right]_{,x} - \tfrac{1}{2}bd_{t1}\tau^{c_j}_{,x}\Big|_{z_c=0} - b\sigma^{c_j}_{zz}\Big|_{z_c=0} + m^{t_i}_{,x} - q^{t_i} = 0 \tag{12}$$

Analogous equations are found for the lower/bottom skin.

Core:

$$-b\tau^{c_j}_{,z_c}(x, z_c) = 0, \qquad -b\sigma^{c_j}_{zz,z_c}(x, z_c) - b\tau^{c_j}_{,x}(x, z_c) = 0 \tag{13}$$

where $i = 1, 4$ and $j = 1, 3$ represent the corresponding values for zone 1 or zone 3.

Because zone 2 includes the delamination, the set of governing equations has a different form, as follows.

Upper/top skin:

$$-N^{t_2}_{xx,x}(x) - n^{t_2}_{xx}(x) = 0 \tag{14}$$

$$-M^{t_2}_{xx,xx}(x) - \left[N^{t_2}_{xx}\left(w^{t_2}_{,x} + \bar{w}^{t_2}_{,x}\right)\right]_{,x} + m^{t_2}_{,x} - q^{t_2} = 0 \tag{15}$$

$$-N^{t_3}_{xx,x}(x) - b\tau^{c_2}(x, 0) - n^{t_3}_{xx}(x) = 0 \tag{16}$$

$$-M^{t_3}_{xx,xx}(x) - \left[N^{t_3}_{xx}\left(w^{t_3}_{,x} + \bar{w}^{t_3}_{,x}\right)\right]_{,x} - \tfrac{1}{2}bd_{t3}\tau^{c_2}_{,x}(x, 0) - b\sigma^{c_2}_{zz}(x, 0) + m^{t_3}_{,x} - q^{t_3} = 0 \tag{17}$$

with analogous equations derived for the lower/bottom skin.

Core:

$$-b\tau^{c_2}_{,z_c}(x, z_c) = 0, \qquad -b\sigma^{c_2}_{zz,z_c}(x, z_c) - b\tau^{c_2}_{,x}(x, z_c) = 0 \tag{18}$$

Equation (18) shows that the shear stress is only a function of x.

The corresponding boundary conditions read as follows. At the ends of the structure, that is, at $x = x_1$ and $x = x_4$,

$$u^{t_i}_0 - u^{t_i}_{\text{ext}} = 0, \quad \text{or} \quad -N^{t_i}_{xx} - N^{t_i}_{\text{ext}} = 0$$

$$w^{t_i} - w^{t_i}_{\text{ext}} = 0, \quad \text{or} \quad -M^{t_i}_{xx,x} - N^{t_i}_{xx}w^{t_i}_{,x} + m^{t_i} - \left(bd_{t_i}/2\right)\tau_1 = 0$$

$$w^{t_i}_{,x} - w^{\text{ext}}_{,x} = 0, \quad \text{or} \quad -M^{t_i}_{xx} - M^{t_i}_{\text{ext}} = 0$$

$$u^{b_j}_0 - u^{b_j}_{0\,\text{ext}} = 0, \quad \text{or} \quad -N^{b_j}_{xx} - N^{b_j}_{\text{ext}} = 0$$

$$w^{b_j} - w^{b_j}_{\text{ext}} = 0, \quad \text{or} \quad -M^{b_j}_{xx,x} + N^{b_j}_{xx}w^{b_j}_{,x} + m^{b_j} - (bd_{b_j}/2)\tau_1 = 0$$

$$w^{b_j}_{,x} - w^{\text{ext}}_{,x} = 0, \quad \text{or} \quad -M^{b_j}_{xx} - M^{b_j}_{\text{ext}} = 0$$

$$w^{c_j} - w^{c_j}_{\text{ext}} = 0, \quad \text{or} \quad -\tau_1 = 0 \tag{19}$$

where $i = 1, 4$ and $j = 1, 3$.

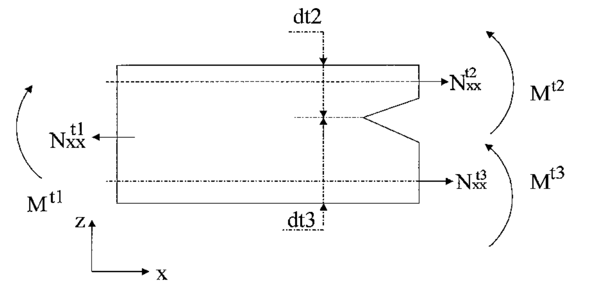

Fig. 2 Resultants at the first crack tip.

Conditions at the Delamination Tips and the Ends

The conditions at the crack tips can be grouped into displacement compatibility and resultant force (Fig. 2) equilibrium.

The displacement compatibility conditions at the first crack tip $x = x_2$ are as follows:

$$-u^{t_2}_0 + u^{t_1}_0 + \left[\left(d_{t_1} - d_{t_2}\right)/2\right]w^{t_1}_{,x} = 0$$

$$-u^{t_3}_0 + u^{t_1}_0 - \left[\left(d_{t_1} - d_{t_3}\right)/2\right]w^{t_1}_{,x} = 0$$

$$w^{t_1} - w^{t_2} = 0, \qquad w^{t_1} - w^{t_3} = 0, \qquad w^{t_2}_{,x} - w^{t_1}_{,x} = 0$$

$$w^{t_2}_{,x} - w^{t_3}_{,x} = 0, \qquad u^{b_2}_0 - u^{b_1}_0 = 0, \qquad w^{b_2} - w^{b_1} = 0$$

$$w^{b_2}_{,x} - w^{b_1}_{,x} = 0, \qquad w^{c_1} - w^{c_2} = 0 \tag{20}$$

The resultant force equilibrium conditions at the first crack tip $x = x_2$ are

$$N^{t_1}_{xx} - N^{t_2}_{xx} - N^{t_3}_{xx} = 0$$

$$M^{t_1}_{xx} - M^{t_2}_{xx} - M^{t_3}_{xx} + d_{t_3}N^{t_2}_{xx}/2 - d_{t_2}N^{t_3}_{xx}/2 = 0$$

$$-M^{t_2}_{xx,x} - N^{t_2}_{xx}w^{t_2}_{,x} + M^{t_1}_{xx,x} - N^{t_1}_{xx}w^{t_1}_{,x} - M^{t_3}_{xx,x}N^{t_3}_{xx}w^{t_3}_{,x} + \left(bd_{t_1}/2\right)\tau_1 - \left(bd_{t_3}/2\right)\tau_2 + m^{t_3} + m^{t_3} + m^{t_2} - m^{t_1} = 0$$

$$N^{b_1}_{xx} - N^{b_2}_{xx} - N^{b_2}_{\text{ext}} = 0, \qquad M^{b_1}_{xx} + M^{b_2}_{xx} - M^{b_2}_{\text{ext}} = 0$$

$$M^{b_1}_{xx,x} + N^{b_1}_{xx}w^{b_1}_{,x} - M^{b_2}_{xx,x} + N^{b_2}_{xx}w^{b_2}_{,x} + \left(bd_{b_1}/2\right)\tau_1 - \left(bd_{b_2}/2\right)\tau_2 + m^{b_2} - m^{b_1} = 0, \qquad \tau_2 - \tau_1 = 0 \tag{21}$$

Similar equations are written for the second crack tip. The end boundary conditions are as follows. In displacement form

$$w^{t_i} - w^{b_j} = 0, \qquad w^{t_i}_{,x} - w^{b_j}_{,x} = 0$$

$$w^{t_i}_{,x} + \frac{-u^{t_i}_0 + u^{b_j}_0}{z_{et} + z_{eb}} = 0, \qquad \frac{u^{t_i}_0 z_{eb} + u^{b_j}_0 z_{et}}{z_{et} + z_{eb}} - u^{\text{tot}}_{0\text{ext}} = 0$$

$$w^{\text{tot}}_{,x\text{ext}} + \frac{-u^{t_i}_0 + u^{b_j}_0}{z_{et} + z_{eb}} = 0, \qquad w^{\text{tot}} - w^{\text{tot}}_{\text{ext}} = 0, \qquad \frac{\tau^1_{,x}c^3}{12E_c} = 0 \tag{22}$$

or in resultant force form

$$N^{b_j}_{xx} - N^{b_j}_{\text{ext}} + N^{t_i}_{xx} - N^{t_i}_{\text{ext}} = 0$$

$$-M^t_{\text{ext}} - N^t_{\text{ext}}z_{et} + N^b_{\text{ext}}z_{eb} - N^{b_j}_{xx}z_{eb} + N^{t_i}_{xx}z_{et} - M^{t_i}_{xx} - M^b_{\text{ext}} - M^{b_j}_{xx} = 0$$

$$M^{t_i}_{xx,x} + M^{b_j}_{xx,x} + N^{t_i}_{xx}w^{t_i}_{,x} + N^{b_j}_{xx}w^{b_j}_{,x} + [(d_{t4} + d_b)/2]b\tau^3 - m^{t_i} - m^{b_j} - p^t_{\text{ext}} - p^b_{\text{ext}} = 0, \qquad b\tau^j = 0 \tag{23}$$

where $i = 1$ or 4 and $j = 1$ or 3.

In this section we have derived all governing equations and corresponding boundary conditions and continuity conditions for the general problem of a sandwich panel with a delaminated face sheet. Theoretically, solving these equations would yield all of the information needed to describe the buckling and postbuckling behavior of a sandwich panel with delaminations. However, after substituting the constitutive relations (7) into the preceding formulas, which are then written in terms of the displacements, it is seen that this is a set of coupled nonlinear equations. The buckling behavior and postbuckling can be conveniently studied by using the perturbation technique, which has been successfully applied previously to bifurcation analyses.[9,10]

III. Imperfection Analysis

Imperfections are very common in the face sheets of a sandwich construction due to the difficulty of manufacturing the core materials.[5] The imperfection considered is assumed to be a small transverse (thickness) deformation of the face sheets. The governing equations for imperfection analysis can be obtained by substituting Eq. (8) into the nonlinear equations (12–18) together with the corresponding boundary conditions. These equations can be written as follows.

Upper/top skin in zone 1:

$$-N^{t1}_{xx,x} - b\tau_1 - n^{t1}_{xx} = 0 \tag{24}$$

$$-M^{t1}_{xx,xx} - \left[N^{t1}_{xx}\left(w^{t1}_{,x} + \bar{w}^{t1}_{,x}\right)\right]_{,x} - \tfrac{1}{2}b(d_{t1} + c)\tau_{1,x} + [b(w^{t1} - w^{b1})/c]E_c + m^{t1}_{,x} - q^{t1} = 0 \tag{25}$$

Similar equations hold for the lower/bottom skin in zone 1.

Core in zone 1:

$$\left(-c^3/12E_c\right)\tau_{1,xx} + (c\tau_1/G_c) - \tfrac{1}{2}\left(c + d_{t_1}\right)w^{t1}_{,x} - \tfrac{1}{2}(c + d_b)w^{b1}_{,x} + u^{t1}_0 - u^{b1}_0 = 0 \tag{26}$$

Upper/top skin in zone 2:

$$-N^{t2}_{xx,x} - n^{t1}_{xx} = 0 \tag{27}$$

$$-M^{t2}_{xx,xx} - \left[N^{t2}_{xx}\left(w^{t2}_{,x} + \bar{w}^{t1}_{,x}\right)\right]_{,x} + m^{t2}_{,x} - q^{t2} = 0 \tag{28}$$

$$-N^{t3}_{xx,x} - b\tau_3 - n^{t3}_{xx} = 0 \tag{29}$$

$$-M^{t3}_{xx,xx} - \left[N^{t3}_{xx}\left(w^{t3}_{,x} + \bar{w}^{t3}_{,x}\right)\right]_{,x} - \tfrac{1}{2}b(d_{t3} + c)\tau_{2,x} + [b(w^{t2} - w^{b2})/c]E_c + m^{t2}_{,x} - q^{t2} = 0 \tag{30}$$

Analogous equations can be written for the lower/bottom skin in zone 2.

Core in zone 2:

$$\left(-c^3/12E_c\right)\tau_{2,xx} + (c\tau_2/G_c) - \tfrac{1}{2}\left(c + d_{t_3}\right)w^{t3}_{,x} - \tfrac{1}{2}(c + d_b)w^{b2}_{,x} + u^{t2}_0 - u^{b2}_0 = 0 \tag{31}$$

In a similar fashion the equations for zone 3 can be derived.

The boundary conditions and continuity conditions have the same forms as Eqs. (19–21). If we adopt the commonly used assumption[3,5,6] that imperfection does not affect the membrane state, then the $N^{t_i}_{xx}$, $i = 1, 2, 3, 4$, and N^{bj}_{xx}, $j = 1, 2, 3$, can be replaced by membrane state resultant force $N^{t_i(0)}_{xx}$, $i = 1, 2, 3, 4$, and $N^{bj(0)}_{xx}$, $j = 1, 2, 3$, respectively, in the preceding equations, as well as the corresponding boundary conditions.

IV. Results and Discussion

A numerical scheme for solving the nonlinear equations of simply supported sandwich beams has been formulated. The geometry of this investigated structure is shown in Fig. 1, where the length of the sandwich beam $L = 300$ mm, $(X_2 - X_1)/L = 22/65$, $(X_3 - X_2)/L = 15/65$, $(X_4 - X_3)/L = 28/65$; the core height $c = 20$ mm, beam width is 50 mm; the face sheet thickness $d_{t_1} = d_{t_4} = d_b = 1.5$ mm, $d_{t_2} = 0.5$ mm, and $d_{t_3} = 1.0$ mm. The face sheets are made from graphite/epoxy with $E_f = E_{\mathrm{ft}} = E_{\mathrm{fb}} = 1.38 \times 10^{11}$ N/m^2, the honeycomb core with $E_c = 0.002E_f$, and $E_c/G_c = 2.66$. The unsymmetrical imperfection modes in the face sheets are assumed to be $\bar{w}^t(x) = W^t_{\mathrm{imp}} \sin(n\pi x/L)$ and $\bar{w}^b(x) = W^b_{\mathrm{imp}} \sin(-n\pi x/L)$, with $n = 1$, and $W^t_{\mathrm{imp}} = W^b_{\mathrm{imp}} = c/10$, where

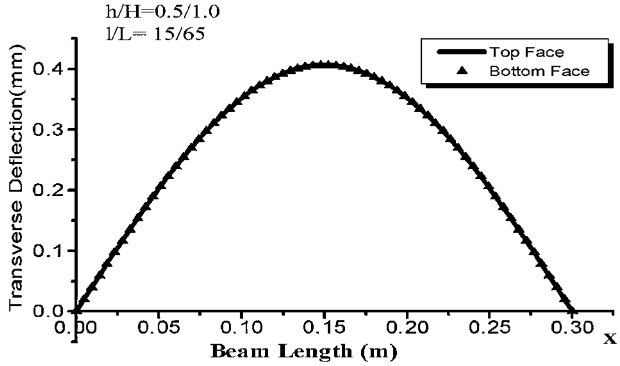

Fig. 3 Deflections for the top and bottom face sheets of simply supported sandwich beam without a delaminations for $P/P_{\mathrm{cr}} = 0.2$.

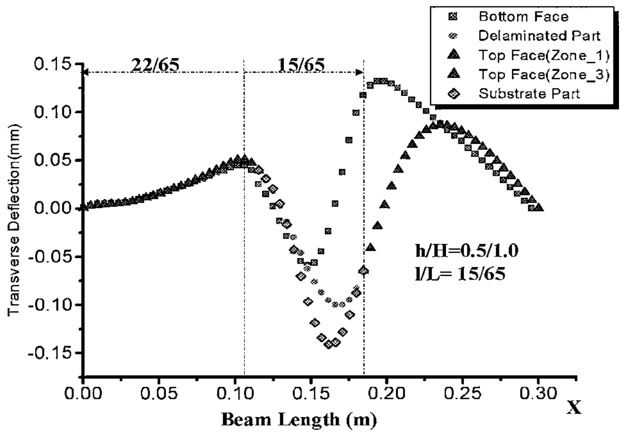

Fig. 4 Deflections for the top and bottom face sheets of simply supported sandwich beam with a delamination inside the top face sheet for $P/P_{\mathrm{cr}} = 0.2$.

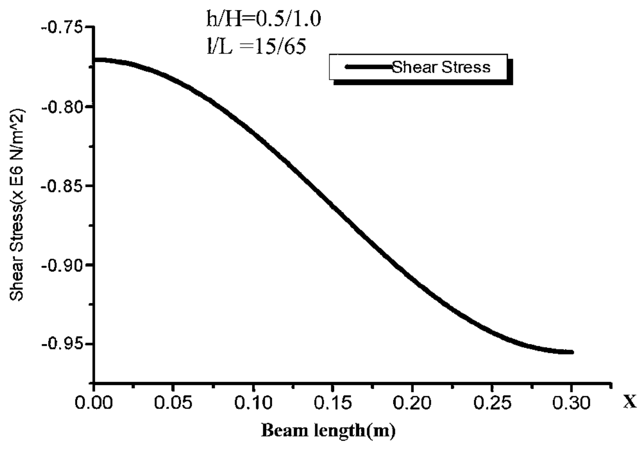

Fig. 5 Distribution of shear stress for simply supported sandwich beam without delaminations for $P/P_{\mathrm{cr}} = 0.2$.

c is the height of the core. The results shown in Figs. 3–8 and Figs. 9–14 are calculated under two loading levels, $P/P_{\mathrm{cr}} = 0.2$ and $P/P_{\mathrm{cr}} = 0.435$, respectively, where P_{cr} is the classical critical load of the sandwich structure without delaminations.

Figures 4 and 10 are the deflections of the skins. It can be seen that the buckled shapes of zones 1 and 3 form a global buckling mode and that of zone 2, which includes delamination, represents a local buckling mode. A coupling of the local and global buckling modes occurs in this structure. However, under these same loading levels, the structure without a delamination buckled only in the global mode (Figs. 3 and 9). It is also shown that the displacements in zone 2, which includes the delamination, are separated that is, the delaminated part of the skin in zone 2 does not overlap with the substrate. This separation pattern justifies that the approaches employed in the present work may be an appropriate method in dealing

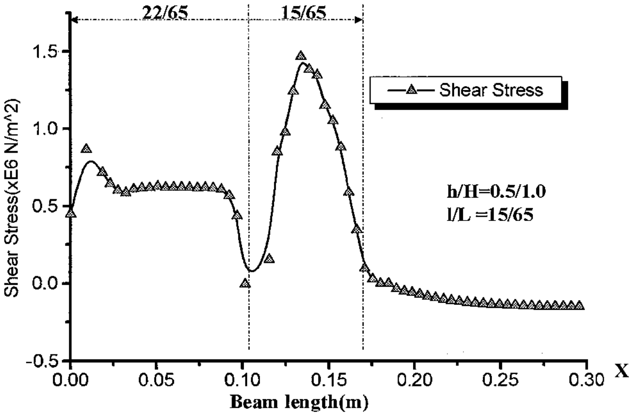

Fig. 6 Distribution of shear stress for a simply supported sandwich beam with a delamination inside the top face sheet for $P/P_{cr} = 0.2$.

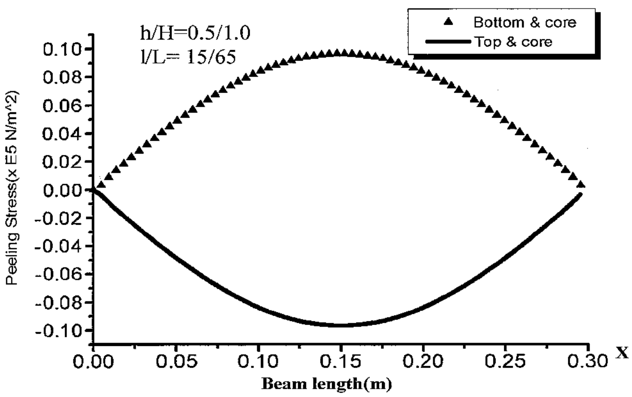

Fig. 7 Peeling stress distribution of simply supported sandwich beam without delaminations for $P/P_{cr} = 0.2$.

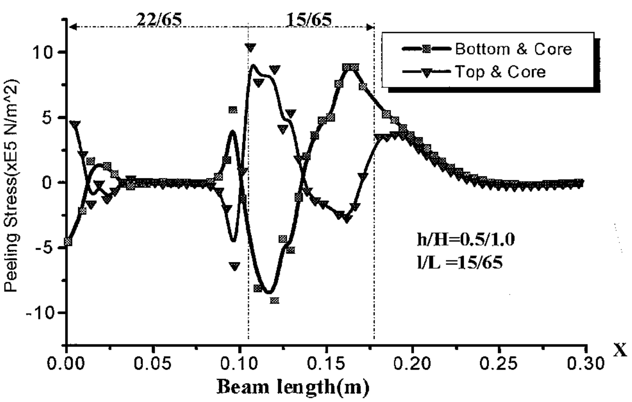

Fig. 8 Peeling stress distribution of a simply supported sandwich structure with a delamination inside the top face sheet for $P/P_{cr} = 0.2$.

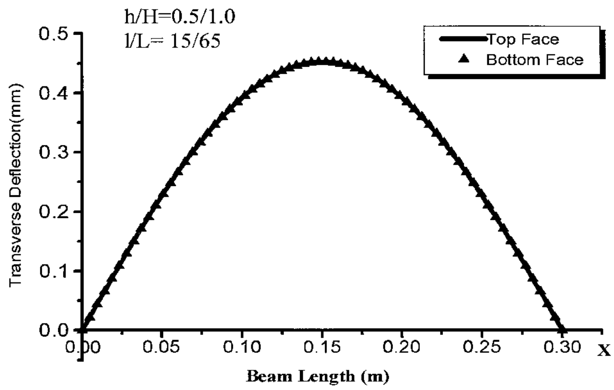

Fig. 9 Deflections for the top and bottom face sheets of simply supported sandwich beam without delaminations for $P/P_{cr} = 0.435$.

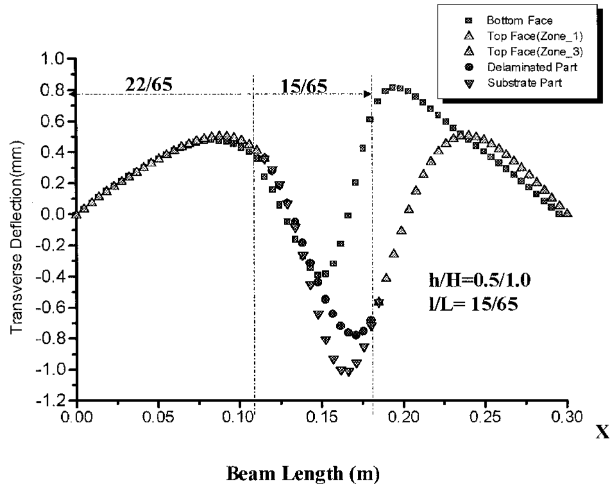

Fig. 10 Deflections for the top and bottom face sheets of simply supported sandwich beam with a delamination inside the top face sheet for $P/P_{cr} = 0.435$.

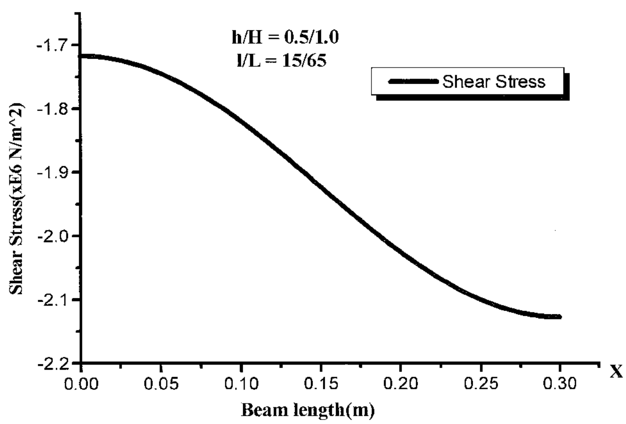

Fig. 11 Distribution of shear stress for simply supported sandwich beam without delaminations for $P/P_{cr} = 0.435$.

with delaminations in sandwich constructure. From a study of the literature,[5,11,12] it can be found that the behavior of the sandwich panels and composite panels are quite different even when no delaminations are involved. In the case of composites, the response of the structure due to in-plane compressive loading is limited to the rigidities of the composites only. However, for a sandwich panel with a soft core, there is an interaction between the top and bottom face sheets and the core. The structure resists bending through coupling (compression in one face sheet and tension in the other) and shear in the face sheets and the core. Therefore, buckling of delaminated face sheets may occur in the presence of bending moments as opposed to the case of delamination in monolithic composite panels where it occurs as a result of compressive loading only. In the current example of a compressed sandwich panel, the in-plane resultants depend not only on the compressive loading, but also simultaneously on the bending as the result of the buckling of the sandwich constructure.

The distribution of shear stresses between the interface of the face sheets and core are presented Figs. 6 and 12 and Figs. 5 and 11 for the panels with delamination and without delamination, respectively. These results show that when the load is increased by 2.175 times, the maximum shear increased by 6.533 times for the panel with delamination but by 2.263 times without delamination. There is a similar tendency in the peeling stresses between the face sheets and the core (Table 1). Figures 7 and 8 and Figs. 13 and 14 are peeling stress distributions for the cases of delamination and no delamination in the structure, respectively. Because of the introduction of delamination, there is a high stress concentration both on the peeling stresses and the shear stresses in zone 2 where the face sheet delaminated. Under the load level $P/P_{cr} = 0.435$, the

Table 1 Stress-loading sensitiveness ($\times 10^5$ N/m^2)

	Shear stress		Peeling stress	
P/P_{cr}	No delamination	With delamination	No delamination	With delamination
0.2	0.95	1.5	0.085/0.085[a]	6.95/10.05
0.435	2.15	9.8	0.225/0.225	37.5/62.5
Ratio	2.26	6.53	2.64/2.64	5.4/6.22

[a]Values for top and bottom face, respectively.

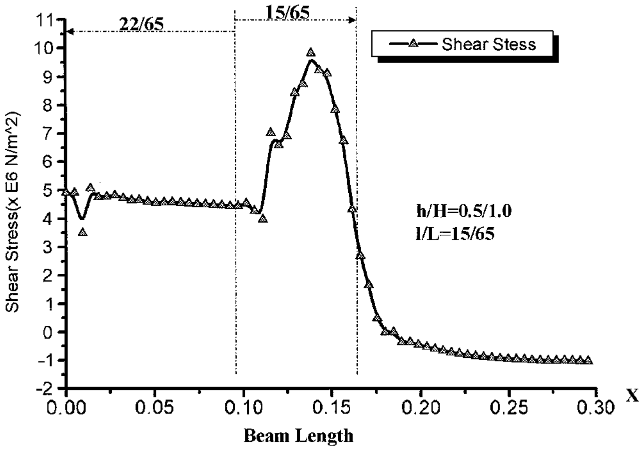

Fig. 12 Distribution of shear stress for simply supported sandwich beam with a delamination inside the top face sheet for $P/P_{cr} = 0.435$.

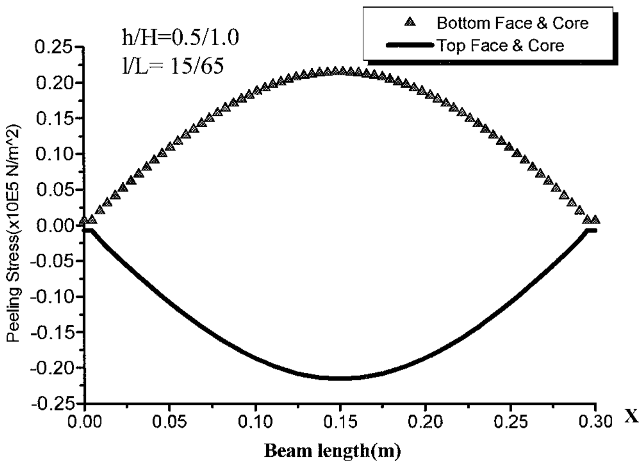

Fig. 13 Peeling stress distribution of simply supported sandwich beam without delaminations for $P/P_{cr} = 0.435$.

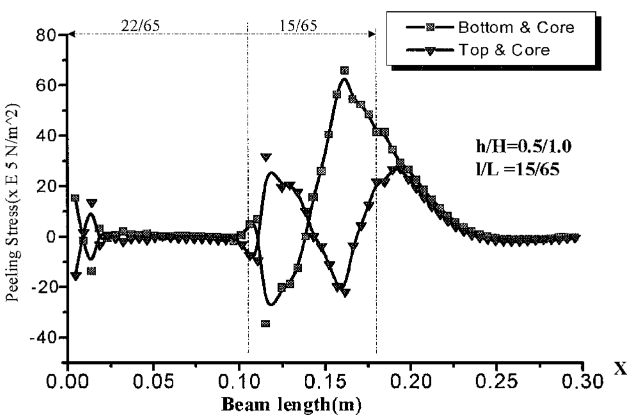

Fig. 14 Peeling stress distribution of simply supported sandwich structure with a delamination inside the top face sheet for $P/P_{cr} = 0.435$.

Table 2 Peeling stress changes ($\times 10^5$ N/m^2)

P/P_{cr}	Maximum absolute value	No delamination	With delamination	Ratio
0.2	Top face	0.085	6.95	81.8
0.2	Bottom face	0.085	10.05	118
0.435	Top face	0.225	37.5	167
0.435	Bottom face	0.225	62.5	278

maximum peeling stress of the sandwich panel with delamination is 278 times that of the structure with no delaminations (Table 2). The dramatic increases on both shear and peeling stresses between the face sheets and the core may make the structure much easier to be debonded. Once the debonding occurs, the sandwich panel, which was designed principally as an efficient integral structure, behaves like two separate thin composite sheets lying on a very soft foundation; thus, it can not maintain its exceptional sandwich character.

V. Conclusions

Because of the strong interaction between the face sheets and core, the theories and formulations for composite delamination buckling cannot be directly applied to the case of a delaminated sandwich structure. However, the approach proposed in the paper can be used to handle this interaction properly. Using our model, which is based on a higher-order core theory, we find that the local and global buckling modes can be more easily coupled due to the presence of the delaminations in the face sheets. This coupling of local–global buckling modes may contribute to the analytical complexity of this phenomenon. Furthermore, the shear stresses and the peeling stresses of the sandwich structure become more sensitive to the applied load levels, and high stress concentrations in the delaminated zone increase the possibility of complete debonding of the face sheets and the core. Therefore, delamination in a sandwich panel can be a kind of severe failure mode for sandwich construction.

Acknowledgments

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